Physics Letters B 756 (2016) 354–357

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Holographic complexity in gauge/string superconductors

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ARTICLE INFO

Article history:
Received 17 February 2016
Accepted 11 March 2016
Available online 16 March 2016
Editor: N. Lambert

ABSTRACT

Following a methodology similar to [1], we derive a holographic complexity for two dimensional holographic superconductors (gauge/string superconductors) with backreactions. Applying a perturbation method proposed by Kanno in Ref. [2], we study behaviors of the complexity for a dual quantum system near critical points. We show that when a system moves from the normal phase ($T > T_c$) to the superconductor phase ($T < T_c$), the holographic complexity will be divergent.

1. Introduction

The anti-de Sitter/conformal field theories (AdS/CFT) correspondence provides us a holographic dual description of the strong interacting systems in various fields of physics, especially in condensed matter physics. More precisely, this correspondence establishes a dual relationship between the d dimensional strongly interacting theories on the boundary and the $d+1$ dimensional weakly coupled gravity theories in the bulk [3]. One of the most widely investigated objects is the holographic superconductors. The simple holographic superconductor model dual to gravity theories was made by applying a scalar field and a Maxwell field coupled in an AdS black hole background [4–6]. Then a lot of works have been carried out for investigating holographic superconductors in other complicated gravity theories such as Einstein–Gauss–Bonnet gravity, Horava–Lifshitz gravity, non-linear electrodynamics gravity and so on [7–9].

According to AdS/CFT duality, instability of the bulk black hole leads to exist a conductor and superconductor phase transition in holographic superconductor models. A holographic superconductor/insulator phase transition model was also built at zero temperature [10]. Then, in Ref. [11] authors investigated a complete phase diagram for a holographic s-wave superconductor/conductor/insulator system by mixing the conductor/superconductor phase transition with the insulator/superconductor phase transition.

On the other hand, the entanglement entropy plays a main role in distinguishing different phases and corresponding phase transitions. It is also considered as a useful tool for keeping track of the degrees of freedom of strongly coupled systems.

In framework of AdS/CFT duality, a holographic method for evaluating the entanglement of quantum systems has been proposed by Ryu and Takayanagi [12]. Following this conjecture, the entanglement entropy of CFT's states living on the boundary of an AdS spacetime is associated with the area of a minimal surface defined in the bulk of that spacetime. Namely, the holographic entanglement entropy of subsystem A with its complement is given by:

$$S_A = \frac{Area(\gamma_A)}{4G_{d+1}} \tag{1}$$

where G and γ_A are the gravitational constant in the bulk and the $(d-1)$-minimal surface extended into the bulk with the same boundary ∂A of subsystem A, respectively. Recently, the behavior of entanglement entropy for holographic superconductor models has been studied in investigating conductor/superconductor phase transitions [13,14]. We also obtained an exact form of the holographic entropy due to an advantage of approximate solutions inside normal and superconducting phases with backreactions for 2D holographic superconductors [15].

Recently, Susskind has found a new quantity related to *complexity* in CFTs which is dual to a volume of a codimension one time slice in anti-de Sitter (AdS) [16,17]. The time slice connects two boundaries dual to the thermofield doubled CFTs, through the Einstein–Rosen bridge [18]. Following Refs. [16,17], the holographic complexity can be defined as:

$$\mathcal{C}_A = \frac{V(\gamma)}{8\pi R G_{d+1}}, \tag{2}$$

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http://dx.doi.org/10.1016/j.physletb.2016.03.031

where R and $V(\gamma)$ are the radius of the curvature and the volume of the part in the bulk geometry enclosed by the minimal hypersurface appearing in the computation of entanglement entropy, respectively. This quantity can be carried out for certain holographic models [1]. Today it is used to get more information about the quantum systems in black hole physics and cosmology [19,20]. In the framework of AdS/CFT correspondence, holographic complexity in bulk might provide a dual description of the fidelity defined in quantum information [1,21]. Because the fidelity is purely a quantum information concept, it would be a great advantage if one used it to characterize the quantum phase transitions [22,23]. Moreover, if one extends the fidelity to thermal states, the leading term (fidelity susceptibility) of the fidelity between two neighboring thermal states is simply the specific heat [24]. Roughly speaking, the fidelity approach is a powerful tool to talk about quantum (or thermal) phase transitions [22,23]. In this paper, we will investigate behaviors of phase transitions for 2D holographic superconductors by calculating the holographic complexity. The case of 2D holographic superconductivity of our interest is based on the AdS_3/CFT_2 correspondence. We also consider the influence of the backreaction on the dynamics of perturbation in the background spacetime. Then by applying the domain wall approximation analysis [25], we will get the holographic complexity which would be divergent at critical points.

The paper is organized as follows. In the next section, we will have a brief glance at 2D holographic superconductors. In Section 3, we will calculate the holographic complexity to analyze the phase transition in such superconductors. Finally, Section 4 is devoted to conclusions.

2. 2D holographic superconductor with backreactions

In this section, we begin with a brief review of 2D holographic superconductor away from the probe limit by considering the backreaction. The dual gravity description of these superconductors is defined by the following action [26]:

$$S=\int d^3x\sqrt{-g}\Big[\frac{1}{2\kappa^2}(R+\frac{2}{l^2})-\frac{1}{4}F^{ab}F_{ab} \\ -|(\nabla-ieA)\psi|^2-m^2|\psi|^2\Big], \tag{3}$$

in which $\kappa^2=8\pi G_3$, $F_{\mu\nu}=\nabla_\mu A_\nu-\nabla_\nu A_\mu$ where A_μ are Maxwell's fields, and e and m represent the charge and the mass of the scalar field ψ. In order to regard the effect of the backreaction of the holographic superconductor, we consider a metric ansatz as follows:

$$ds^2=-f(r)e^{-\beta(r)}dt^2+\frac{dr^2}{f(r)}+\frac{r^2}{l^2}dx^2\,, \tag{4}$$

and the electromagnetic field and the scalar field can be chosen as:

$$A_\mu dx^\mu=\phi(r)dt,\quad \psi\equiv\psi(r). \tag{5}$$

Furthermore we consider ψ as a real function without loss of generality. The Hawking temperature of this black hole, which is equivalent to the temperature of the CFT, is given by:

$$T=\left.\frac{f'(r)e^{-\beta(r)/2}}{4\pi}\right|_{r=r_+}. \tag{6}$$

Employing the ansatz (4) and (5), the equations of motion can be easily obtained by the following relations [27]:

$$\psi''(r)+\psi'(r)\left[\frac{1}{r}+\frac{f'(r)}{f(r)}-\frac{\beta'(r)}{2}\right] \\ +\psi(r)\left[\frac{e^2\phi(r)^2e^{\beta(r)}}{f(r)^2}-\frac{m^2}{f(r)}\right]=0\,, \\ \phi''(r)+\phi'(r)\left[\frac{1}{r}+\frac{\beta'(r)}{2}\right]-\frac{2e^2\phi(r)\psi(r)^2}{f(r)}=0\,, \\ f'(r)+2\kappa^2r\Big[\frac{e^2\phi(r)^2\psi(r)^2e^{\beta(r)}}{f(r)}+f(r)\psi'(r)^2 \\ +m^2\psi(r)^2+\frac{1}{2}e^{\beta(r)}\phi'(r)^2\Big]-\frac{2r}{l^2}=0, \\ \beta'(r)+4\kappa^2r\left[\frac{e^2\phi(r)^2\psi(r)^2e^{\beta(r)}}{f(r)^2}+\psi'(r)^2\right]=0. \tag{7}$$

Here the prime denotes the derivative with respect to r. In Ref. [27], to find the effect of the backreaction on the scalar condensation, Yunqi Liu et al. did numerical calculations. They showed that numerical results in solving equation (7) for various values of the backreaction κ^2 lead to drop the critical temperatures ($T_c\sim\rho$, where ρ is the dual chemical potential in CFT) consistently when the backreaction grows. It means that the backreaction makes the condensation harder to occur. They also confirmed that the gap of the condensation operator $<\mathcal{O}_+>$ becomes bigger if the backreaction increases. Moreover near the phase transition the condensation operator shows itself a behavior like $<\mathcal{O}_+>\sim\sqrt{1-\frac{T}{T_c}}$ which is the same as expected results from mean field theory. The exponent $\frac{1}{2}$ implies that the second order phase transition occurs. Furthermore, at the zero temperature limit $T=0$, the condensate $<\mathcal{O}_+>$ tends to infinity that it justifies the same results from the BCS theory. (See Ref. [28] for an analytic description of phase transitions.)

3. Holographic complexity and phase transitions

Now, we study the holographic complexity for 2D holographic superconductors. By defining $z=r_+/r$ in the Poincaré's coordinate, and replacing it into the metric (Eq. (4)), the volume function yields:

$$V(\gamma)=\frac{r_+^2}{l}\int\frac{x(z)dz}{z^3\sqrt{f(z)}}. \tag{8}$$

In order to find $x(z)$ in above formula, let us consider an entangling region (subsystem A) in the shape of a strip [12,29]. The minimal surface γ_A is a one dimensional hypersurface (geodesic) at $t=0$ when Eq. (1) is employed. It should be noted that none of the coordinates $(z;x)$ is independent of the other. Therefore, considering z as a function of x, the surface area becomes:

$$A(z(x))=r_+\int\frac{dx}{z^2}\sqrt{\frac{z'^2}{f(z)}+\frac{z^2}{l^2}}. \tag{9}$$

It is noteworthy that we have not imposed the minimality condition on the surface area yet. We now use the Hamiltonian approach to minimize the surface area. Therefore, we get to the first order differential equation as follows,

$$\frac{z'^2}{f(z)}+\frac{z^2}{l^2}=(Cl^2)^2. \tag{10}$$

Defining a turning point z_* such that $z'|_{z=z_*}=0$, one can obtain the following minimal path for $x(z)$ as: