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Extremal RN/CFT in both hands revisited

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ABSTRACT

We study RN/CFT correspondence for four dimensional extremal Reissner–Nordstrom black hole. We uplift the 4d RN black hole to a 5d rotating black hole and make a geometric regularization of the 5d space–time. Both hands central charges are obtained correctly at the same time by Brown–Henneaux technique.

1. Introduction

To fully understand Bekenstein–Hawking entropy of a black hole in a microscopic point of view is still a challenge. An important progress has been made in [1] by using the Brown–Henneaux technique [2], i.e. the Kerr/CFT correspondence. The central charge of the dual CFT reproduces the exact Bekenstein–Hawking entropy of the 4-dimensional extremal Kerr black hole by Cardy's formula. This method has been extensively studied for Kerr black hole and other more general rotating black holes [3–16]. However, in the original Kerr/CFT method, the Virasoro algebra was realized only from an enhancement of the rotational $U(1)$ isometry, which corresponds to left hand central charge, not from the $SL(2,R)$. Later, it was found that the right hand central charge of rotating black holes can be obtained by delicately choosing appropriate boundary conditions, [17–28].

Nevertheless, the method in Kerr/CFT correspondence cannot be directly applied to non-rotating charge black holes, such as Reissner–Nordstrom (RN) black hole. One found that the central charge for 4d RN black hole vanishes by directly using the Brown–Henneaux technique. One possible way to solve the problem is to uplift the space–time to higher dimensions. The left hand central charge $c_L = 6Q^2$ of 4d extremal RN black hole was obtained by uplifting it to 5 dimensions [4,9,24,29–31]. The other possible way is to consider a 2-dimensional effective theory of 4d extremal RN black hole via a dimensional reduction following the idea in [19], by which the right hand central charge $c_R = 6Q^2$ of 4d extremal RN was obtained in [31,32]. Although both hands central charges can be obtained, but one has to use difference method. It is an interesting question whether we are able to calculate both hands central charges for extremal RN black hole at the same time by choosing appropriate boundary conditions as in the case of Kerr/CFT. The answer is yes, but with a geometric regularization.

In this paper, we calculate both hands central charges for 4d extremal RN black hole by using the Brown–Henneaux technique. We first uplift the 4d RN black hole to a 5d black hole, following the idea in [27], we then deform the 5d black hole by a geometric regularization. Both hands central charges $c_R = c_L = 6Q^2$ are obtained in this way at the same time.

The paper is organized as follows. In section 2, we briefly review the near horizon geometry of 4d extremal RN black hole. We then uplift 4d RN black hole to 5d and calculate both hands asymptotic Killing vectors in section 3. In section 4, we deform the 5d black hole by the geometric regularization to obtain both hands central charges. Our conclusion and discussion are included in section 5.

2. Near horizon geometry of 4d extremal RN black hole

In four dimensions, the Einstein–Maxwell theory

$$S_4 = \frac{1}{16\pi G_4}\int d^4x\sqrt{-g_4}\left(R_4 - F^2\right), \tag{1}$$

admits a unique spherical electro-vacuum solution, the RN black hole. The metric of 4d non-extremal RN black hole is

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2, \tag{2}$$

where $d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2$ and

$$f(r) = 1 - \frac{2m}{r} + \frac{q^2}{r^2}, \qquad A = -\frac{q}{r}dt. \tag{3}$$

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Two horizons are obtained by solving $f(r)=0$,

$$r_\pm = m \pm \sqrt{m^2-q^2}. \tag{4}$$

The Hawking temperature and black hole entropy can be calculated as follows,

$$T_H = \frac{\kappa}{2\pi} = \frac{r_+ - r_-}{4\pi r_+^2}, \qquad S_{BH} = \frac{A}{4G_4} = \frac{\pi r_+^2}{G_4}. \tag{5}$$

In the extremal limit of RN black hole, i.e. $r_+ = r_- = m = q$, we have

$$f(r) = \left(1-\frac{q}{r}\right)^2, \tag{6}$$

and

$$T_H = 0, \qquad S_{BH} = \frac{\pi q^2}{G_4} = \pi Q^2, \tag{7}$$

where we defined $Q \equiv q/\sqrt{G_4}$.

To consider the near horizon limit, $r \to r_+$, we make the following coordinates transformation,

$$\rho = \frac{r-r_+}{\epsilon q}, \qquad \tau = \frac{t}{q}\epsilon, \tag{8}$$

and take the limit $\epsilon \to 0$. Finally, the metric of 4d near horizon extremal RN black hole is obtained as

$$ds^2 = q^2\left(-\rho^2 d\tau^2 + \frac{d\rho^2}{\rho^2} + d\Omega_2^2\right), \tag{9}$$

$$A = q\rho d\tau. \tag{10}$$

3. Central charges by uplifting to five dimensions

Direct calculation following the method of Kerr/CFT in [1] shows that the central charges of the 4d extremal RN black hole always vanish. This is because the $U(1)$ symmetry of rotation in Kerr black hole does not exist in the RN black hole, in which the $U(1)$ symmetry behaves as gauge symmetry. To get non-zero central charge, a possible way is to uplift the 4d RN black hole into 5-dimensional space–time with the extra dimension compactified on a circle as in [4,31]. The metric of the uplifted 5-dimensional black hole can be expressed as,

$$\begin{aligned} ds^2 &= q^2\left(-\rho^2 d\tau^2 + \frac{d\rho^2}{\rho^2} + d\Omega_2^2\right) + (dy+A)^2 \\ &= \frac{q^2 d\rho^2}{\rho^2} + q^2 d\Omega_2^2 + dy^2 + 2q\rho d\tau dy, \end{aligned} \tag{11}$$

$$A_{(2)} = \sqrt{3} q\rho d\tau \wedge dy, \tag{12}$$

where $A_{(2)}$ is a gauge 2-form and the new coordinate y is compactified on a circle with a proper period [30] $y = y + 2\pi q$. The metric (11) with the 2-form (12) is a solution of 5d Einstein–Maxwell theory,

$$S_5 = \frac{1}{16\pi G_5}\int d^4x\sqrt{-g_5}\left(R_5 - \frac{1}{12}F_{(3)}^2\right), \tag{13}$$

where $G_5 = 2\pi q G_4$ and $F_{(3)} = dA_{(2)}$. We should notice that the 2-form field $A_{(2)}$ has no contribution to the central charge based on the argument in [6,29].

To get the expected leading behavior of asymptotic Killing vector (AKV) with both hands, we carefully choose the following boundary condition,

$$\begin{pmatrix} h_{\tau\tau}=\mathcal{O}(1) & h_{\tau\rho}=\mathcal{O}(\rho^{-2}) & h_{\tau\theta}=\mathcal{O}(\rho^{-1}) & h_{\tau\phi}=\mathcal{O}(\rho^{-1}) & h_{\tau y}=\mathcal{O}(1) \\ & h_{\rho\rho}=\mathcal{O}(\rho^{-3}) & h_{\rho\theta}=\mathcal{O}(\rho^{-2}) & h_{\rho\phi}=\mathcal{O}(\rho^{-2}) & h_{\rho y}=\mathcal{O}(\rho^{-1}) \\ & & h_{\theta\theta}=\mathcal{O}(\rho^{-1}) & h_{\theta\phi}=\mathcal{O}(\rho^{-1}) & h_{\theta y}=\mathcal{O}(\rho^{-1}) \\ & & & h_{\phi\phi}=\mathcal{O}(\rho^{-1}) & h_{\phi y}=\mathcal{O}(\rho^{-1}) \\ & & & & h_{yy}=\mathcal{O}(1) \end{pmatrix}. \tag{14}$$

The AKV is obtained by solving the asymptotic Killing equation $L_\zeta(g_{\mu\nu}+h_{\mu\nu}) \sim h_{\mu\nu}$ with the corresponding boundary condition. We note that a weaker boundary condition will cause the AKV being ill-defined and further restriction will rule out the interesting excitations. The AKV associated to the above boundary condition (14) is

$$\begin{aligned} \zeta &= \epsilon(\tau)\partial_\tau + \left[-\rho\partial_\tau\epsilon(\tau) - \rho\partial_y\eta(y)\right]\partial_\rho \\ &\quad + \left[\eta(y) - q\partial_\tau^2\epsilon(\tau)/\rho\right]\partial_y, \end{aligned} \tag{15}$$

where $\epsilon(\tau)$ and $\eta(y)$ are arbitrary functions of τ and y, respectively. The left hand and right hand AKVs read

$$\zeta^L = -\rho\partial_y\eta(y)\partial_\rho + \eta(y)\partial_y, \tag{16a}$$

$$\zeta^R = \epsilon(\tau)\partial_\tau - \rho\partial_\tau\epsilon(\tau)\partial_\rho - \frac{q}{\rho}\partial_\tau^2\epsilon(\tau)\partial_y. \tag{16b}$$

Since $y = y + 2\pi q$ is periodic, we can express the function $\eta(y)$ as its Fourier bases $e^{-iny/q}$. Similarly, if we assign a period for τ with $\tau = \tau + 2\pi\beta$ with β an arbitrary real number, the function $\epsilon(\tau)$ can be expressed by $e^{-in\tau/\beta}$. With appropriate normalizations, we can write

$$\eta_n(y) = -qe^{-in\frac{y}{q}}, \qquad \epsilon_n(\tau) = -\beta e^{-in\frac{\tau}{\beta}}. \tag{17}$$

The corresponding AKVs

$$\zeta_n^L = -in\rho e^{-in\frac{y}{q}}\partial_\rho - qe^{-in\frac{y}{q}}\partial_y, \tag{18a}$$

$$\zeta_n^R = -\beta e^{-in\frac{\tau}{\beta}}\partial_\tau - in\rho e^{-in\frac{\tau}{\beta}}\partial_\rho - \frac{n^2 q}{\rho\beta}e^{-in\frac{\tau}{\beta}}\partial_y, \tag{18b}$$

compose two copies of Virasoro algebra without central extension as expected,

$$i\left[\zeta_m^L, \zeta_n^L\right] = (m-n)\zeta_{m+n}^L, \tag{19a}$$

$$i\left[\zeta_m^R, \zeta_n^R\right] = (m-n)\zeta_{m+n}^R. \tag{19b}$$

To obtain the central charges, we define the asymptotic charge Q_ζ associated to the AKV ζ as,

$$Q_\zeta = \frac{1}{8\pi G_5}\int_{\partial\Sigma} k_\zeta[h,g], \tag{20}$$

where the 2-form k_ζ is defined for a perturbation $h_{\mu\nu}$ around the background metric $g_{\mu\nu}$ [3],

$$\begin{aligned} k_\zeta[h,g] = \frac{1}{2}\Big[& \zeta_\nu\nabla_\mu h - \zeta_\nu\nabla_\sigma h_\mu^\sigma + \zeta_\sigma\nabla_\nu h_\mu^\sigma \\ & + \frac{1}{2}h\nabla_\nu\zeta_\mu - h_\nu^\sigma\nabla_\sigma\zeta_\mu \\ & + \frac{1}{2}h_{\nu\sigma}\left(\nabla_\mu\zeta^\sigma + \nabla^\sigma\zeta_\mu\right)\Big] * \left(dx^\mu \wedge dx^\nu\right), \end{aligned} \tag{21}$$

where $*$ denotes the Hodge dual, and Σ is the 4-dimensional equal-time hypersurface at $\tau = const$.

The Dirac bracket of the asymptotic charge is given by

$$\left\{Q_{\zeta_m}, Q_{\zeta_n}\right\}_{DB} = Q_{[\zeta_m,\zeta_n]} + \frac{1}{8\pi G_5}\int_{\partial\Sigma} k_{\zeta_m}\left[L_{\zeta_n}\bar{g}, \bar{g}\right]. \tag{22}$$