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Bounds on topological Abelian string-vortex and string-cigar from information-entropic measure

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ABSTRACT

In this work we obtain bounds on the topological Abelian string-vortex and on the string-cigar, by using a new measure of configurational complexity, known as configurational entropy. In this way, the information-theoretical measure of six-dimensional braneworlds scenarios is capable to probe situations where the parameters responsible for the brane thickness are arbitrary. The so-called configurational entropy (CE) selects the best value of the parameter in the model. This is accomplished by minimizing the CE, namely, by selecting the most appropriate parameters in the model that correspond to the most organized system, based upon the Shannon information theory. This information-theoretical measure of complexity provides a complementary perspective to situations where strictly energy-based arguments are inconclusive. We show that the higher the energy the higher the CE, what shows an important correlation between the energy of the a localized field configuration and its associated entropic measure.

1. Introduction

In 1948, in a seminal work, Shannon [1] introduced the information theory, whose main goal was to introduce the concepts of entropy and mutual information, using the communication theory. Therein, the entropy was defined to be a measure of "randomness" of a random phenomenon. Thus, if a little deal of information concerning a random variable is received, the uncertainty decreases, which makes it possible to measure the decrement in the uncertainty, related to the quantity of transmitted information. Inspired by Shannon, Gleiser and Stamatopoulos (GS) latterly introduced a measure of complexity of a localized mathematical function [2]. GS proposed that the Fourier modes of square-integrable, bounded, mathematical functions can be used to construct a measure, the so-called configurational entropy (CE). A single mode system has zero CE, whereas that one where all modes contribute with equal weight has maximal CE. In order to apply such ideas to physical models, GS used the energy density of a given spatially-localized field configuration, as a solution of the related partial differential equation (PDE). Hence the CE can be used to choose the best fitting trial function with energy degeneracy.

The CE has been already employed to acquire the stability bound for compact objects [3], to investigate the non-equilibrium dynamics of spontaneous symmetry breaking [4], to study the emergence of localized objects during inflationary preheating [5] and to discern configurations with degenerate-energy spatial profiles [6]. Moreover, solitons were studied in a Lorentz symmetry violating (LV) framework with the aid of CE [7–10]. In this context, the CE associated to travelling solitons in LV frameworks plays a prominent role in probing systems wherein the parameters are somehow arbitrary. Furthermore, the CE identifies critical points in continuous phase transitions [11]. Moreover, the CE can be used to measure the informational organization in the structure of the system configuration for five-dimensional (5D) thick scenarios. In particular, the CE plays an important role to decide the most appropriate intrinsic parameters of sine-Gordon braneworld models [12], being further studied both in $f(R)$ [13] and $f(R,T)$ [14] theories of gravity. In what follows, we present a brief discussion of 5D braneworld models to treat the CE in six-dimensional (6D) scenarios.

Randall–Sundrum (RS) models [15,16] proposed a warped braneworld scenario, wherein the gauge hierarchy problem is

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explained and the gravity zero mode is localized, reproducing four-dimensional (4D) gravity on the brane. The 5D bulk gravitons provide a small correction in the Newton law [16]. However, this thin model presents singularities and drawbacks concerning the non-localization of spin gauge and fermion fields [17]. To solve these problems, some thick models were proposed [18].

Soon after the works of RS, an axially symmetric warped 6D model was proposed by Gergheta–Shaposhnikov [19], called *string-like defect* (SD). This scenario further provided the resolution of the mass hierarchy and a smaller correction to the Newtonian potential [19], besides the non-requirement of fine tuning between the bulk cosmological constant and the brane tension, for the cancellation of the 4D cosmological constant [19]. Besides, the localization of gauge zero modes is spontaneous even in the thin brane case [20,21]. Fermions fields are trapped through a minimal coupling with an $U(1)$ gauge background field [22,23]. Later, other 6D, spherically symmetric, models were employed to explain the generations of fundamental fermions [24,25] and the resolution of the mass hierarchy of neutrinos as well [26]. Nevertheless, the SD model is a thin model and it leads to some irregularities [27]. Due to it, some 6D thick models were proposed to solve these remaining issues [28–42]. In Ref. [28], a topological abelian Higgs vortex was used to construct a regular scenario in which the dominant energy conditions hold, however solely numerical solutions have been found. Similarly, Refs. [31,32], looking for an exact vortex solution, show that the energy density and the angular pressure are similar. This condition is likewise verified for the Resolved Conifold scenario [37–39]. Finally, for the String-Cigar [33–36], the transverse space is represented by a cigar soliton, which is a stationary solution for the Ricci flow [43–45]. The dominant energy conditions are also satisfied in this model.

Therefore, in this paper we investigate the entropic measure both in the Torrealba topological Abelian string (TA) [31,32] and String-Cigar (HC) [33–36] in 6D scenarios due to its analytic properties. The main aims of our work are to find bounds for 6D string defects based upon the CE concept and to establish a value for the thickness of the configuration responsible for extremizing the CE.

This paper is organized as follows: in Sect. 2 a briefly review of string-like defects is present, whereas in Sect. 3 the CE bounds the parameters of TA and HC scenarios. We expose the conclusions and perspectives accordingly in Sect. 4.

2. String-like defect in warped six dimensions

A metric *ansatz* for 6D string-like models reads [19,20]

$$ds_6^2 = \sigma(r)\eta_{\mu\nu}dx^\mu dx^\nu - dr^2 - \gamma(r)d\theta^2 \tag{1}$$

where $\eta_{\mu\nu} = diag(+1, -1, -1, -1)$. The radial coordinate is limited to $r \in [0, \infty)$, whereas the angular coordinate is restricted to $\theta \in [0, 2\pi)$. The $\sigma(r)$ represents the dimensionless warp factor and $\gamma(r)$ has length squared dimension.

The 4D Planck mass (M_P) and the bulk Planck mass (M_6) are related through the volume of the transverse of space as [19,33,35, 36]:

$$M_P^2 = 2\pi M_6^4 \int_0^\infty \sigma(r)\sqrt{\gamma(r)}dr\,. \tag{2}$$

In addition, the energy-momentum tensor $T_M^N = diag(t_0, t_0, t_0, t_0, t_r, t_\theta)$ components are given by [19,33]

$$t_0(r) = -\frac{1}{\kappa}\left(\frac{3\sigma''}{2\sigma} + \frac{3\sigma'\gamma'}{4\sigma\gamma} + \frac{\gamma''}{2\gamma} - \frac{\gamma'^2}{4\gamma^2}\right) - \Lambda, \tag{3a}$$

$$t_r(r) = -\frac{1}{\kappa}\left(\frac{3\sigma'^2}{2\sigma^2} + \frac{\sigma'\gamma'}{\sigma\gamma}\right) - \Lambda, \tag{3b}$$

$$t_\theta(r) = -\frac{1}{\kappa}\left(\frac{2\sigma''}{\sigma} + \frac{\sigma'^2}{2\sigma^2}\right) - \Lambda, \tag{3c}$$

where the $\kappa = \frac{8\pi}{M_6^4}$ is the 6D gravitational constant, Λ is the 6D (negative) cosmological constant and the prime denotes the derivative with respect to the radial coordinate r.

To obtain a regular geometry, the conditions [19,28,33,42]

$$\sigma(r)\Big|_{r=0} = const., \qquad \sigma'(r)\Big|_{r=0} = 0,$$

$$\gamma(r)\Big|_{r=0} = 0, \qquad \left(\sqrt{\gamma(r)}\right)'\Big|_{r=0} = 1, \tag{4}$$

must hold.

For the vacuum solution, the warp factor for the Gergheta–Shaposhnikov *String Like Defect* (SD) model is proposed as [19–23]:

$$\sigma_{SD}(r) = e^{-cr}, \quad \gamma_{SD}(r) = R_0^2\sigma_{SD}(r) \tag{5}$$

where the parameters c is a constant, which connects the 6D Newtonian constant and the 6D cosmological constant, and R_0 is the radius of compactification of transverse space. See that, in the limit where $r \to 0$, only the first condition of Eq. (4) holds.

Following the perspective pointed by Ref. [19], Giovannini in adopted a 6D action [28], wherein the matter Lagrangian is an Abelian–Higgs model and the transverse space obeys the Abrikosov–Nielsen–Olesen *ansatz* [28,31,32]:

$$\phi(r,\theta) = v f(r)\, e^{-il\theta} \qquad l \in \mathbb{Z}\,,$$

$$\mathcal{A}_\theta(r) = \frac{1}{q}\left[l - P(r)\right], \quad \mathcal{A}_\mu = \mathcal{A}_r = 0\,,$$

where ϕ and $\mathcal{A}_M$ are scalar and gauge fields, respectively. The condition $v = 1$ is a length dimension L^{-2} constant. The functions $f(r)$ and $P(r)$ are such that $f(r \to 0) = 0$, $f(r \to \infty) = 1$, whereas $P(r \to 0) = l$ and $P(r \to \infty) = 0$.

From constraints by this *ansatz* and the regular conditions in the Eq. (4), the solutions of fields and warp factors are numerically obtained in Ref. [28]. On the other hand, by imposing conditions on the function $P(r) \equiv 0$, Torrealba [31,32] obtained an analytical solution, named *Topological Abelian Higgs string* (TA):

$$\sigma_{TA}(r) = \cosh^{-2\delta}\left(\frac{\beta r}{\delta}\right), \quad \gamma_{TA}(r) = R_0^2\sigma_{TA}(r)\,, \tag{6}$$

where the parameter β is similar to the parameter c in the SD model, and δ is a thickness parameter which, for small values, reproduces the thin Gergheta–Shaposhnikov model in Eq. (5). Moreover, Ref. [31] concludes that, for the localization of gauge fields zero mode, the thickness of the model cannot exceed the value

$$\delta < \frac{5\beta}{4\pi}q^2v^2\,. \tag{7}$$

Now, in the TA (6) string, two of the conditions (4) are verified.

In another approach, the transverse space can also be built for a cigar soliton solution of Ricci flow [33–36]

$$\frac{\partial}{\partial\lambda}g_{MN}(\lambda) = -2R_{MN}(\lambda)\,,$$

with λ being a metric parameter Refs. [33–36] constructed the geometry named *Hamilton String Cigar* (HC), where the warp factors read

$$\sigma_{HC}(r) = e^{-cr+\tanh(cr)}, \quad \gamma_{HC}(r) = \frac{\tanh^2 cr}{c^2}\sigma_{HC}(r). \tag{8}$$

In this case, all conditions of Eq. (4) do hold.