Physics Letters B 761 (2016) 203–206

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Gravitationally coupled electroweak monopole

Y.M. Cho [a,b], Kyoungtae Kimm [c], J.H. Yoon [d]

[a] Administration Building 310-4, Konkuk University, Seoul 143-701, Republic of Korea
[b] School of Physics and Astronomy, Seoul National University, Seoul 151-742, Republic of Korea
[c] Faculty of Liberal Education, Seoul National University, Seoul 151-747, Republic of Korea
[d] Department of Physics, College of Natural Sciences, Konkuk University, Seoul 143-701, Republic of Korea

ARTICLE INFO

Article history:
Received 3 June 2016
Received in revised form 15 August 2016
Accepted 16 August 2016
Available online 18 August 2016
Editor: A. Ringwald

Keywords:
Gravitationally coupled electroweak monopole
Monopole star
Magnetically charged black hole
Cho–Maison monopole
Mass of electroweak monopole
Finite energy electroweak monopole

ABSTRACT

We present a family of gravitationally coupled electroweak monopole solutions in Einstein–Weinberg–Salam theory. Our result confirms the existence of globally regular gravitating electroweak monopole which changes to the magnetically charged black hole as the Higgs vacuum value approaches to the Planck scale. Moreover, our solutions could provide a more accurate description of the monopole stars and magnetically charged black holes.

Ever since Dirac has proposed the Dirac monopole generalizing the Maxwell's theory, the monopole has become an obsession theoretically and experimentally [1]. After Dirac we have had the Wu–Yang monopole [2], the 't Hooft–Polyakov monopole [3], the grand unification (Dokos–Tomaras) monopole [4], and the electroweak (Cho–Maison) monopole [5–7]. But none of them except the electroweak monopole might become realistic enough to be discovered.

Indeed the Dirac monopole in electrodynamics should transform to the electroweak monopole after the unification of the electromagnetic and weak interactions, and Wu–Yang monopole in QCD is supposed to make the monopole condensation to confine the color. Moreover, the 't Hooft–Polyakov monopole exists in an unphysical theory, and the grand unification monopole which could have existed at the grand unification scale probably has become completely irrelevant at present universe after the inflation.

This makes the electroweak monopole the only realistic monopole we could ever hope to detect, which has made the experimental confirmation of the electroweak monopole one of the most urgent issues in the standard model after the discovery of the Higgs particle at LHC. In fact the newest MoEDAL ("the magnificient seventh") detector at LHC is actively searching for the monopole [8]. But to detect the electroweak monopole at LHC, we have to ask the following questions.

First, does the electroweak monopole really exist? This, of course, is the fundamental question. As we know, the Dirac monopole in electrodynamics does not have to exist, because there is no reason why the electromagnetic U(1) gauge group has to be non-trivial. So we must know if the standard model predicts the monopole or not.

Fortunately, unlike the Dirac monopole, the electroweak monopole must exist. This is because the electromagnetic U(1) in the standard model is obtained by the linear combination of the U(1) subgroup of SU(2) and the hypercharge U(1), but it is well known that the U(1) subgroup of SU(2) is non-trivial. In this case the mathematical consistency requires the electromagnetic U(1) non-trivial, so that the electroweak monopole must exist if the standard model is correct [9,10]. But this has to be confirmed by experiment. This makes the discovery of the monopole, not the Higgs particle, the final (and topological) test of the standard model.

Second, what (if any) is the characteristic feature of the electroweak monopole which is different from the Dirac monopole? This is an important question for us to tell if the monopole (when discovered) is the Dirac monopole or the electroweak monopole. The characteristic difference is the magnetic charge. The elec-

E-mail address: ymcho7@konkuk.ac.kr (Y.M. Cho).

http://dx.doi.org/10.1016/j.physletb.2016.08.033

troweak monopole has the magnetic charge twice bigger than that of the Dirac monopole. The magnetic charge of the Dirac monopole becomes a multiple of $2\pi/e$, because the period of the electromagnetic U(1) is 2π.

On the other hand, the magnetic charge of the electroweak monopole becomes a multiple of $4\pi/e$, because the period of electromagnetic U(1) in the standard model becomes 4π [5,9,10]. The reason is that this U(1) is (again) given by the linear combination of the U(1) subgroup of SU(2) and the hypercharge U(1), but the period of the U(1) subgroup of SU(2) is well known to be 4π.

Third, can we estimate the mass of the electroweak monopole? This is the most important question from the experimental point of view. There was no way to predict the mass of the Dirac monopole theoretically, which has made the search for the monopole a blind search in the dark room without any theoretical lead.

Remarkably the mass of the electroweak monopole can be predicted. Of course, the Cho–Maison monopole has a singularity at the origin which makes the energy divergent [5]. But there are ways to regularize the energy and estimate the mass, and all point consistently to 4 to 10 TeV [9–11]. This, however, is tantalizing because the upgraded 14 TeV LHC can produce the electroweak monopole pairs only when the monopole mass becomes below 7 TeV. So we need a more accurate estimate of the monopole to see if LHC can actually produce the monopole.

The purpose of this Letter is discuss how the gravitational interaction affects the electroweak monopole. We show that, when the gravity is turned on, the monopole becomes a globally regular gravitating electroweak monopole which looks very much like the non-gravitating monopole, but turns to the magnetic black holes as the Higgs vacuum value approaches to the Planck scale. This confirms that the change of the monopole mass due to the gravitational interaction is negligible, which assures that the present LHC could produce the electroweak monopole.

Before we discuss the modification of the monopole induced by the gravitation, we briefly review the non-gravitating electroweak monopole and explain how we can estimate the monopole mass. Consider the following effective Lagrangian of the standard model,

$$\mathcal{L}_{eff} = -|\mathcal{D}_\mu \phi|^2 - \frac{\lambda}{2}\Big(\phi^2 - \frac{\mu^2}{\lambda}\Big)^2 - \frac{1}{4}\vec{F}_{\mu\nu}^2 - \frac{\epsilon(\phi)}{4} G_{\mu\nu}^2,$$
$$\mathcal{D}_\mu \phi = \big(\partial_\mu - i\frac{g}{2}\vec{\tau}\cdot\vec{A}_\mu - i\frac{g'}{2}B_\mu\big)\phi, \qquad (1)$$

where $\epsilon(\phi)$ is a positive dimensionless function of the Higgs doublet which approaches to unit asymptotically. Obviously when $\epsilon = 1$, the Lagrangian reproduces the standard model. In general ϵ modifies the permeability of the hypercharge U(1) gauge field, but the effective Lagrangian still retains the SU(2) × U(1) gauge symmetry.

When $\epsilon = 1$, we can obtain the Cho–Maison monopole with the ansatz [5]

$$\phi = \frac{1}{\sqrt{2}}\rho\xi, \quad \rho = \rho(r), \quad \xi = i\begin{pmatrix} \sin\theta/2\, e^{-i\varphi} \\ -\cos\theta/2 \end{pmatrix},$$
$$\vec{A}_\mu = \frac{1}{g}(f(r)-1)\hat{r}\times\partial_\mu\hat{r},$$
$$B_\mu = -\frac{1}{g'}(1-\cos\theta)\partial_\mu\varphi. \qquad (2)$$

Notice that $\vec{A}_\mu$ has the structure of the 't Hooft–Polyakov monopole, but B_μ has the structure of the Dirac monopole. This tells that the Cho–Maison monopole is a hybrid between 't Hooft–Polyakov and Dirac.

The ansatz clearly shows that the U(1) point singularity in B_μ makes the energy of the Cho–Maison infinite, so that classically the monopole mass is undetermined. But, unlike the Dirac monopole, here we can estimate the mass of the electroweak monopole. A simplest way to do so is to realize that basically the monopole mass comes from the same mechanism which generates the weak boson mass, i.e., the Higgs mechanism, except that here the coupling becomes magnetic (i.e., $4\pi/e$) [9,10]. This must be clear from (2). So, from the dimensional reasoning the monopole mass M should be of the order of $M \simeq (4\pi/e^2)\times M_W$, or roughly about 10 TeV.

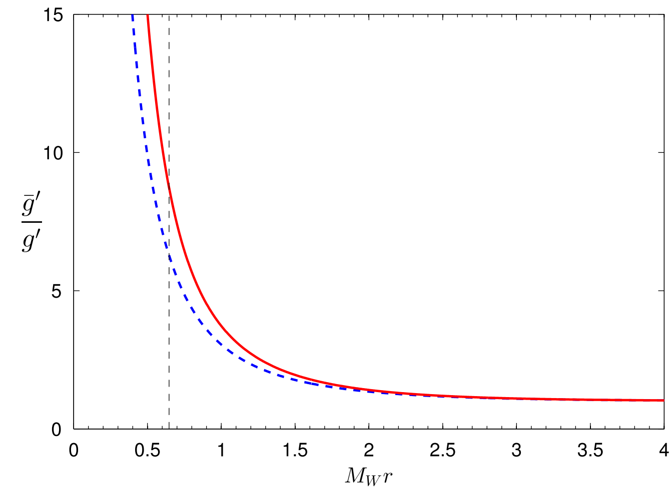

Fig. 1. The running coupling $\bar{g}'$ of the hypercharge U(1) induced by ϵ. The dotted (blue) curve is obtained with $\epsilon = (\rho/\rho_0)^8$, and the solid (red) curve is obtained with ϵ proposed by Ellis et al. The vertical line indicates the Higgs mass scale. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

A better way to estimate the mass is to notice that the Cho–Maison monopole energy consists of four parts, the SU(2) part, the hypercharge U(1) part, the Higgs kinetic part, and the Higgs potential part, but only the U(1) part is divergent [5,9]. Now, assuming that this divergent part can be regularized by the ultra-violet quantum correction, we can derive a constraint among the four parts which minimizes the monopole energy using the Derrick's theorem. From this we can deduce the monopole energy to be around 3.96 TeV [9,10].

Moreover, we can regularize the Cho–Maison monopole introducing non-vacuum permeability ϵ which can mimic the quantum correction. This is because, with the rescaling of B_μ to B_μ/g', g' changes to $g'/\sqrt{\epsilon}$, so that ϵ changes the U(1) gauge coupling g' to the "running" coupling $\bar{g}' = g'/\sqrt{\epsilon}$. So, by making $\bar{g}'$ infinite (requiring ϵ vanishing) at the origin, we can regularize the monopole. For example, with $\epsilon = (\rho/\rho_0)^8$, the regularized monopole energy becomes 7.19 TeV [9,10].

The monopole energy, of course, depends on the functional form of ϵ, so that we could change the monopole energy changing ϵ. Recently Ellis et al. pointed out that $\epsilon = (\rho/\rho_0)^8$ is unrealistic because it makes the Higgs to two photon decay rate larger than the experimental value measured by LHC. Moreover, they have argued that the monopole mass can not be larger than 5.5 TeV if we choose a more realistic ϵ which reproduces the experimental value of the Higgs to two photon decay rate [11]. The effective couplings induced by two different ϵ are shown in Fig. 1.

Now we discuss the gravitational modification of the monopole. Intuitively, the gravitational modification is expected to be negligible. But there is the possibility that the gravitational attraction could change the monopole to a black hole and make the monopole mass arbitrary [12]. We show that this happens only when the Higgs vacuum value approaches to the Planck scale.