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Relativistic corrections to the algebra of position variables and spin-orbital interaction

Alexei A. Deriglazov [a,b,*], Andrey M. Pupasov-Maksimov [a]

[a] Departamento de Matemática, ICE, Universidade Federal de Juiz de Fora, MG, Brazil
[b] Laboratory of Mathematical Physics, Tomsk Polytechnic University, 634050 Tomsk, Lenin Ave. 30, Russian Federation

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ABSTRACT

In the framework of vector model of spin, we discuss the problem of a covariant formalism [35] concerning the discrepancy between relativistic and Pauli Hamiltonians. We show how the spin-induced non-commutativity of a position accounts the discrepancy on the classical level, without appeal to the Dirac equation and Foldy–Wouthuysen transformation.

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1. Introduction

In series of previous works [1–4] we developed a Poincare-invariant variational formulation describing particle with spin. This classical model provides a unified description of both Frenkel and BMT equations [5]. The latter are considered as a basic tool in the analysis of the polarization precession measurements [6]. In [7] we extend the variational formulation to the general relativity, where the classical models of a spinning particle are widely used to describe a rotating body in pole-dipole approximation [8–16]. Another possible application can be related with the kinetic theory of chiral medium, where, in the regime of weak external fields and weak interactions between spinning (quasi)-particles, each particle can be considered as moving along a classical trajectory [17].

For variational formulations provide a striating point to the canonical quantization [18], they have incredible theoretical importance connecting classical and quantum descriptions of nature. Canonical quantization of the free relativistic spinning particle (within our variational formulation, [19]) leads to the positive-energy part of the Dirac equation in the Foldy–Wouthuysen representation. It also identifies [19] the non-commutative Pryce's d-type center of mass operator[1] as the quantum observable which corresponds to the classical position variable. Non-commutativity of (physically meaningful) position operators for relativistic spinning particles was noticed already by Pryce [20]. He shown that coordinates of the relativistic center-of-mass have to obey non-trivial Poisson brackets. As a result, the corresponding quantum observables do not commute. Therefore a physically meaningful position operators of a spin-1/2 should be non-commutative.

Recent theoretical studies revive Snyder's attempts [21] to solve fundamental physical problems by introducing non-commutativity of the space [22]. It is believed that this fundamental non-commutativity may be important at Plank length scale λ_P. Extensive studies of non-commutativity cover both classical and quantum theories, as well relativistic and non-relativistic situations. Postulating non-commutative deformation of position operators [31] one can study physical consequences and estimate possible effects. Calculations of the hydrogen spectrum corrections strongly limit possible non-commutativity of coordinate parameters in the Dirac equation [26–30].

* Corresponding author at: Departamento de Matemática, ICE, Universidade Federal de Juiz de Fora, MG, Brazil.
E-mail addresses: alexei.deriglazov@ufjf.edu.br (A.A. Deriglazov), pupasov.maksimov@ufjf.edu.br (A.M. Pupasov-Maksimov).

[1] See also [32], where the same result was obtained for the classical particle with anticommuting spin variables.

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In the present work we will study effects of a natural non-commutativity of Pryce's d-type center of mass (at both classical and quantum levels) in the description of electron interacting with an electromagnetic background. Our considerations extend results of [19] towards a quantization of interacting spinning particle.

In the free theory, different candidates for the position operator are almost indistinguishable. All these operators obey the same Heisenberg equations (uniform rectilinear motion), and the difference in their expectation values is of Compton wave length order, λ_C. In the interacting case, the problem of the identification of quantum position observables becomes more complicated.[2]

Fleming [25] noted:

"The simplest form of interaction is that due to a static potential which may be expressed in terms of the position operator of the particle. For a relativistic particle, however, the important question arises of which position operator should be used. The conventional approach, in which the position operator is assumed to be local, forces the choice of the center of spin."[3]

He also observed, that a formal substitution of Pryce d-type operator into the potential leads to some reasonable corrections:

"The first correction term to a spherically symmetric local potential will be recognized as the spin-orbit coupling that Thomas derived many years ago as a consequence of classical relativity and which appears in the nonrelativistic limit of the Dirac equation for spin particles."

Analogous situation was observed in general relativity, [35, 37–39] where a formal substitution of a non-local position variable into potential results in correct equations of motion for the spinning particle. Restricting ourselves to the case of special relativity, in the present work we provide some theoretical grounds for such substitution.

The paper is organized as follows. In the first section we present general considerations of the structure of classical and quantum Hamiltonians for a spinning particle. In the second section we give a brief description of the vector model for the classical description of a relativistic spinning particle. In the third section we will realize classical algebra of Dirac brackets by quantum operators in the case of a stationary electro-magnetic background. This realization will deform free Foldy–Wouthusen Hamiltonian and at low energies will give Pauli Hamiltonian with correct spin-orbital interaction. In the conclusion we discuss obtained results.

2. Model independent discussion of the quantum and classical Hamiltonians of a spinning particle

From quantum point of view, at low energies an electron interacting with a background electromagnetic field is described by the two-component Schrödinder equation. Pauli Hamiltonian[4] includes spin-orbital and Zeeman interactions

$$\hat{H}_{ph} = \frac{1}{2m}(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A})^2 - eA_0 + \frac{e(g-1)}{2m^2c^2}\hat{\mathbf{S}}[\hat{\mathbf{p}} \times \mathbf{E}] - \frac{eg}{2mc}\mathbf{B}\hat{\mathbf{S}}$$
$$= \hat{H}_{charge} + \hat{H}_{spin-em} \,. \qquad (1)$$

Gyromagnetic ratio g is a coupling constant of spin with an electromagnetic field. In principle, in non-relativistic theory one can expect different coupling constants for the third and the fourth terms of the Hamiltonian. Experimental observations of the hydrogen spectrum lead to the factor $g-1$ in the third term and to the factor g in the last term. Thus, Hamiltonian explains Zeeman effect and reproduces fine structure of the energy levels of the hydrogen atom. This Hamiltonian follows also from the non-relativistic limit of the Dirac equation in the Foldy–Wouthuysen representation [18, 36].

From classical point of view, models of spinning particles are based on a Lagrangian or Hamiltonian mechanics, both in the relativistic and non-relativistic regime [23]. In a covariant formulation, the spin part of the Hamiltonian describing an interaction between spin S and electromagnetic field reads

$$H_{spin-em-cov} \sim \frac{eg}{2m^2c^2}\mathbf{S}[\mathbf{p} \times \mathbf{E}] - \frac{eg}{2mc}\mathbf{BS}\,. \qquad (2)$$

We emphasize that the expression (2) follows from the analysis of all possible terms in covariant equations of motion and thus is a model-independent [35]. It can also be predicted from symmetry considerations on the level of a Hamiltonian. For instance, if we take the Frenkel spin-tensor $S^{\mu\nu}$, the only Lorentz-invariant combination that could give the desired terms is $F_{\mu\nu}S^{\mu\nu} = 2E^iS^{i0} + \epsilon^{ijk}S^{ij}B^k$ (see our notations in Appendix).

For the classical gyromagnetic ratio $g=2$, the classical spin-orbital interaction in (2) differs by the famous and troublesome factor[5] of $\frac{1}{2}$ from its quantum counterpart in (1). It seems quantization of $H_{spin-em-cov}$ will not reproduce quantum behavior given by $\hat{H}_{spin-em}$. The issue about this difference was raised already in 1926 [34] and still remains under discussion [35].

In principle, Hamiltonian $H_{spin-em}$ can be obtained, if one impose a non-covariant supplementary condition on spin, $2S^{i0}p_0 + S^{ij}p_j = 0$, where $p_0 = -mc$ in the non-relativistic limit. On a first glance, any covariant spin-supplementary condition [8,34,42–44] would give $H_{spin-em-cov}$ and the discrepancy factor of $\frac{1}{2}$.

In the next section we study this issue in the framework of vector model of a spinning particle [4]. We show that the vector model provides an answer on a pure classical ground, without appeal to the Dirac equation. In a few words, it can be described as follows. The relativistic vector model involves a second-class constraints, which should be taken into account by passing from the Poisson to Dirac bracket. The emergence of a higher non-linear classical brackets that accompany the relativistic Hamiltonian (2) is a novel point, which apparently has not been taken into account in literature. If we pretend to quantize the model, it is desirable to find a set of variables with the canonical brackets. The relativistic Hamiltonian (2), when written in the canonical variables, just gives (1).

[2] Another related problem is in the identification of spin operator, since a change of the center of mass definition leads to the modification if the spin definition. [24] compares Pauli, Foldy–Wouthuysen, Czachor, Frenkel, Chakrabarti, Pryce, and Fradkin–Good spin operators in different physical situations and concluded that interaction with electromagnetic potentials allows to distinguish between various spin operators experimentally.

[3] Fleming calls the Newton–Wigner position operator as the center of spin, while Pryce d-type operator is called as the center of mass.

[4] We will write quantum Hamiltonians and other operators using the hat, the same observables without the hat correspond to the classical theory. Thus (1) defines also classical Pauli-like Hamiltonian.

[5] This factor is often referred to Thomas precession [33]. We will not touch this delicate and controversial issue [34,40] since the covariant formalism automatically accounts the Thomas precession [41].

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