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Electron–positron pair production in ion collisions at low velocity beyond Born approximation

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ABSTRACT

We derive the spectrum and the total cross section of electromagnetic e^+e^- pair production in the collisions of two nuclei at low relative velocity β. Both free–free and bound-free e^+e^- pair production is considered. The parameters $\eta_{A,B} = Z_{A,B}\alpha$ are assumed to be small compared to unity but arbitrary compared to β ($Z_{A,B}$ are the charge numbers of the nuclei and α is the fine structure constant). Due to a suppression of the Born term by high power of β, the first Coulomb correction to the amplitude appears to be important at $\eta_{A,B} \gtrsim \beta$. The effect of a finite nuclear mass is discussed. In contrast to the result obtained in the infinite nuclear mass limit, the terms $\propto M^{-2}$ are not suppressed by the high power of β and may easily dominate at sufficiently small velocities.

1. Introduction

The process of electromagnetic e^+e^- pair production in heavy-ion collisions plays an essential role in collider experiments. It has a long history of experimental and theoretical investigations. The process takes place in two different flavors dubbed as “free–free” and “bound-free” production, depending on whether the final electron is in the continuous spectrum or in the bound state with one of the nuclei.

As for the free–free pair production, the pioneering papers [1,2] appeared already in 1930-s and dealt with the high-energy asymptotics of the process. In late 1990-s the interest to the process has been revived due to the RHIC experiment and approaching launch of the LHC experiment. In particular, the contribution of the higher orders in the parameters $\eta_{A,B} = Z_{A,B}\alpha$ (the Coulomb corrections) in the high-energy limit has been discussed intensively, see Refs. [3–7] and the review [8].

The interest to the lepton pair production in collisions of slow nuclei appeared long ago in connection with the supercritical regime taking place when the total charge of the nuclei is large enough (at least larger than 173), see Ref. [9] and references therein.

Recently, in Ref. [10] the total Born cross section of the free–free pair production has been calculated exactly in the relative velocity β of the colliding nuclei. It turns out that the cross section is strongly suppressed as β^8 at $\beta \ll 1$. A natural question arises whether such a suppression also holds for the Coulomb corrections (the higher terms in $\eta_{A,B}$).

In the present paper we show that the Coulomb corrections are less suppressed with respect to β than the Born term. We assume that $\beta \ll 1$ and $\eta_{A,B} \ll 1$ and take into account the higher-order terms in $\eta_{A,B}$ amplified with respect to β. We consider both free–free and bound-free pair production. In the next section we perform calculations in the approximation in which both nuclei have constant velocities, i.e., we treat the nuclei as infinitely heavy objects and neglect the Coulomb interaction between them. This approach has severe restrictions with respect to values of β. These restrictions are discussed in the third section together with the qualitative modification of the results in the region where the constant-velocity approximation is not valid.

2. Pair production cross section

Let us first assume that the parameter η_A is sufficiently small to be treated in the leading order. In particular, we assume that $\eta_A \ll \eta_B, \beta$. We neglect the Coulomb interaction between the nuclei and work in the rest frame of the nucleus B with z axis directed along the momentum of the nucleus A. Since our primary goal is the total cross section of the process, we find it convenient to use the eigenfunctions of angular momentum as a basis. In this basis the cross section has the form

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$$d\sigma = \frac{1}{\beta} 2\pi\delta\left(\beta q_z - \varepsilon - \tilde{\varepsilon}\right) \sum |M|^2 \frac{d^3\boldsymbol{q}}{(2\pi)^3} \frac{dp}{2\pi} \frac{d\tilde{p}}{2\pi} \tag{1}$$

where β is the relative velocity of the nuclei, $\boldsymbol{q}$ is the space components of the momentum transfer to nucleus A, $\varepsilon = \sqrt{p^2 + m^2}$ is the electron energy, m is the electron mass, and the corresponding quantities with tildes are related to a positron. The summation in Eq. (1) is performed over all discrete quantum numbers related to the states of both particles, i.e., over the total angular momentum J, its projection M, and two possible values of $L = J \pm 1/2$, related to the parity of the state. Within our accuracy, the matrix element M reads

$$M = \frac{4\pi\eta_A}{\omega \boldsymbol{q}^2} \int d\boldsymbol{r} e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \boldsymbol{q}\cdot\boldsymbol{J}\,,$$
$$\boldsymbol{J} = U^+\left(\eta_B, \kappa, \varepsilon|\boldsymbol{r}\right) \boldsymbol{\alpha}\, V\left(\eta_B, \tilde{\kappa}, \tilde{\varepsilon}|\boldsymbol{r}\right). \tag{2}$$

Here $\omega = \varepsilon + \tilde{\varepsilon}$, $U\left(\eta_B, \kappa, \varepsilon|\boldsymbol{r}\right)$ is the electron wave function with the energy ε, the total angular momentum $J = |\kappa| - \frac{1}{2}$, and $L = J + \frac{1}{2}\mathrm{sgn}\,\kappa$. This wave function is the solution of the Dirac equation in the attractive potential $-\eta_B/r$. The function $V\left(\eta_B, \tilde{\kappa}, \tilde{\varepsilon}|\boldsymbol{r}\right)$ is the negative-energy solution of the Dirac equation corresponding to the charge conjugation of the positron wave function, so that

$$V\left(\eta_B, \tilde{\kappa}, \tilde{\varepsilon}|\boldsymbol{r}\right) = i\gamma_2 U^*\left(-\eta_B, \tilde{\kappa}, \tilde{\varepsilon}|\boldsymbol{r}\right).$$

In the derivation of Eq. (2) we have used the gauge in which the photon propagator has the form

$$D^{ab} = -\frac{4\pi\left(\delta^{ab} - q^a q^b/\omega^2\right)}{\omega^2 - q^2}\,, \quad D^{0a} = D^{00} = 0\,.$$

The limit $\beta \ll 1$ is quite special. From kinematic constraints, it is easy to conclude (cf. Ref. [10]) that the characteristic momentum transfers to both nuclei are of the order of $m/\beta \gg m$. A simple estimate $r \sim \beta/m \ll 1/m\eta_B$ justifies using the small-r asymptotics of the Coulomb wave functions:

$$U\left(\eta_B, \kappa, \varepsilon|\boldsymbol{r}\right) = \begin{pmatrix} f(r)\,\Omega_{\kappa M} \\ -ig(r)\,(\boldsymbol{\sigma n})\,\Omega_{\kappa M} \end{pmatrix},$$
$$V\left(\eta_B, \tilde{\kappa}, \tilde{\varepsilon}|\boldsymbol{r}\right) = \begin{pmatrix} \tilde{g}(r)\,\Omega_{-\tilde{\kappa} M} \\ -i\tilde{f}(r)\,(\boldsymbol{\sigma n})\,\Omega_{-\tilde{\kappa} M} \end{pmatrix}, \tag{3}$$

where $\boldsymbol{n} = \boldsymbol{r}/r$, $\Omega_{\kappa M} = \Omega_{JLM}$ is the spherical spinor, and the radial wave functions read

$$\left.\begin{matrix} f(r) \\ g(r) \end{matrix}\right\} = C r^{\gamma-1} \begin{cases} \frac{\kappa-\gamma}{\eta_B} + \underline{\frac{r}{2\gamma+1}\left[\varepsilon\left(2\gamma - 2\kappa + 1\right) + m\right]} \\ 1 - \underline{\frac{r(\kappa-\gamma)}{(2\gamma+1)\eta_B}\left[\varepsilon\left(2\gamma + 2\kappa + 1\right) - m\right]} \end{cases}$$
$$\left.\begin{matrix} \tilde{f}(r) \\ \tilde{g}(r) \end{matrix}\right\} = \tilde{C} r^{\tilde{\gamma}-1} \begin{cases} -\frac{\tilde{\kappa}-\tilde{\gamma}}{\eta_B} + \underline{\frac{r}{2\tilde{\gamma}+1}\left[\tilde{\varepsilon}\left(2\tilde{\gamma} - 2\tilde{\kappa} + 1\right) + m\right]} \\ 1 + \underline{\frac{r(\tilde{\kappa}-\tilde{\gamma})}{(2\tilde{\gamma}+1)\eta_B}\left[\tilde{\varepsilon}\left(2\tilde{\gamma} + 2\tilde{\kappa} + 1\right) - m\right]} \end{cases} \tag{4}$$

Here

$$C = \frac{p}{\varepsilon}\sqrt{\frac{1 + \frac{\gamma}{\kappa}}{1 + \frac{m\gamma}{\varepsilon\kappa}}}\, e^{\pi\nu/2} \left|\frac{\Gamma\left(\gamma + 1 + i\nu\right)}{\Gamma\left(2\gamma + 1\right)}\right| (2p)^\gamma\,,$$
$$\gamma = \sqrt{\kappa^2 - \eta_B^2}\,, \quad \nu = \varepsilon\eta_B/p\,,$$
$$\tilde{C} = C\left(\eta_B \to -\eta_B,\ \varepsilon \to \tilde{\varepsilon},\ \kappa \to \tilde{\kappa}\right).$$

Let us assume for the moment that $\beta \ll \eta_B \lesssim 1$. Then the underlined terms can be safely neglected due to the estimate $r \sim \beta/m$. Moreover, due to the same estimate, the leading contribution to the sum in Eq. (1) is given by the terms with $\kappa = \pm 1$ and $\tilde{\kappa} = \pm 1$. If we also assume that $\eta_B \ll 1$, then only the contributions of two states with

$$\left(\kappa, \tilde{\kappa}\right) = (+1, -1) \text{ and } \left(\kappa, \tilde{\kappa}\right) = (-1, +1)$$

survive. The underlined terms in Eq. (4) become important for $\eta_B \lesssim \beta$. In this region, in addition to the two states mentioned above, the states with $\left(\kappa, \tilde{\kappa}\right)$ equal to

$$(+1, +2)\,, \quad (-1, -2)\,, \quad (+2, +1)\,, \quad \text{and} \quad (-2, -1)$$

also should be taken into account. Integrating over $\boldsymbol{r}$ in Eq. (2), substituting the result in Eq. (1), and integrating over $\boldsymbol{q}$, we obtain the cross section σ_{ff} of the free–free pair production:

$$\begin{aligned} \frac{d\sigma_{ff}}{d\varepsilon d\tilde{\varepsilon}} = \frac{\eta_A^2\eta_B^2\beta^6 p\tilde{p}}{\pi\left(\tilde{\varepsilon} + \varepsilon\right)^8} \Bigg\{ & \pi^2\eta_B^2\left(\varepsilon\tilde{\varepsilon} - m^2\right) \\ & - \frac{128\pi\eta_B\beta(\tilde{\varepsilon} - \varepsilon)}{27(\tilde{\varepsilon} + \varepsilon)}\left(\varepsilon\tilde{\varepsilon} - 2m^2\right) \\ & + \frac{16\beta^2}{45\left(\tilde{\varepsilon} + \varepsilon\right)^2}\Big[\left(33\varepsilon\tilde{\varepsilon} - 49m^2\right)\left(\varepsilon^2 + \tilde{\varepsilon}^2\right) \\ & - 14\varepsilon^2\tilde{\varepsilon}^2 + 78m^2\varepsilon\tilde{\varepsilon} - 32m^4\Big]\Bigg\}. \end{aligned} \tag{5}$$

The relative order of the three terms in braces is regulated by the ratio η_B/β. When this ratio is small, the last term dominates. This term coincides with the Born result obtained in Ref. [10], as should be. The parameter η_B/β appears due to the "accidental" suppression of the Born amplitude of pair production and has nothing to do with the Sommerfeld–Gamov–Sakharov factor.

The bound-free pair production can be treated exactly in the same way as the free–free pair production. It appears that an electron is produced mostly in $ns_{1/2}$ states ($\kappa = -1$). The positron spectrum reads

$$\begin{aligned} \frac{d\sigma_{bf}}{d\tilde{\varepsilon}} = \eta_A^2\eta_B^5\beta^6 \frac{2m^3\left(\tilde{\varepsilon} - m\right)\tilde{p}}{\left(\tilde{\varepsilon} + m\right)^8}\zeta_3 \Bigg\{ & \pi^2\eta_B^2 - \frac{128\pi\eta_B\beta(\tilde{\varepsilon} - 2m)}{27(\tilde{\varepsilon} + m)} \\ & + \frac{16\beta^2}{15\left(\tilde{\varepsilon} + m\right)^2}\left[11\tilde{\varepsilon}^2 - 10m\tilde{\varepsilon} + 27m^2\right]\Bigg\}, \end{aligned} \tag{6}$$

where the Riemann zeta function $\zeta_3 = \sum_{n=1}^{\infty}\frac{1}{n^3}$ comes from summation over the principal quantum number. It is quite remarkable that Eq. (6) can be obtained from Eq. (5) by the simple substitution $\frac{p\varepsilon d\varepsilon}{2\pi^2} \to \sum_n |\psi_{ns}(0)|^2 = \sum_n \frac{m^3\eta_B^3}{\pi n^3}$ followed by the replacement $\varepsilon \to m$. This substitution works because of the factorization of hard-scale $r \sim \beta/m$ and soft-scale $r \sim 1/m\eta_B$ contributions.

The total cross sections are obtained by the direct integration over energies (energy)[1]:

$$\sigma_{ff} = \frac{\eta_A^2\eta_B^2\beta^6}{1050\pi m^2}\left\{\pi^2\eta_B^2 + \frac{592}{105}\beta^2\right\},$$
$$\sigma_{bf} = \frac{16\eta_A^2\eta_B^5\beta^6}{15015m^2}\zeta_3\left\{\pi^2\eta_B^2 + \frac{976}{153}\beta^2\right\}. \tag{7}$$

[1] Note that Eq. (7) is in obvious contradiction with the results of Refs. [11,12]. The origin of discrepancy is different for these two papers. As it concerns free–free pair production, in Ref. [11] two definitions for the total momentum transfer from the nuclei (differing by the relative sign between momentum transfers from each nucleus) appear to be mixed. Meanwhile, in Ref. [12] the space components of momentum transfer from the projectile nucleus (of the order of $m/\beta \gg m$!) are totally omitted in the annihilation current.