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Non-perturbative fixed points and renormalization group improved effective potential

A.G. Dias [a,*], J.D. Gomez [a], A.A. Natale [a,b], A.G. Quinto [a], A.F. Ferrari [a]

[a] Universidade Federal do ABC, Centro de Ciências Naturais e Humanas, Rua Santa Adélia, 166, 09210-170, Santo André, SP, Brazil
[b] Instituto de Física Teórica, UNESP, Rua Dr. Bento T. Ferraz, 271, Bloco II, 01140-070, São Paulo, SP, Brazil

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ABSTRACT

The stability conditions of a renormalization group improved effective potential have been discussed in the case of scalar QED and QCD with a colorless scalar. We calculate the same potential in these models assuming the existence of non-perturbative fixed points associated with a conformal phase. In the case of scalar QED the barrier of instability found previously is barely displaced as we approach the fixed point, and in the case of QCD with a colorless scalar not only the barrier is changed but the local minimum of the potential is also changed.

1. Introduction

The discovery of a Higgs-like particle at the CERN-LHC, and the fact that this particle is "lighter" than what could be expected for the Higgs boson in several extensions of the Standard Model (SM) is leading to a deeper investigation of the mass generation mechanism as it is known in the SM. Many recent papers are discussing the Higgs mechanism under new points of view, such as the naturality of the model [1–3], its stability [4–6], and studying possible alternatives or extensions of the model.

Several years ago the possibility that a conformal classical symmetry could be important in the mass generation mechanism was discussed by Meissner and Nicolai [7]. At that time they proposed an extension of the SM where the radiative symmetry breaking calculated with the help of the effective potential, as first suggested by Coleman and Weinberg (CW) [8], was compatible with the experimental data then available. The example of [7] was already giving an answer to the questions of naturality and stability of the Higgs mechanism, and the possibility that the SM symmetry breaking could be implemented through radiative corrections is still in discussion [9].

How the CW effective potential calculation is applicable and reliable in a realistic theory is a motive of debate. Meissner and Nicolai discussed the applicability of a renormalization group (RG) improved version of the one-loop CW effective potential in simple models [10], where the behavior of the coupling constants could be easily calculated. The inclusion of the coupling constants evolution extends the validity range of the effective potential. One of the criteria for applicability of the RG improved CW effective potential proposed in [10] was that the running coupling constants, expressed as functions of the classical field, should stay small. Of course, away from the origin the RG β functions will depend on the renormalization scheme and how the coupling is defined. However, we should expect the stability of the effective potential certainly to depend on the coupling constants RG behavior in a much larger range of values [11].

In the examples of classically conformal theories of [10] the effective potential stability is connected to ultraviolet (UV) and infrared (IR) barriers caused by the presence of a Landau pole in the QED or QCD couplings. However, the existence of a Landau pole in these couplings has been questioned, and instead of a pole they may present a non-perturbative fixed point. Due to the phenomenon of dynamical symmetry breaking in QED and QCD, the coupling constants may freeze after they reach a certain critical value, whereas in the QCD case, as will be discussed later, such critical value is even not so large. It is the effect of non-perturbative fixed points of these types in the RG improved effective potential calculation, applied to the models of [10], that we want to discuss in this work. Their effect has not been discussed in the context of the CW potential and they may even modify the potential stability conditions.

In QED the non-perturbative fixed point that we referred to above implies a critical coupling $\alpha_c \approx \pi/3$ [12,13], where α_c is the UV critical value of the fine structure constant ($\alpha \equiv e^2/4\pi$). This behavior is a consequence of dynamical chiral symmetry breaking, in a mechanism similar to the fall into the Coulomb center

* Corresponding author.

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for large charge [14], with a β function that is approximated by

$$\beta_\alpha = -2(\alpha - \alpha_c). \tag{1}$$

It is not clear whether this fixed point indeed exists, and QED already does not make sense at the physical scale of such critical value, which happens to be above the Planck scale. Nevertheless, the study of such possibility can be instructive.

On the other hand, there are many plausible evidences that QCD develops an infrared non-perturbative fixed point. For instance, the study of dynamical mass generation in QCD indicates that the coupling constant may freeze in the infrared as [15,16]

$$\bar{g}^2(k^2) = \frac{1}{\beta_0 \ln[(k^2 + 4m_g^2)/\Lambda^2]}, \tag{2}$$

where $\beta_0 = (11N - 2n_q)/48\pi^2$ with n_q quark flavors, Λ is the characteristic QCD scale, and m_g is a dynamically generated "effective mass" for the gluon, whose preferred value is $m_g \approx 2\Lambda$ [15,17]. The IR value of Eq. (2) as well as the compilation of other IR values for the QCD coupling obtained in different phenomenological applications can be seen in [18], and they do not indicate an abrupt transition to the non-perturbative regime. We can also quote theoretical estimates of $\alpha_s(0)$ through the functional Schrödinger equation, which suggest $\alpha_s(0) \approx 0.5$ [19]. The infrared finite effective charge of QCD, in the context of Schwinger–Dyson equations, has also been discussed in [20] and is associated with an infrared finite gluon propagator. Actually, finite IR gluon propagators have been confirmed in lattice simulations [21,22], and they do lead to a non-perturbative IR fixed point [23]. The effect of such non-perturbative coupling constant has not been explored in the case of a CW potential calculation involving QCD.

The organization of this work is the following: In Section 2 we discuss the RG improved potential of scalar QED in the presence of a non-perturbative fixed point. This section is a simple example of what we will calculate in the more elaborated case of the CW potential for QCD with a colorless scalar, which is going to be presented in Section 3. In Section 4 we draw our conclusions.

2. CW potential in scalar QED with a fixed point

The RG improved effective potential of [10] for an ordinary scalar field φ theory with quartic self-interaction and no classical mass term is given by

$$W_{eff}(\varphi, g, v) = \hat{g}_1(L)\varphi^4 \exp\left[2\int_0^L \bar{\gamma}\big(\hat{g}(t)\big)dt\right], \tag{3}$$

where $\hat{g}$ may indicate a set of coupling constants, v is some renormalization mass scale,

$$L \equiv \ln\frac{\varphi^2}{v^2}, \tag{4}$$

and $\bar{\gamma}(\hat{g})$ is an anomalous dimension associated with the coupling constants.

We will discuss the effective potential of Eq. (3) in the case of massless scalar QED. The scalar self-coupling and the gauge coupling are respectively given by

$$y = \frac{g}{4\pi^2}, \qquad u = \frac{e^2}{4\pi^2}, \tag{5}$$

and their RG equations are

$$2\frac{dy}{dL} = a_1 y^2 - a_2 yu + a_3 u^2, \qquad 2\frac{du}{dL} = 2bu^2, \tag{6}$$

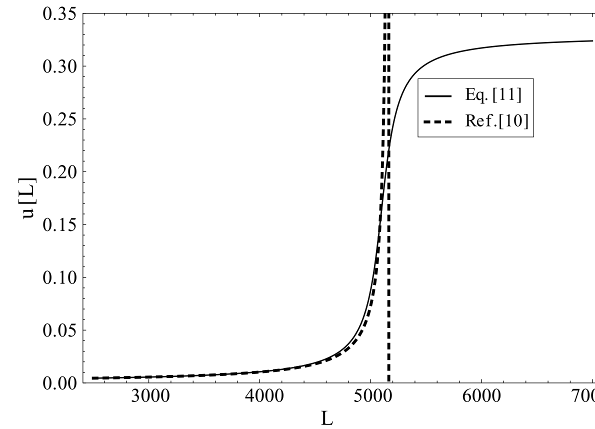

Fig. 1. Evolution of the QED gauge coupling giving by Eq. (7) (dashed curve) and the fit of Eq. (11) (continuous curve), which assumes the existence of a non-perturbative fixed point. For small L values the fit agrees with the perturbative result.

where $a_1 = 5/6$, $a_2 = 3$, $a_3 = 9$ and $b = 1/12$. The anomalous dimension is $\gamma(y, u) = cu$, with $c = 3/4$. The solutions of Eq. (6) are

$$u(L) = \frac{u_0}{1 - bu_0 L}, \tag{7}$$

$$y(L) = \frac{(a_2 + 2b)}{2a_1}u(L) + \frac{Au(L)}{2a_1}\tan\left(\frac{A}{8b}\ln u(L) + C\right), \tag{8}$$

where

$$A = \sqrt{4a_1 a_3 - (a_2 + 2b)^2} \tag{9}$$

is a positive quantity and C is a constant chosen to satisfy $y(0) = y_0$.

The RG improved effective potential at one loop is

$$W_{eff} = \frac{\pi^2\varphi^4 y(L)}{(1 - bu_0 L)^{2c/b}}. \tag{10}$$

Eq. (10) is the result obtained in [10]. This result has one striking difference in relation to the unimproved potential, which is the presence of two barriers, one related to the UV Landau pole and an IR one where the potential becomes unbounded from below.

Let us now suppose that the theory has a fixed point at $\alpha_c(L) \approx 1$. Note that this would be a possibility for QED with fermions as discussed in Refs. [12,13], but here this is just an *ad hoc* supposition to exemplify what may happen with the effective potential in the case of a possible freezing of the coupling constant. We see in Fig. 1 that the critical point of the perturbative coupling will occur for $L > 5000$, while for much smaller L values the coupling follows Eq. (7). As a consequence, for small values of L we have $u = e^2/4\pi^2$, and at large L the coupling freezes at $u \approx 1/\pi$.

The main difference between this work and previous calculations of the RG improved effective potential is the introduction of an interpolating coupling constant joining the perturbative to the non-perturbative regime. The best interpolation formula between these values is given by a $\tan^{-1}$ function. We make a fit for the gauge coupling assuming the RG standard solution for $L < 4500$, and interpolate it in the region $4500 < L < 5300$ with a $\tan^{-1}$ formula, such that it will be joined to the frozen value of the coupling for $L > 5300$. A fit that is reasonable from $L = 0$ up to $L < 4000$ at 0.1% level is given by

$$u_{Fit}(L) = 0.105\tan^{-1}\left(\frac{L - 5100}{110}\right) + 0.165. \tag{11}$$