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Remarks on pole trajectories for resonances

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ABSTRACT

We discuss in general terms pole trajectories of resonances coupling to a continuum channel as some strength parameter is varied. It is demonstrated that, regardless of the underlying dynamics, the trajectories of poles that couple to the continuum in a partial wave higher than s-wave are qualitatively the same, while in case of s-waves the pole trajectory can reveal important information on the internal structure of the resonance. In addition we show that only molecular (or extraordinary) states appear near thresholds naturally, while more compact structures need a significant fine tuning in the parameters.

This study is of current relevance especially in strong interaction physics, since lattice QCD may be employed to deduce the pole trajectories for hadronic resonances as a function of the quark mass thus providing additional, new access to the structure of s-wave resonances.

1. Introduction

If all mesons were $\bar{q}q$ states then there would be no natural reason for poles in scattering amplitudes to occur very close to thresholds. At large values of N_c, the number of colors in QCD, all $\bar{q}q$ mesons become narrow with (nearly) unchanged mass [1]. Thus, their masses have no relation to the masses of the mesons to which they couple.[1] Accordingly, the ρ mass is not related to $2m_\pi$, nor is the K^* mass related to $m_K + m_\pi$. So the mere fact that the $f_0(980)$ and $a_0(980)$ appear very near $K\bar{K}$ threshold is already reason to be suspicious that they are not simple $\bar{q}q$ states. The same applies to unusual charmonium states that have been found near charm–anticharm meson thresholds like the famous $X(3872)$ located very close to the $D^0\bar{D}^{0*}$ threshold — for a recent review see [5].

On the other hand, there is good reason for "extraordinary" hadrons, often called hadronic molecules, to have masses close to thresholds [6]. In this paper we look carefully at the way that the manifestations of poles in scattering amplitudes change as the poles approach thresholds as some strength parameter is varied — here one may think of varying the quark masses in lattice QCD calculations. This has acquired a renewed interest after the trajectory of the σ or $f_0(500)$ resonance pole as a function of the quark mass was predicted by us within unitarized Chiral Perturbation Theory [7]. A similar trajectory as that of the σ was soon shown to be followed by the controversial κ or $K(800)$ resonance in the isospin 1/2 scalar πK scattering partial wave, including the appearance of a virtual state at sufficiently large pion masses [8]. The subtleties of the extraction of resonance parameters from lattice QCD simulations performed at a finite volume are outlined in detail in Refs. [9,10] and will not be discussed further here. Recently the existence of such a virtual bound state at high pion masses has been confirmed by lattice calculations [11].

While finishing this work, we became aware of a theoretical study [13] of the scaling of hadron masses near an s-wave threshold, showing that the bound state energy is not continuously connected to the real part of the resonance energy. In this paper we have another look at this issue which allows us to provide various additional, non-trivial insights. In particular, we demonstrate that there is a qualitative difference between the pole trajectories of resonances that couple to the relevant continuum channel in an s-wave or in a higher partial wave: As a consequence of analyticity a resonance is characterized by two poles on the second sheet, one located at $s = s_R$ and one located at $s = s_R^*$. For narrow resonances only one of them is close to the physical region. As some strength parameter is increased, the two poles start to approach each other. We demonstrate on general grounds below that while for higher partial waves the poles meet at the corresponding two meson threshold, for s-waves the poles can still be located inside the complex plane even for the real part of the

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[1] The same is true for a straightforward extension of tetraquarks to large N_c [2] — in case they existed at large N_c [3] — although other possible extensions of tetraquarks to $N_c \neq 3$ lead to masses that grow when N_c is increased [4].

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pole position at or below threshold. As a consequence, s-wave-trajectories are controlled by an additional dimensionful parameter, namely the value of s where the two poles meet below threshold which may be related to the structure of the state. In other words, generic trajectories of s-wave resonances do lead to poles whose real part of the position is below threshold, but whose imaginary part of the position does not vanish, before giving rise to virtual bound states, and then bound states, as some strength parameter is varied. While this observation is in line with the findings of Refs. [7,14], it is in vast conflict with "common wisdom" that the imaginary part of a pole has to be identified with one half of its decaying width, for this implies that, if the "resonance mass" — identified with the real part of the pole position — lies below threshold, the pole necessarily has to lie on the real axis.

The paper is organized as follows: in the next section we discuss general properties of the poles that appear in the S-matrix, paying particular attention to poles that occur in partial waves with angular momenta higher than 0, especially to the role of the centrifugal barrier which is absent in the scalar partial waves. Next we consider the trajectories of resonance poles in the complex plane as a function of some strength parameter, and how they can become bound states. In the next section we briefly review Weinberg's compositeness criterion and reformulate it in terms of the parameters introduced in the previous section. The possible behaviors are then illustrated with two models of scattering in separable potentials within non-relativistic scattering theory, one with a single channel and another one in a two-channel system. In Section 4 we analyze, the realistic examples of the pole trajectories of the σ or $f_0(500)$ scalar meson and the $\rho(770)$ as functions of the quark masses, obtained from the combination of Chiral Perturbation Theory and a single channel dispersion relation obtained in [7]. We show how the generic features discussed in this paper show up in these two cases. In particular, we can conclude that the $f_0(500)$ or sigma meson would have a predominantly molecular nature, if the pion mass were of the order of 450 MeV or higher. In the final section we summarize our results.

2. General properties of S-matrix poles

In this work we only consider one continuum channel. This implies that the S-matrix has one right hand cut, starting at $s = (2m)^2$ — the so-called unitarity cut.[2] As a consequence there are two sheets and, as usual, we call first or physical sheet the one corresponding to a momentum with a positive imaginary part. The S matrix evaluated on sheet I (II) is written as $S_I(s)$ ($S_{II}(s)$). If no subscript is given, the expression holds for both sheets. It follows directly from unitarity and analyticity that [15]

$$S_I(s) = 1/S_{II}(s) \quad \text{and} \quad \left[S(s)\right]^* = S\left(s^*\right). \tag{1}$$

As a consequence a pole on the second sheet immediately implies a zero on the first and vice-versa. In addition, if there is a pole at $s = s_0$, there must also be a pole at $s = s_0^*$, i.e., poles outside the real axis occur in conjugate pairs. Furthermore, it can be shown that the only poles allowed on the physical sheet are bound state poles, namely, those located on the real axis below threshold.

A different, but equivalent, way to discuss the pole structure of the S-matrix is to use the k-plane: instead of the Mandelstam variable s, the center of mass momentum k is used to characterize the energy of the system. The two quantities are related via

$$k = \sqrt{s/4 - m^2}. \tag{2}$$

[2] For simplicity we only consider the case of scattering of two particles with equal mass, however, the generalization to unequal masses is straightforward.

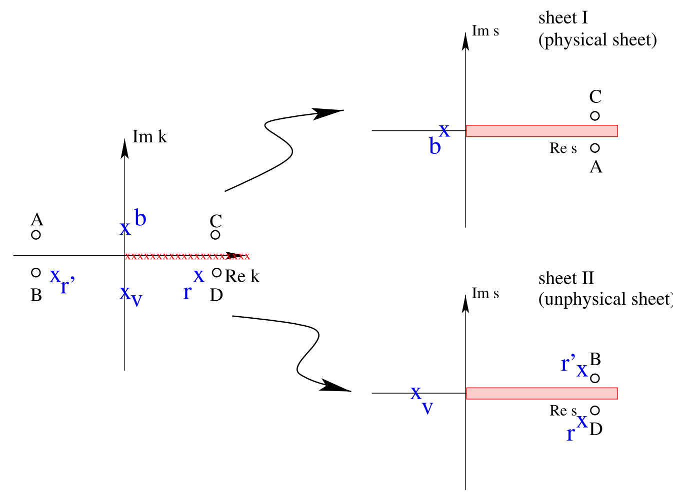

Fig. 1. Relation between k-plane and s-plane: on the left the k-plane is shown. The (red) xs denote the physical axis. On the right the two s-plane sheets are shown. Here the broad band indicates the position of the unitarity cut. The upper (lower) half plane of the k-plane maps onto the first (second) sheet in the s-plane such that the points A–D get transferred as indicated in the figure. In addition, the allowed pole positions in the complex plane are also shown as x. They are labeled as b for the bound state, v for the virtual state, and r and r' for the two conjugate poles of the resonance state.

The obvious advantage is that there is no right hand cut with respect to k and correspondingly there is only one sheet. It follows directly from the definition that the upper (lower) half plane of the complex k-plane, defined by positive (negative) values of the imaginary part of k, maps onto the first (second) sheet of the s-plane. The conditions derived above from Eq. (1) translate into the k-plane as follows: the only poles allowed in the upper half plane are on the imaginary axis and in the lower half plane appear as mirror images with respect to the imaginary axis. The relation between the different planes is illustrated in Fig. 1. On the one hand, it becomes clear from the figure that the resonance pole located at r is the one closest to the physical axis and therefore physically more relevant than the one at r' in the vicinity of the pole. On the other hand, at the threshold both poles are equally relevant regardless where they are located in the second sheet. Finally, in the k plane virtual states appear as poles on the negative imaginary axis (labeled as v in the figure) and bound states as poles on the positive imaginary axis (labeled as b in the figure).

Now, assuming that there is at least one resonance pole, and that it is not too far away from threshold, we are now in the position of writing down the most general expression for the S-matrix in the vicinity of that pole or its conjugate partner. For the derivation it is easier to use the k plane, and thus we assume that there is a resonance pole at $k = k_p - i\gamma$ with $\gamma > 0$. For a resonance, k_p is a real number and we choose $k_p > 0$, for, as commented above, it corresponds to the pole closest to the physical axis. Then, from the above considerations it follows that there is in addition a pole at $k = -k_p - i\gamma$ and zeros at $k = \pm k_p + i\gamma$. We may therefore, dropping terms of higher order in k, and for a particular partial wave ℓ, write the following general expression for the S-matrix element in the vicinity of the pole [15]:

$$S_\ell(k) = e^{i\phi(k)} \frac{(k - k_p - i\gamma)(k + k_p - i\gamma)}{(k - k_p + i\gamma)(k + k_p + i\gamma)}, \tag{3}$$

where $\phi(k)$ is a smooth function, real valued for real, positive values of k. For simplicity this phase factor will be dropped in what