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Quantum gates via relativistic remote control

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ABSTRACT

We harness relativistic effects to gain quantum control on a stationary qubit in an optical cavity by controlling the non-inertial motion of a different probe atom. Furthermore, we show that by considering relativistic trajectories of the probe, we enhance the efficiency of the quantum control. We explore the possible use of these relativistic techniques to build 1-qubit quantum gates.

1. Introduction

The study of the interface between quantum mechanics, field theory and general relativity has led to results where, in principle, relativistic features can be used to gain advantage over non-relativistic settings in the processing of quantum information [1–4].

To implement quantum gates, or even quantum simulators, we need to very accurately control the degrees of freedom we use as qubits as well as the dynamics of the quantum mechanical systems that contain them. Such degree of control has been achieved, for instance, in NMR devices [5] which have been largely employed to implement quantum computing algorithms on nuclear spins.

In these devices, electrical currents are used to generate magnetic fields that ultimately influence the state of the nuclear spin qubit. The microscopic mechanism of how the accelerated charges interact with the nuclear spin is commonly simplified by treating the field classically. However, it is not unreasonable to think that detailed study of the interaction of the moving charges with the qubit degrees of freedom – mediated by a fully quantum EM field – may enhance our ability to control the qubit. Moreover, treating this setting in a relativistic framework may allow us to see how (or if) relativistic effects can actually improve our capacity to control the qubit beyond what classical models predict.

From the fundamental high-energy physics point of view this analysis may prove interesting in the following way: We will show that the relativistic motion of a probe induces high-energy relativistic effects that can be used to control a logical qubit stored in a stationary atom. Hence, this suggests a connection between high energy physics and quantum optics and information. For instance, based on these results, one could think of using charged beams generated by particle colliders to control the state of atomic qubits, and maybe recast some of the problems of measurement of the outcome of particle colliders in terms of quantum informational variables. As we will highlight, the phenomena described in this paper already manifests at the scales of energies present in the LHC.

It is already known that non-inertial motion can be used to implement universal single qubit gates on atomic systems [2] and Gaussian two-qubit gates on cavity field modes [3]. In more detail, [2] showed that control over the acceleration of atoms can be used to perform quantum gates as a direct consequence of relativistic quantum effects. However, these schemes require control over both the internal degrees of freedom of an atom and over the non-inertial motion of its center-of-mass, which may prove challenging in a practical experimental setting. For instance, the force that accelerates the atom may also induce transitions between the energy levels that constitute the logical qubits.

In this paper we explore how controlling the trajectory of an accelerated atom (the probe atom) allows us to garner control over a different atomic qubit (the target qubit) that sits stationary inside an optical cavity. Namely, we will show that it is indeed possible to perform arbitrary rotations on the Bloch sphere of the state of the target qubit with only a small decoherence effect. We obtain such effects already in the simplified case of uniformly accelerated trajectories of the probe atom, even in the relatively simple scenario where we consider only atoms (one probe and one target) coupled through the interaction with the quantum field.

Furthermore, we show that when the probe is allowed to attain high speeds, relativistic effects start to influence the target atom. Remarkably, and maybe against intuition [6,7], these effects

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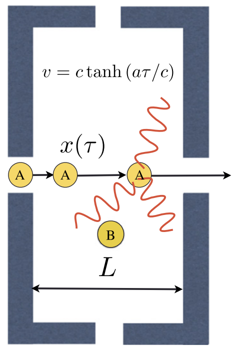

Fig. 1. The probe atom (A) is shot through the cavity and the target atom (B) is stationary at $x = L/2$, they interact only via the field. We control the probe's trajectory, and this gives us control over the target qubit. The probe's worldline is given by $t(\tau) = a^{-1}\sinh a\tau$, $x(\tau) = a^{-1}(\cosh a\tau - 1)$.

allow for better control of the target qubit. We will quantitatively show how we can effectively get larger controlled Bloch sphere rotations when the probe's motion is relativistic as opposed to non-relativistic.

2. Setup

We will consider a target atom at rest inside a stationary optical cavity of purely reflective walls as illustrated in Fig. 1. The probe atom will fly through the cavity describing a constantly accelerated motion. Both atoms couple locally (along their respective worldlines) to the quantum field inside the cavity.

We will use the Unruh–DeWitt Hamiltonian [8] to model the light-matter interaction. This model, often used to model relativistic particle detectors [9], is identical to the Jaynes–Cummings model of light-matter interaction [10] but without taking the single mode approximation nor the rotating wave approximation. Although simple, the model captures all the features of the light-matter interaction when no orbital angular momentum exchange transitions are considered [11,12].

The Hamiltonian for a single detector will be of the form $H = H_0^{(d)} + H_0^{(f)} + H_I$, where $H_0^{(d)}$ and $H_0^{(f)}$ are the detector and field free Hamiltonians. The interaction Hamiltonian H_I is of the form $H_I = \lambda\xi(\tau)\mu(\tau)\Phi[x(\tau)]$ where $\lambda\xi(\tau)$ is a time dependent coupling strength controlling the interaction time, $\mu(\tau) = (\sigma^+ e^{-\mathrm{i}\Omega\tau} + \text{H.c.})$ is the monopole moment operator (in the interaction picture) where Ω is the energy gap between the two levels of the atom, $x(\tau)$ is the worldline of the atom parametrized in terms of its proper time and $\Phi[x(\tau)]$ is the field operator which we expand in terms of stationary wave modes. Throughout the paper we will use natural units $c = \hbar = 1$ and we will take the scale Ω as our reference. How our results translate into dimensionful units is explained in the section 'Experimental feasibility' below.

Since there are two atoms with different states of motion (thus different proper reference frames) we need to choose with respect to what time parameter we want the full Hamiltonian to generate evolution. We choose the proper time of the stationary atom; consequently there is a redshift factor in front of the accelerated atom term of H_I. This is a somewhat subtle point which is discussed in-depth in [13]. Taking all this into account we finally obtain $H_I(t) = \frac{\mathrm{d}\tau}{\mathrm{d}t} H_I^{(\mathrm{A})}[\tau(t)] + H_I^{(\mathrm{B})}(t)$, where the individual $H_I^{(\mathrm{d})}$ are given by the single detector interaction Hamiltonian shown above and t is the cavity rest frame time.

We initially prepare the quantum field in the cavity such that one of the field modes is in a coherent state of complex amplitude α, and the rest of the modes are lowly-populated. Preparing a near-resonant coherent state reduces the amount of entanglement acquired between the atoms and the field. This in turn helps screen out the mixedness effects on the target qubit produced by the 'Unruh noise' generated by the probe's relativistic motion [2,14]. Thus the initial atoms-field density matrix can be written as $\rho_0 = \rho_{A,0} \otimes \rho_{B,0} \otimes |\alpha_{\omega_1}\rangle\langle\alpha_{\omega_1}| \otimes_{\omega_n \neq \omega_1} |0_{\omega_n}\rangle\langle 0_{\omega_n}|$. Notice that, since we are in a cavity, the frequencies $\omega_n = n\pi/L$ form a discrete set.

When the probe enters the cavity, it becomes coupled to the field. We take a perturbative approach (valid for small couplings and short times) to analyze the system dynamics. The time evolution under this Hamiltonian from a time $t = 0$ to time $t = T$ is given by

$$U(T,0) = \mathbb{1} - \mathrm{i}\int_0^T \mathrm{d}t_1 H_I(t_1) - \int_0^T \mathrm{d}t_1 \int_0^{t_1} \mathrm{d}t_2 H_I(t_1) H_I(t_2),$$

plus terms $\mathcal{O}(\lambda^3)$, where the notation $\mathcal{O}(\lambda^n)$ refers to powers of the coupling strengths of both the probe-field λ_A and target-field λ_B, so that $\lambda_A\lambda_B$ is an $\mathcal{O}(\lambda^2)$ term. The density matrix after a time T will be given by the perturbative expansion $\rho_T = \rho_0 + \rho_T^{(1)} + \rho_T^{(2)} + \mathcal{O}(\lambda^3)$ where

$$\rho_T^{(1)} = U^{(1)}\rho_0 + \rho_0 U^{(1)\dagger}, \tag{1}$$

$$\rho_T^{(2)} = U^{(1)}\rho_0 U^{(1)\dagger} + U^{(2)}\rho_0 + \rho_0 U^{(2)\dagger}. \tag{2}$$

Recall that we are interested in the target's final state, and so we will trace out the field modes as well as the probe's state to obtain: $\rho_{T,B} = \mathrm{Tr}_A(\mathrm{Tr}_f(\rho_T))$. We will compare this to the target's initial density matrix, and quantitatively assess our ability to control the target qubit by controlling the probe's motion.

3. Performing 1-qubit rotations

In [2], one-qubit gates were obtained through the non-inertial motion of the atom which supported the logical qubit. Arbitrary rotations on the Bloch sphere were achieved introducing no decoherence to leading order in perturbation theory. The price to pay is that logical quantum operations are performed on the qubit whose non-inertial trajectory had to be controlled. As opposed to [2] we use the motion of a *different* probe atom to gain control over the target qubit, physically supported on a different atom which rests in the cavity. Hence, we are not required to keep under control both the trajectory and the internal state of one atom simultaneously. While advantageous in this sense, there is a trade-off on the quality of the quantum gates that we could implement with this setting. As the 'remote control' appears as a second order effect, it is impossible to perform a 100% clean Bloch sphere rotation via this mechanism and, unavoidably, some mixedness will be introduced in the target state. In contrast, in [2] the dynamics were fully unitary to leading order in perturbation theory. However, we will show that the mixedness introduced in the stationary qubit is always small as compared to the magnitude of the rotations that we can obtain on the target's Bloch sphere vector. Moreover, we will show that it is indeed advantageous to consider regimes where the probe's trajectory is relativistic in order to more efficiently manipulate the target's qubit.

First order contributions to the target's time evolution cannot be influenced by the interaction of the field and the probe: At first order in perturbation theory we will only have contributions to the target dynamics which are proportional to λ_B, and thus these effects are only dependent on the initial state of the field and the target. The leading order contributions to the remote control of the target have to be proportional to $\lambda_A\lambda_B$.