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Holographic quantum phase transitions and interacting bulk scalars

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ABSTRACT

We consider a system of two massive, mutually interacting probe real scalar fields, in zero temperature holographic backgrounds. The system does not have any continuous symmetry. For a suitable range of the interaction parameters adhering to the interaction potential between the bulk scalars, we have shown that as one turns on the source for one scalar field, the system may go through a second order quantum critical phase transition across which the second scalar field forms a condensate. We have looked at the resulting phase diagram and numerically computed the condensate. We have also investigated our system in two different backgrounds: AdS_4 and AdS soliton, and got similar phase structure.

1. Introduction

Holography [1] or gauge/gravity duality enables us to understand phases of strongly coupled gauge theory from a string theory/gravity calculation. This philosophy has been used in the past decade to model various kinds of field theoretic phase transitions in asymptotically anti-de Sitter space (AdS). For example it was suggested in [2], that the black hole horizons could exhibit spontaneous breaking of an Abelian gauge symmetry if gravity were coupled to matter Lagrangian including a charged scalar (EYMH) that condenses near the horizon. This could be thought of as a superfluid/superconducting like transition in the dual gauge theory [3]. Such condensed matter inspired systems have been studied intensively over the last few years [4–6]. Similarly, EYMH like systems with two or more bulk fields have been studied in [7–17].

To understand the spectrum of possible second order holographic phase transitions, we look at simplistic models which do not even have a local gauge symmetry (conserved particle number in the dual boundary theory). Many condensed matter systems including classic examples like Ising models, have phase transitions which are not necessarily due to breaking of global or local continuous symmetries. In this context the first thing to try is to look at a Lagrangian consisting of a single scalar field. However, such examples are limited. One known example is a scalar field in a near extremal black hole background whose mass is close to BF bound [18]. Another is by turning on double trace deformations [19]. In this regard we study a generic system with two massive, mutually interacting, real scalar fields in zero temperature global AdS_4 space–time (and also in AdS soliton background) and we establish that such system goes through an interesting quantum critical phase transition. For the system we proceed with turning on the source for one scalar field (which may be thought as an impurity density) and look at the condensation of another field. We find that for suitable attractive interactions between the two bulk scalar fields one may obtain a condensate for the latter field through a second order phase transition.

2. Model

In order to describe our model we consider a bulk gravitational action with two mutually interacting and self-gravitating, real scalar fields in asymptotically AdS_d spacetime. The form of bulk gravitational action may be written down as,

$$S = \frac{1}{2\kappa^2}\int dx^d\sqrt{-g}(\mathcal{L}_G) + \int dx^d\sqrt{-g}(\mathcal{L}_M)$$
$$\mathcal{L}_M = -(\nabla_\mu\psi_1)^2 - (\nabla_\mu\psi_2)^2 - m_1^2\psi_1^2 - m_2^2\psi_2^2 - \left(\frac{\alpha_1}{2}\psi_1^4 + \frac{\alpha_2}{2}\psi_2^4\right) + \beta\left(\psi_1^2\psi_2^2\right), \quad (1)$$

where, $\kappa^2 = 8\pi G$ is related to the gravitational constant in the bulk. This model has two Z_2 symmetries corresponding to $\psi_1 \to -\psi_1$ and $\psi_2 \to -\psi_2$. In principle one may choose a less symmetric model to realize the same phase transition we are looking at. We

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will be working in the probe limit ($\kappa \to 0$) for which we can neglect the back-reaction of the bulk scalar fields on the background geometry. In this limit, by rescaling the equations of the motion for the bulk scalar fields one may argue that phase diagram for the model depends on the ratio of parameters $\frac{\alpha_2}{\alpha_1}, \frac{\beta}{\alpha_1}$. We will further assume that, $\alpha_1, \alpha_2 > 0$. Also the condition, $\beta < \sqrt{\alpha_1 \alpha_2}$ must be satisfied for the boundedness of the interaction potential.

Near the boundary the space time looks like AdS, and consequently we can expand a scalar field as a leading source (J) and a vacuum expectation value term ($\langle \mathcal{O} \rangle$). We further assume that $m_1^2 = m_2^2 < 0$. Now when we turn on the source J_1 it generates a condensate $\psi_1(x)$. The effective mass of ψ_2 is given by,

$$m_{2,eff}^2(x) = m_2^2 - \beta \psi_1^2(x) \tag{2}$$

For $\beta > 0$, as we increase J_1 beyond a certain critical value J_1^c, the effective mass of ψ_2 decreases and a condensation becomes possible for the second bulk scalar field (ψ_2). Alternatively one may say that the formation of a zero mode of $\psi_2 = \psi_2^0(x)$ occurs exactly at $J_1 = J_1^c$. For the bulk scalar fields it may be noted that as $J_1 \to \infty$, ψ_1^2 approaches the attractor value $-\frac{2m_1^2}{\alpha_1}$. To have an instability the limiting value of $m_{2,eff}^2$ for the scalar field ψ_2 must be less than the BF bound. Hence the condition for instability may be written down as,

$$-\beta \frac{2m_1^2}{\alpha_1} > -m_{bf}^2 + m_2^2. \tag{3}$$

For $d = 4$ and $m_1^2 = m_2^2 = -2$ we have,

$$\frac{\beta}{\alpha_1} > \frac{1}{16}. \tag{4}$$

Thus to achieve a possible condensation of the second bulk scalar field (ψ_2), the parameters (β, α_1) must satisfy the bound (4) in $(3+1)$ dimensions.

Secondly, assuming a small condensate $\psi_2 = \epsilon \psi_2^0(x)$ and $J_1 = J_1^c + \delta J_1$, it can be estimated that, $\epsilon \sim O(\delta J_1^{\frac{1}{2}})$. One can also estimate the free energy of the new phase to be negative ($\sim -\epsilon^4$) [20,21], which is consistent with a second order phase transition. It may also be noted that unlike the holographic superfluid [18] case, the instability of ψ_2 can be dynamical in nature. Thus our model might have implications for studying the quenching dynamics described in [22].

3. Global AdS background

We begin with describing the equations of motion for the bulk scalar fields in the $(3+1)$ dimensional global AdS background. In the probe limit one may write down the global AdS space metric in $(3+1)$ dimensions as,

$$ds^2 = \frac{L^2}{\cos^2 x}\left(-dt^2 + dx^2 + \sin^2 x \, d\Omega^2\right), \tag{5}$$

where, $d\Omega^2$ is the standard metric on the round unit two-sphere. The ranges of the coordinates are $-\infty < t < \infty$ and $0 \le x < \pi/2$. We also consider the following ansatz for the two bulk scalars as,

$$\psi_1(x^\mu) = \psi_1(x), \qquad \psi_2(x^\mu) = \psi_2(x). \tag{6}$$

Now using the metric ansatz (5) and the ansatz for the bulk scalar fields (6), one may write down the independent equations of motion for the bulk scalar fields (ψ_1, ψ_2) in the global AdS space background as,

$$\psi_1''(x) = -\frac{4}{\sin 2x}\psi_1'(x) + m_1^2 \sec(x)^2 \psi_1(x) + \sec(x)^2\left(\alpha_1 \psi_1(x)^2 - \beta \psi_2(x)^2\right)\psi_1(x), \tag{7}$$

$$\psi_2''(x) = -\frac{4}{\sin 2x}\psi_2'(x) + m_2^2 \sec(x)^2 \psi_2(x) + \sec(x)^2\left(\alpha_2 \psi_2(x)^2 - \beta \psi_1(x)^2\right)\psi_2(x), \tag{8}$$

here the prime denotes derivative with respect to coordinate x. The asymptotic form of the functions $\Theta = \{\psi_1(x), \psi_2(x)\}$ near the AdS boundary $x \to \pi/2$ may be written as,

$$\psi_1(x) = J_1 \bar{x}^{\Delta_-} + \langle \mathcal{O}_1 \rangle \bar{x}^{\Delta_+} + \cdots,$$
$$\psi_2(x) = J_2 \bar{x}^{\Delta_-} + \langle \mathcal{O}_2 \rangle \bar{x}^{\Delta_+} + \cdots, \tag{9}$$

where, $\bar{x} = (x - \pi/2)$ and $\Delta_\pm = (3 \pm \sqrt{9 + 4m^2})/2$. Here the dots represent the higher order terms in the powers of $\bar{x}$. Now if one takes the masses of scalar fields as $m_1^2 = m_2^2 = -2$, then from the asymptotic forms of the fields given in Eq. (9) we observe that one can have dual boundary CFT operators of scaling dimension one (J_1, J_2) acting as source terms for the bulk scalar fields. Also we have dual boundary CFT operators of scaling dimension two ($\langle \mathcal{O}_1 \rangle, \langle \mathcal{O}_2 \rangle$) acting as vacuum expectation values for the bulk scalar fields.

3.1. Numerical results

We attempt to numerically solve the equations of motion via shooting method. Here we require that the functions corresponding to the bulk scalar fields, $\Theta = \{\psi_1(x), \psi_2(x)\}$ must be regular at the horizon ($x = 0$). This implies that all the functions must admit finite values and a taylor series expansion near the boundary at ($x = 0$) as,

$$\Theta(x) = \Theta(0) + \Theta'(0)x + \cdots \tag{10}$$

Analyzing the equations of motion and the expansions of the functions near ($x = 0$), it may be clearly identified that one must have two independent parameters at the horizon ($x = 0$), namely $\psi_1(0)$ and $\psi_2(0)$. Out of these two we will use one of them as shooting parameter to get the boundary condition for the sources as $\{J_1 \neq 0, J_2 = 0\}$. The remaining quantities like $\langle \mathcal{O}_1 \rangle$ and $\langle \mathcal{O}_2 \rangle$ may be obtained by reading off the corresponding coefficients in the asymptotic forms given in Eq. (9) for the bulk scalar fields. The boundary condition on the source terms described above implies that for the numerical analysis we only turn on the source J_1 for the first scalar field ψ_1 and look for the vacuum expectation value $\langle \mathcal{O}_2 \rangle$ to arise spontaneously for the second scalar field ψ_2.

For the masses $m_1^2 = m_2^2 = -2$ of the bulk scalar fields, in Figs. 1, 2, 3 we plot the vacuum expectation value $\langle \mathcal{O}_2 \rangle$ with respect to the source J_1 for varying values of the parameters β, α_1 and α_2.

From Fig. 1 we observe that vacuum expectation value $\langle \mathcal{O}_2 \rangle$ exists for values of the source J_1 above a certain critical value $J_{1c} = 28.896$ for varying values of α_2. Here the values of the parameters α_1 and β are kept to be a fixed constant. From Fig. 2 we observe that critical value of J_1 increases for increasing values of α_1 where the values of the parameters α_2 and β are kept to be a fixed constant. Similarly from Fig. 3 we observe that critical value of J_1 decreases for increasing values of β where the values of the parameters α_1 and α_2 are kept to be a fixed constant. The most remarkable observation that can be easily interpreted from the figures is that, when we turn on the source J_1 for the scalar field ψ_1 the second scalar field ψ_2 spontaneously acquires a vacuum expectation value above certain critical value of the source J_1. This implies that when one scalar is turned on to act as the source then