Physics Letters B 739 (2014) 196–200

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Rotating black holes can have short bristles

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ARTICLE INFO

Article history:
Received 17 September 2014
Received in revised form 17 October 2014
Accepted 26 October 2014
Available online 30 October 2014
Editor: B. Grinstein

ABSTRACT

The elegant 'no short hair' theorem states that, if a spherically-symmetric static black hole has hair, then this hair must extend beyond 3/2 the horizon radius. In the present paper we provide evidence for the failure of this theorem beyond the regime of spherically-symmetric static black holes. In particular, we show that rotating black holes can support extremely short-range stationary scalar configurations (linearized scalar 'clouds') in their exterior regions. To that end, we solve analytically the Klein–Gordon–Kerr–Newman wave equation for a *linearized* massive scalar field in the regime of large scalar masses.

1. Introduction

Within the framework of classical general relativity, the black-hole horizon acts as a one-way membrane which irreversibly absorbs matter fields and radiation. This remarkable property of the black-hole horizon suggests, in particular, that static matter configurations cannot be supported in the spacetime region outside the black-hole horizon. This expectation is nicely summarized in Wheeler's famous dictum "a black hole has no hair" [1,2], which suggests that the spacetime geometries of all asymptotically flat stationary black holes are uniquely described by the three-parameter family [3] of the Kerr–Newman electrovacuum solution [4–6].

The 'no-hair' conjecture [1,2] has attracted much attention over the years from both physicists and mathematicians. Early investigations of the conjecture have ruled out the existence of static hairy black-hole configurations made of scalar fields [7], spinor fields [8], and massive vector fields [9]. However, the early 90s have witnessed the discovery of a variety of regular [10] hairy black-hole configurations, the first of which were the 'colored' black holes which are solutions of the coupled Einstein–Yang–Mills equations [11]. It has soon been realized that many non-linear matter fields [12], when coupled to the Einstein field equations, can lead to the formation of hairy black-hole configurations [13–23].

The validity of the original no-hair conjecture [1,2] has become highly doubtful since the discovery of these non-linear [11,13–23] hairy black-hole configurations [24]. The current situation naturally gives rise to the following question: Is it possible to formulate a more modest (and robust) alternative to the original no hair conjecture?

A very intriguing attempt to reveal the generic characteristics of hairy black-hole configurations was made in [25]: A '*no short hair*' theorem was proved, according to which static spherically-symmetric black holes cannot support short hair. In particular, it was shown in [25] that, in all Einstein-matter theories in which static hairy black-hole configurations have been discovered, the effective length of the outside hair is bounded from below by [26]

$$r_{\text{hair}} > \frac{3}{2} r_{\text{H}}, \tag{1}$$

where r_{H} is the horizon-radius of the black hole. This 'no short hair' theorem was suggested [25] as an alternative to the original [1,2] 'no hair' conjecture.

It is worth emphasizing that the formal proof of the lower bound (1) provided in [25] is restricted to the static sector of spherically-symmetric black holes. Nevertheless, it was conjectured [25] that the 'no short hair' bound (1) can be generalized in the form

$$r_{\text{hair}} > \frac{3}{2}\sqrt{\frac{A_{\text{H}}}{4\pi}}; \quad A_{\text{H}} \equiv \text{horizon area} \tag{2}$$

to include the cases of non-spherically-symmetric stationary hairy black-hole configurations.

The main goal of the present paper is to test the validity of the 'no short hair' conjecture beyond the regime of spherically symmetric static black holes. In particular, we shall explore here the physical properties of non-spherically-symmetric rotating black holes coupled to linearized stationary (rather than static) scalar

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http://dx.doi.org/10.1016/j.physletb.2014.10.062

matter configurations. (It should be emphasized that the scalar fields we consider have a time dependence of the form $e^{-i\omega t}$ [see Eq. (10) below]. However, physical quantities, like the energy–momentum tensor itself, are time-*independent*.)

2. Composed black-hole–scalar-field configurations

While early no hair theorems have shown that asymptotically flat black holes cannot support regular static scalar configurations in their exterior regions [7], they have not ruled out the existence of non-static composed black-hole–scalar-field configurations. In fact, it has recently [27] been demonstrated that rotating black holes can support linearized stationary scalar configurations (scalar 'clouds' [28,29]) in their exterior regions. Since non-linear (self-interaction) effects tend to stabilize the outside hair [25,30], we conjectured in [27] the existence of rotating black hole solutions endowed with genuine non-static scalar hair. These non-static hairy black-hole–scalar-field configurations are the non-linear counterparts of the linear scalar clouds studied analytically in [27]. In a very interesting letter, Herdeiro and Radu [31] have recently solved numerically the non-linear coupled Einstein-scalar equations, and confirmed the existence of these non-static hairy black-hole configurations.

The composed black-hole–scalar-field configurations [32] explored in [27,31] are intimately related to the intriguing phenomenon of superradiant scattering of bosonic fields in rotating black-hole spacetimes [33–36]. In particular, the linearized stationary scalar configurations studied in [27,31] are characterized by orbital frequencies which are integer multiples of the central black-hole angular frequency [37]:

$$\omega_{\text{field}} = m\Omega_{\text{H}} \quad \text{with } m = 1, 2, 3, \ldots . \tag{3}$$

It is well-established [33–36] that the energy flux of the field into the central spinning black hole vanishes for bosonic modes which satisfy the relation (3). In this case, the bosonic field is not swallowed by the central black hole. This suggests that stationary bosonic configurations which are in resonance with the central spinning black hole (that is, bosonic fields with orbital frequencies $\omega_{\text{field}} = m\Omega_{\text{H}}$) may survive in the spacetime region exterior to the black-hole horizon.

In order to have genuine stationary (non-decaying) field configurations around the central black hole, one should also prevent the field from escaping to infinity. A natural confinement mechanism is provided by the gravitational attraction between the massive field and the central black hole. In particular, for a scalar field of mass μ, low frequency field modes in the regime [38]

$$\omega^2 < \mu^2 \tag{4}$$

are confined to the vicinity of the central black hole.

As discussed above, the main goal of the present paper is to test the validity of the 'no short hair' conjecture (1) [25] beyond the regime of spherically-symmetric static black holes. To that end, we shall analyze the physical properties of the non-static (rotating) black-hole–scalar-field configurations [27,31] in the eikonal regime

$$M\mu \gg 1, \tag{5}$$

where M is the mass of the central spinning black hole.

3. Description of the system

The physical system we consider consists of a massive scalar field Ψ linearly coupled [39] to an extremal Kerr–Newman black hole of mass M, angular-momentum per unit mass a, and electric charge Q. In Boyer–Lindquist coordinates (t, r, θ, ϕ) the spacetime metric is given by [4–6]

$$ds^2 = -\frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\phi\right)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\left[adt - \left(r^2 + a^2\right)d\phi\right]^2 \tag{6}$$

where $\Delta \equiv r^2 - 2Mr + a^2 + Q^2$ and $\rho \equiv r^2 + a^2\cos^2\theta$. The extremality condition implies that the degenerate horizon of the black hole is located at

$$r_{\text{H}} = M = \sqrt{a^2 + Q^2}. \tag{7}$$

The angular velocity of the black hole is given by [4–6]

$$\Omega_{\text{H}} = \frac{a}{M^2 + a^2}. \tag{8}$$

The dynamics of the linearized massive scalar field Ψ in the Kerr–Newman black-hole spacetime is governed by the Klein–Gordon (Teukolsky) wave equation

$$\left(\nabla^\nu\nabla_\nu - \mu^2\right)\Psi = 0. \tag{9}$$

It proves useful to use the ansatz [40]

$$\Psi(t, r, \theta, \phi) = \int \sum_{l,m} e^{im\phi} S_{lm}(\theta; s\epsilon) R_{lm}(r; s, \mu, \omega) e^{-i\omega t} d\omega \tag{10}$$

for the scalar wave field in (9), where

$$s \equiv \frac{a}{M} \tag{11}$$

is the dimensionless angular-momentum (spin) of the black hole, and

$$\epsilon \equiv M\sqrt{\mu^2 - \omega^2}. \tag{12}$$

The angular equation for $S_{lm}(\theta; s\epsilon)$, which is obtained from the substitution of (10) into (9), is given by [41–46]

$$\frac{1}{\sin\theta}\frac{d}{\theta}\left(\sin\theta\frac{dS_{lm}}{d\theta}\right) + \left[K_{lm} + (s\epsilon)^2\sin^2\theta - \frac{m^2}{\sin^2\theta}\right]S_{lm} = 0. \tag{13}$$

This angular equation is supplemented by the requirement that the angular functions $S_{lm}(\theta; s\epsilon)$ [47] be regular at the poles $\theta = 0$ and $\theta = \pi$. These boundary conditions single out the discrete set of angular eigenvalues $\{K_{lm}(s\epsilon)\}$ with $l \geq |m|$ [45]. We shall henceforth consider equatorial scalar modes in the eikonal regime

$$l = m \gg 1 \quad \text{and} \quad s\epsilon \gg 1, \tag{14}$$

in which case the angular eigenvalues are given by [48,49]

$$K_{mm}(s\epsilon) = m^2 - (s\epsilon)^2 + O(m). \tag{15}$$

The radial equation for R_{lm}, which is obtained from the substitution of (10) into (9), is given by [41,42]

$$\Delta\frac{d}{dr}\left(\Delta\frac{dR_{lm}}{dr}\right) + \left[\left[\left(r^2 + a^2\right)\omega - ma\right]^2 + \Delta\left[2ma\omega - \mu^2\left(r^2 + a^2\right) - K_{lm}\right]\right]R_{lm} = 0. \tag{16}$$

Note that the radial equation (16) for R_{lm} is coupled to the angular equation (13) for S_{lm} through the angular eigenvalues $\{K_{lm}(s\epsilon)\}$ [50].