Physics Letters B 739 (2014) 387–390

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Level-rank duality in Chern–Simons theory from a non-supersymmetric brane configuration

Adi Armoni, Edwin Ireson *

Department of Physics, Swansea University Singleton Park, Swansea, SA2 8PP, UK

ARTICLE INFO

Article history:
Received 22 August 2014
Accepted 10 November 2014
Available online 13 November 2014
Editor: L. Alvarez-Gaumé

ABSTRACT

We derive level-rank duality in pure Chern–Simons gauge theories from a non-supersymmetric Seiberg duality by using a non-supersymmetric brane configuration in type IIB string theory. The brane configuration consists of fivebranes, N D3 antibranes and an O3 plane. By swapping the fivebranes we derive a 3d non-supersymmetric Seiberg duality. After level shifts from loop effects, this identifies the IR of $Sp(2N)_{2k-2N+2}$ and $Sp(2k-2N+2)_{-2N}$ pure Chern–Simons theories, which is a level-rank pair. We also derive level-rank duality in a Chern–Simons theory based on a unitary group.

1. Introduction

Level-rank duality between two Chern–Simons theories has been a well known relation for many years [1]. It has recently been found that it is closely related to 3d Seiberg duality [2–4,6,7]. The 3d Seiberg duality had been motivated by a realization of the gauge theory on a type IIB brane configuration and the fivebrane swapping procedure [3,4,8].

So far the study of 3d Seiberg duality for Yang–Mills Chern–Simons theories was restricted to supersymmetric theories with various degree of supersymmetry (the case with minimum supersymmetry was studied in [6]). Breaking supersymmetry is hard and typically leads to instabilities. Moreover, in the absence of holomorphicity it is difficult to prove or even motivate the duality. The case of level-rank duality in pure Chern–Simons theory is exceptional in that respect, since the duality is known to hold in the absence of supersymmetry.

In this paper we consider non-supersymmetric brane configurations that consist of anti-branes and an orientifold plane, also known as Sugimoto model [9]. While this class of brane configurations breaks supersymmetry explicitly, one may hope that it preserves properties such as S-duality [10,11] and Seiberg duality [12,13]. In particular, in this paper we consider a certain brane configuration that leads to a non-supersymmetric Seiberg duality. We show that in the IR this duality becomes the celebrated level-rank duality between two Chern–Simons theories based on Sp gauge group

$$Sp(2N)_{2k-2N+2} \sim Sp(2k-2N+2)_{-2N}. \qquad (1)$$

In addition, we also comment on non-supersymmetric Seiberg duality and level-rank duality between theories based on a unitary gauge group. In this case we use a type 0B brane configuration to support our claim.

The organization of the paper is as follows: in Section 2 we present the brane setup and propose a non-supersymmetric Seiberg duality. In Section 3 we study the dynamics of the gauge theory. In Section 4 we consider the case of a unitary group. Finally, in Section 5 we discuss our results.

2. Brane configuration and duality

We study brane configuration similar to those that give rise to 3d $\mathcal{N}=2$ Yang–Mills Chern–Simons theories [4]. The brane configuration consists of

- an NS5 brane along the 012345 directions,
- an $O3$ plane along the 0126 directions and N *anti* D3 branes (and their mirrors) along the 012|6| directions,
- a tilted $(1,2k)$ fivebrane along 012(37)89 directions. The latter fivebrane is a bound state of an NS5 brane and $2k$ *anti* D5 branes, it is tilted in the (37) plane by an angle θ such that $\tan(\theta) = -2g_s k$. With this choice of angle, we would preserve $\mathcal{N}=2$ supersymmetry, had we not had an orientifold. The minus sign reflects the presence of anti-branes in the construction.

* Corresponding author.
E-mail addresses: a.armoni@swan.ac.uk (A. Armoni), pyireson@swansea.ac.uk (E. Ireson).

http://dx.doi.org/10.1016/j.physletb.2014.11.013
0370-2693/

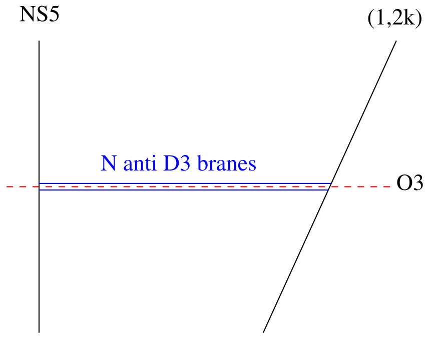

Fig. 1. The electric theory. It is a non-supersymmetric $Sp(2N)$ Yang–Mills theory with a level $2k$ Chern–Simons term. In this plot $N=1$.

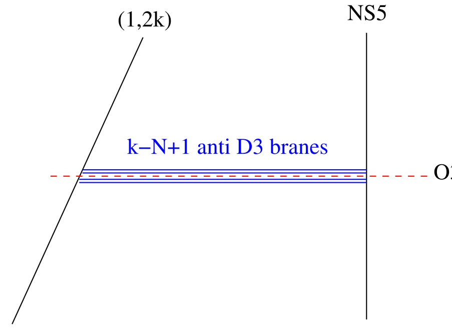

Fig. 2. The magnetic theory. It is a non-supersymmetric $Sp(2k-2N+2)$ Yang–Mills theory with a level $-2k$ Chern–Simons term. In this plot $k-N+1=2$.

The orientifold plane is of $O3^+$ type in between the fivebranes. When it crosses the fivebranes it becomes $O3^-$. The brane configuration is depicted in Fig. 1. We refer to it as the "electric" theory.

Despite of the fact that the brane configuration breaks supersymmetry explicitly, it is stable. As we shall see, there is an attractive potential between the $O3$ plane and the anti $D3$ branes that ensures that the branes will not move away.[1]

The gauge theory on the electric brane configuration is an $Sp(2N)$ gauge theory, with two Majorana fermions λ and ψ that transform in the two-index antisymmetric representation and an additional real scalar C that transforms in the two-index symmetric representation of the gauge group. Changing the fermionic representation breaks supersymmetry explicitly, this is a consequence of taking anti-branes with an orientifold, as in [10–12].

The Lagrangian of the theory is given by

$$\begin{aligned}\mathcal{L} = \frac{1}{g^2}\,\mathrm{Tr}\Big\{&-\frac{1}{2}F_{\mu\nu}^2 + (D_\mu C)^2 + D^2 + i\bar{\lambda}\not{D}\lambda + i\bar{\psi}\not{D}\psi \\ &+ i[\bar{\psi},\lambda]C - i[\bar{\lambda},\psi]C\Big\} \\ &+ 2k\,\mathrm{Tr}\Big\{\epsilon^{\mu\nu\rho}\Big(A_\mu\partial_\nu A_\rho - \frac{2}{3}iA_\mu A_\nu A_\rho\Big) - \bar{\lambda}\lambda + \bar{\psi}\psi \\ &+ 2CD\Big\},\end{aligned} \tag{2}$$

with g the Yang–Mills coupling and $2k$ the Chern–Simons level.

Since we are mixing representations of the gauge group, it is easier to work with fields in the double-index notation. Instead of expanding the fields in the relevant bases of the Lie algebra, we keep them to be colour matrices, with (anti-)symmetrised indices. Traces are then taken over these colour indices to produce terms in the Lagrangian.

According to the proposal of Elitzur, Giveon and Kutasov [8], by swapping the fivebranes we arrive at the magnetic Seiberg dual of the electric theory. When we swap fivebranes in the presence of an $O3$ brane a single $D3$ antibrane (and its image) is created [3]. This phenomenon is often attributed to the conservation of a particular quantity, calculated from various brane charges, called Linking Number [5]. Together with the k antibranes and the N branes the magnetic gauge group becomes $Sp(2k-2N+2)$. The magnetic theory is depicted in Fig. 2.

Thus we propose that two non-supersymmetric gauge theories form a Seiberg dual pair, namely that the two theories become equivalent in the far IR, based on string theory dynamics. These two theories are Yang–Mills Chern–Simons, both admit a level $|2k|$ at the classical level. The electric theory is based on the gauge group $Sp(2N)$ and the magnetic on $Sp(2k-2N+2)$. In the next section we will argue that in the far IR the electric and magnetic theories flow to a pair of pure Chern–Simons theories that are equivalent to each other by virtue of level-rank duality.

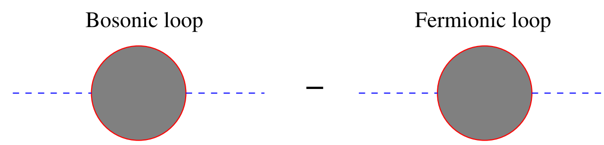

Fig. 3. Perturbative contributions to the scalar mass.

3. Gauge dynamics

Let us consider the gauge dynamics of the electric and magnetic theories.

We start with the electric theory. Since the theory is non-supersymmetric the scalar is expected to acquire a Coleman–Weinberg potential. Let us consider the one-loop contribution to the scalar mass, as depicted in Fig. 3.

The contributions from bosonic one-loop diagrams are

$$+g^2(2N+2)\int\frac{d^3p}{(2\pi)^3}\frac{1}{p^2} = cg^2(2N+2)\Lambda, \tag{3}$$

with g the gauge coupling, Λ the UV cut-off and c some numerical constant. The contributions from fermionic one-loop diagrams are

$$-g^2(2N-2)\int\frac{d^3p}{(2\pi)^3}\frac{1}{p^2} = -cg^2(2N-2)\Lambda. \tag{4}$$

The planar diagram contributions (dependent on N) cancel, but the difference between the contributions of the non-planar diagrams, due to the difference in representations between bosonic and fermionic degrees of freedom, do not cancel. The generated mass for the scalar is therefore

$$M^2 = cg^2\big\{(2N+2)-(2N-2)\big\}\Lambda = 4cg^2\Lambda. \tag{5}$$

Thus, a positive mass^2 proportional to the UV cut-off is generated for the scalar and therefore it decouples from the dynamics of the low-energy field theory.

In the brane side, a potential of the form M^2C^2 is interpreted as an attractive potential between the orientifold $O3^+$ plane and the D3 antibranes. It guarantees the stability of the brane configuration.[2]

[1] An instability develops in the case where the orientifold type between the fivebranes is $O3^-$. Hence, a statement similar to ours, for an SO gauge group, is not obtainable immediately by our method.

[2] In the case of $O3^-$, the gauge theory is $SO(2N)$ and the scalar develops a tachyonic mass. The brane configuration, hence, becomes unstable.