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Sideband mixing in intense laser backgrounds

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ABSTRACT

The electron propagator in a laser background has been shown to be made up of a series of sideband poles. In this paper we study this decomposition by analysing the impact on the sidebands of the residual gauge freedom in the Volkov solution. We show that these gauge transformations do not alter the location of the poles although the wave function renormalisation is gauge dependent. Our identification of the propagator from the diagonal part of the two-point function in the laser background is maintained but we show that the sideband structures mix under residual gauge transformations.

1. Introduction

The recent rapid progress in laser technologies offers a timely testing ground for quantum field theory techniques associated with non-trivial backgrounds [1]. In this paper we are going to study charge propagation in such a background. A novel feature of a propagating charge in a laser is that it is indistinguishable from a charge which absorbs a given number of laser photons and also emits the same number of photons degenerate with the laser. Such laser induced degeneracies have a close parallel with the soft and collinear degeneracies associated with the infrared regime in both QED and QCD [2–5] while the induced mass effects in a laser background should help to refine our understanding of the current versus constituent mass distinction in QCD [6]. Note that building upon experience with QED in a vacuum, the two-point function in a laser background is usually referred to as the propagator. Through interactions with the background this two-point function includes diagrams where the initial and final momenta of the matter field are not the same. The approach we are taking projects out the diagonal part of this two-point function and does not include momentum changing vertex type effects. This is what we mean by the propagator in the rest of what follows.

In QED we usually expand around the free theory but in a laser background we can take the simplest description, the interacting Volkov solution [7,8], as our starting point. This solution is much richer than in the normal perturbative vacuum and, as we will summarise below, alters the propagator which becomes a sum of so-called sideband poles [9–11]. As this is not a free theory, the matter field is not gauge invariant and in this paper we address the effects of local gauge transformations on this solution and the propagator, see also [12].

We recall [7] that the solutions of a scalar field in a plane wave background are distorted. For a linearly polarised background where the vector potential is given by

$$A_\mu(x) = a_\mu \cos(k\cdot x), \tag{1}$$

and where the constant amplitude a_μ is space-like and taking the null vector k^μ to be spatially aligned along the laser direction, the matter field is described by

$$\phi_{\mathrm{V}}(x) = \int \frac{\bar{d}^3 p}{2E_p^*}\big(D(x,p)a_{\mathrm{V}}(p) + D(x,-p)b_{\mathrm{V}}^\dagger(p)\big), \tag{2}$$

where

$$D(x,p) = \mathrm{e}^{-ip\cdot x}\mathrm{e}^{i(eu\sin(k\cdot x)+e^2 v\sin(2k\cdot x))}, \tag{3}$$

and

$$u = -\frac{p\cdot a}{p\cdot k} \quad \text{and} \quad v = \frac{a^2}{8p\cdot k}. \tag{4}$$

In this expression the momentum p appearing in the propagator is on-shell at $m_\star$ where the laser shifted mass [9,13–16] is

$$p^2 = {m_\star}^2 = m^2 - \tfrac{1}{2}e^2 a^2. \tag{5}$$

In this paper we do not explicitly distinguish between on-shell and off-shell momenta as it has no impact on our discussion of gauge dependence. See [17] for a fuller discussion.

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We recall further that (2) may be written as a sum over modes

$$\phi_V(x) = \sum_n \phi_n(x), \tag{6}$$

where

$$\phi_n(x) = \int \frac{d^3p}{2E_p^*}\Big(e^{-iep\cdot x}e^{ink\cdot x}J_n\big(eu, e^2v\big)a_V(p) + e^{iep\cdot x}e^{ink\cdot x}J_n\big(eu, -e^2v\big)b_V^\dagger(p)\Big), \tag{7}$$

and the generalised Bessel function, $J_n(eu, e^2v)$, is defined in terms of Bessel functions via

$$J_n\big(eu, e^2v\big) = \sum_r J_{n-2r}(eu)J_r\big(e^2v\big). \tag{8}$$

The Volkov propagator contains not just the standard pole $i/(p^2 - {m_*}^2)$ familiar from perturbation theory but also infinitely many sideband poles of the form $i/((p+nk)^2 - {m_*}^2)$ where n is any integer [13,18–21]. As we have previously identified [17], the propagator is not given by the two-point function of the full Volkov field but is identified as the diagonal part of the two-point function in the vacuum $|0\rangle_V$ picked out by the Volkov annihilation operators:

$$iD_V(x-y) = \sum_n {}_V\langle 0|T\phi_n(x)\phi_n^\dagger(y)|0\rangle_V. \tag{9}$$

This is to ensure that the propagator represents processes where there is a fixed momentum flow through the matter field. This can also be understood [17] in terms of degenerate processes extending the Lee–Nauenberg [4] characterisation of the infrared problem [5].

The form of the vector potential chosen here requires that $k\cdot a = 0$ which corresponds from (1) to a Landau like gauge as $\partial_\mu A^\mu = 0$. In [17] the propagator was constructed in this gauge. The mass shift and wave function renormalisations were calculated to all orders in an operator formalism. This was further verified to the first few orders by explicit diagrammatic calculations. Each term in the sum (9) generates a separate, so-called sideband structure:

$$\int d^4x\, e^{-ip\cdot(x-y)}{}_V\langle 0|T\big(\phi_n(x)\phi_n^\dagger(y)\big)|0\rangle_V = \frac{Z_2^{(n)}(u,v)}{(p+nk)^2 - {m_*}^2 + i\epsilon}, \tag{10}$$

showing the common mass shift and distinct wave function renormalisations of the sidebands. For the detailed form of Z_2 see [17]. Here p is off-shell. It has been argued that the central sideband, corresponding to $n = 0$ and produced by the $\phi_0(x)$ mode, may dominate in some regimes [18]. In this paper we want to address the issue of the residual gauge freedom which is opened up by the boundary conditions imposed on the plane wave laser background.

Below we will show that, although the Volkov field transforms with the expected phase shift characteristic of a charged matter field under such a residual gauge transformation, the modes (6) actually mix with each other in a non-trivial manner. This mixing of the modes raises a question about whether the above identification of the propagator (9) is consistent with gauge transformations. We will demonstrate below that the construction of the propagator is robust under such transformations and that the overall effect of gauge transformations may be absorbed into shifts of the wave function renormalisation factors.

We therefore now turn to the gauge freedom in the Volkov formalism, its effects on the various modes of the Volkov field and thus build up the diagonal sum (9). Although our conclusions hold to all orders, we shall, for illustrative purposes, demonstrate them perturbatively.

2. Residual gauge transformations

There is in the Landau gauge fixed solution discussed above a residual gauge freedom as we can make the replacement

$$A_\mu(x) \to A_\mu(x) + \partial_\mu\lambda(x), \tag{11}$$

where $\lambda(x) = \lambda\sin(k\cdot x)$ and λ is a constant. This corresponds to the amplitude shift

$$a_\mu \to a_\mu + \lambda k_\mu, \tag{12}$$

which still preserves our Landau gauge choice due to the null nature of k^μ. We note that this gauge freedom preserves the plane wave character of the background laser potential which is why we restrict to it. Under this transformation we have, from (4),

$$u \to u - \lambda \quad \text{and} \quad v \to v. \tag{13}$$

Similarly the distortion factor transforms as

$$D(x,p) \to e^{-ie\lambda(x)}D(x,p). \tag{14}$$

From (2) we see the phase shift

$$\phi_V(x) \to e^{-ie\lambda(x)}\phi_V(x), \tag{15}$$

as would be expected of a charged matter field under gauge transformations. This is a local gauge transformation and, as the field extends to spatial infinity along the laser direction, the transformation does not vanish asymptotically along the laser. This residual gauge transformation is consistent both with our original Landau gauge condition and the boundary conditions of the Volkov solution.

We now want to analyse the impact of the gauge freedom on the various modes of the Volkov field. As the propagator is constructed from the diagonal sum over the modes (9) it is crucial that we know how they transform. In (7) the generalised Bessel functions, through their dependence on u, are responsible for the gauge dependence of the fields

$$J_n\big(eu, e^2v\big) \to J_n\big(e(u-\lambda), e^2v\big) = \sum_m J_m(e\lambda)J_{n+m}\big(eu, e^2v\big), \tag{16}$$

where we used (13). This means that the Volkov modes mix under such a local gauge transformation as

$$\phi_n(x) \to \sum_s J_s(e\lambda)\phi_{n+s}(x)e^{-isk\cdot x}, \tag{17}$$

with a Bessel function dependent weighting. More complicated mixing would presumably occur if a gauge transformation was used which took us outside of the plane wave Volkov solution. It is useful here to verify that this is consistent with the overall transformation of the Volkov field. From (6) we have

$$\phi_V(x) \to \sum_n\sum_s J_s(e\lambda)\phi_{n+s}(x)e^{-isk\cdot x}. \tag{18}$$

Shifting the label n and using the standard result

$$e^{i\ell\sin(k\cdot x)} = \sum_r e^{irk\cdot x}J_r(\ell), \tag{19}$$

we find that the Volkov field