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Revisiting perfect fluid dark matter: Observational constraints from our galaxy

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ABSTRACT

We revisit certain features of an assumed spherically symmetric perfect fluid dark matter halo in the light of the observed data of our galaxy, the Milky Way (MW). The idea is to apply the Faber–Visser approach of combined observations of rotation curves and lensing to a first post-Newtonian approximation to "measure" the equation of state $\omega(r)$ of the perfect fluid galactic halo. However, for the model considered here, no constraints from lensing are used as it will be sufficient to consider only the rotation curve observations. The lensing mass together with other masses will be just computed using recent data. Since the halo has attractive gravity, we shall impose the constraint that $\omega(r) \geq 0$ for $r \leq R_{\rm MW}$, where $R_{\rm MW} \sim 200$ kpc is the adopted halo radius of our galaxy. The observed circular velocity ℓ $(= 2v_c^2/c_0^2)$ from the flat rotation curve and a crucial adjustable parameter D appearing in the perfect fluid solution then yield different numerical ranges of $\omega(r)$. It is demonstrated that the computed observables such as the rotation curve mass, the lens mass, the post-Newtonian mass of our galaxy compare well with the recent mass data. We also calculate the Faber–Visser χ-factor, which is a measure of pressure content in the dark matter. Our analysis indicates that a range $0 \leq \omega(r) \leq 2.8 \times 10^{-7}$ for the perfect fluid dark matter can reasonably describe the attractive galactic halo. This is a strong constraint indicating a dust-like CDM halo ($\omega \sim 0$) supported also by CMB constraints.

1. Introduction

A few years ago, in Ref. [1], a perfect fluid dark matter model was developed that was shown to have many attractive theoretical aspects. The solution may be thought of as a dark matter induced space–time embedded in a static cosmological Friedmann–Lemaître–Robertson–Walker (FLRW) background.[1] The motivation for developing an isotropic perfect fluid model (we leave open the question of particle identity of dark matter) came from the fact that predictions from such model at stellar and cosmic scales have been observationally well corroborated so far. More recently, Harko and Lobo [2] investigated dark matter as a mixture of two non-interacting perfect fluids, with different four-velocities and thermodynamic parameters. González-Morales and Nuñez [3] compared two different dark matter models: one is a perfect fluid and the other is a scalar field [3]. See also [4].

The model considered here assumes that a spherical dark matter distribution is the only gravitating source. This assumption is of course an oversimplification since, although the bulge is quite spherical and is dominated by old stars, the Milky Way has a strongly flattened stellar distribution. However, we know from the vertical velocity dispersion of stars as a function of distance from the disk plane that the local disk mass density is almost identical to the sum of the densities that can be attributed to stars, gas and stellar remnants. Therefore, there is practically little dark matter hidden in the disk. Hence, to explain the rotation curve measurements, we are forced to assume that dark matter resides in the halo region dominating its mass, is spherically distributed and, if

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[1] The reason for the appearance of static FLRW background around the embedded perfect fluid dark matter is already explained in Ref. [1]. The Einstein field equations are solved with perfect fluid stress tensor in both the cases but we sought a static solution from the start. While working on a local problem (flat rotation curve), the scale factor is usually fixed to $R_0 = 1$ today.

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it is non-baryonic, would not be expected to collapse into a disk-like structure.

Specifically, the hypothesis of dark matter arose because the Newtonian circular velocity $v_c^2 = \frac{GM(r)}{r}$ of circularly moving probe particles caused by the luminous mass distribution $M(r)$ is not supported by observations [5,6]. The circular velocity becomes nearly flat, $v_c^2 \simeq$ constant, at distances far away from the center (halo region), which is possible only if $M(r) \propto r$. Therefore, almost every galaxy is assumed to host a large amount of non-luminous matter, the so-called gravitational dark matter, consisting of unknown particles not included in the particle standard model, forming a halo around the galaxy. Naturally, dark matter is at the core of modern astrophysics. Many well known theoretical models for dark matter exist in the literature, for instance, see [7–29] (the list is by no means exhaustive). Some models that do not hypothesize dark matter appear in [30–38]. Well known density profiles originated in [39–41]. Excellent reviews are to be found in [42–45].

In this paper, we shall revisit the model of perfect fluid dark matter, developed in Ref. [1], in the light of the observed/inferred data of our galaxy. Our analysis would require three ingredients: (i) A method, viz., the Faber–Visser [46] method of combined post-Newtonian measurements of rotation curves and gravitational lensing for measuring the equation of state $\omega(r)$ of the dark matter and determining the rotation curve mass (m_{RC}), the lens mass (m_{Lens}) and the post-Newtonian mass (M_{pN}). However, for the perfect fluid solution we consider here, it suffices to consider only the rotation curve as a constraint, while the lens mass will be a result of computation. (ii) Two observed inputs, viz., the circular velocity ℓ $(= 2v_c^2/c_0^2)$ of probe particles, where c_0 is the speed of light in vacuum, and the radius R_{MW} of our galactic halo. (iii) An observational constraint, viz., the one imposed by the attractive nature of dark matter so that $p/\rho = \omega(r) \geq 0$. The nature is attractive because the very existence of dark matter is speculated from observations of the Doppler shifted light emanating from neutral hydrogen clouds moving on stable circular orbits in the galactic halo [20,32,47,48]. Using these ingredients, we shall analyze how choices of the adjustable parameter D appearing in the dark matter metric lead to different types of scenarios.

The following are our new results. Depending on the values of D, we show: (i) The observable masses m_{RC}, m_{Lens} and M_{pN} compare well with the masses inferred by other independent means. (ii) There could appear an intriguing negative pressure matter sector ($\omega < 0$) beyond the halo radius.[2] (iii) The Faber–Visser χ-factor has values near unity so that pressure contribution to the post-Newtonian mass M_{pN} is negligible. Hence the perfect fluid dark matter resembles dust ($\omega \sim 0$) akin to CDM model. (iv) There is flexibility in the halo radius in the sense that our model can accommodate extended radii. All these imply that, fundamentally, the perfect fluid model stands up to actual observations on mass, equation of state and in addition predicts marginal quintessence matter at asymptotic distances, all within a single formulation.

In Sec. 2, we briefly outline the perfect fluid dark matter and in Sec. 3, we display the Faber–Visser post-Newtonian observables. In Sec. 4, we apply the galaxy inputs to those observables and deduce the most suitable range of D that agrees with the observational constraints from our galaxy. In Sec. 5, we conclude the paper. We take $G = 1$, $c_0 = 1$, unless specifically mentioned.

[2] However, it will be shown later that the $\omega < 0$ matter sector is not exotic in nature. It will also be evident that we can shift the values of D to make $\omega < 0$ matter appear at any finite radius beyond halo radius R_{MW}, but we must take care not to violate the attractive nature $\omega \geq 0$ *inside* R_{MW}.

2. Perfect fluid dark matter

The general static spherically symmetric space–time is represented by the following metric

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1}$$

where the functions $\nu(r)$ and $\lambda(r)$ are the metric potentials. For the perfect fluid, the matter energy–momentum tensor T^{α}_{β} is given by $T^t_t = \rho(r)$, $T^r_r = T^{\theta}_{\theta} = T^{\varphi}_{\varphi} = p(r)$, where $\rho(r)$ is the energy density, $p(r)$ is the isotropic pressure. Considering flat rotation curve as an input, an exact solution of Einstein field equations is derived in [1]:

$$e^{\nu(r)} = B_0 r^{\ell}, \tag{2}$$

$$e^{-\lambda(r)} = \frac{c}{a} + \frac{D}{r^a}, \tag{3}$$

$$a = -\frac{4(1+\ell) - \ell^2}{2+\ell}, \tag{4}$$

$$c = -\frac{4}{2+\ell}, \tag{5}$$

$$\ell = 2v_c^2/c_0^2, \tag{6}$$

where $B_0 > 0$, D are integration constants and v_c is the circular velocity of stable circular hydrogen gas orbits treated as proble particles. The exact energy density and pressure are

$$\rho(r) = \frac{1}{8\pi}\left[\frac{\ell(4-\ell)}{4+4\ell-\ell^2}r^{-2} - \frac{D(6-\ell)(1+\ell)}{2+\ell}r^{\frac{\ell(2-\ell)}{2+\ell}}\right] \tag{7}$$

$$p(r) = \frac{1}{8\pi}\left[\frac{\ell^2}{4+4\ell-\ell^2}r^{-2} + D(1+\ell)r^{\frac{\ell(2-\ell)}{2+\ell}}\right]. \tag{8}$$

The free adjustable parameter D, having the dimension of $(\text{length})^{-2}$, in the solution is extremely sensitive and its value can be decided only by observed physical constraints. In the present case, the constraint is that the galactic fluid be non-exotic and attractive, i.e., the equation of state parameter $\omega(r) = \frac{p(r)}{\rho(r)} \geq 0$ must hold within the halo radius. With this information at hand, an interesting aspect of the solution can be found from Eqs. (7) and (8).

It can be seen that the integrated quantity, call it $M_0 = 4\pi\int_0^r \rho(r)r^2dr$ derived from exact $\rho(r)$ given by Eq. (7), is identical with the Newtonian mass M_{N} derived in Eq. (23) below. One could as well call M_{N} the post-Newtonian counterpart of M_0 since $\rho(r)$ in Eq. (23) is expressed as derivatives of post-Newtonian masses. The question then we ask: What quantity derived from the exact solution differs from its measurable post-Newtonian counterpart? One such quantity is the total mass within a radius r with *pressure contribution*, which is defined by Eqs. (7) and (8)

$$M_{\mathrm{total}}(r) = 4\pi\int_0^r(\rho + 3p)r^2dr = \frac{\ell(2+\ell)r}{4+4\ell-\ell^2} + \frac{2D}{\ell-6}r^{\frac{4+4\ell-\ell^2}{2+\ell}} \tag{9}$$

$$= \frac{\ell r}{2} + \frac{D\ell r^3}{3} - \frac{\ell^2 r}{4} + O(\ell^2). \tag{10}$$

We shall see in the next section that its post-Newtonian counterpart is just $M_{\mathrm{pN}}(r) = \frac{\ell r}{2}$. Hence the theoretical and observable masses in principle differ depending on arbitrary values of D, even when $D = 0$. Therefore, let us proceed to define the post-Newtonian observables.