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Simple mass matrices of neutrinos and quarks consistent with observed mixings and masses

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ABSTRACT

We propose a simple phenomenological model of quarks-leptons mass matrices having fundamentally universal symmetry structure. These mass matrices consist of democratic and semi-democratic mass matrix terms commonly to the neutrino and the quark sectors and have only eight free parameters. We show that this mass matrix model well reproduces all the observed values of the MNS lepton and the CKM quark mixing angles, the neutrino mass squared difference ratio, and quark mass ratios, with an excellent agreement. The model also predicts $\delta_{CP}^{\ell} = -94°$ for the leptonic *CP* violating phase and $\langle m \rangle \simeq 0.0073$ eV for the effective Majorana neutrino mass.

The Cabbibo–Kobayashi–Maskawa (CKM) quark mixing matrix is almost diagonal, whereas the Maki–Nakagawa–Sakata (MNS) lepton mixing matrix [1,2] is almost maximally mixed. The origin of nearly maximally lepton mixing has been investigated from the point of neutrino mass matrix structure using a $\mu - \tau$ symmetry [3] which predicts $\theta_{23} = \pi/4$ and $\theta_{13} = 0$. The recent finding [4–7] of a relatively large θ_{13} and their global best fits [8–10] forces us to consider its origin and model extensions. The rather large θ_{13} also opens the possibility of CP violation in lepton sector. There are theoretical discussions on the CP violating phase. In the previous paper [11], we proposed a simple two parameter complex mass matrix for Majorana neutrinos incorporating CP violating phase, although the model predicted small CP violating effect.

In this paper, we propose new phenomenological complex mass matrices M_ν, M_u, and M_d for Majorana neutrinos and up-type and down-type quarks with incorporating almost maximal leptonic CP violating phase and incorporating the quark mass matrices with similar structure as neutrino mass matrix too. The model of the present paper has only eight free parameters to describe mixings and mass ratios for quarks and neutrinos. The mass matrices are assumed to take the following forms in the base that charged lepton mass matrix M_e is diagonal:

$$M_\nu = k_\nu \left[\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \rho_\nu e^{i\frac{\pi}{4}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & e^{-i\varphi_\nu} & e^{i\varphi_\nu} \\ 0 & e^{i\varphi_\nu} & e^{-i\varphi_\nu} \end{pmatrix} + z_\nu \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \right], \tag{1}$$

$$M_u = k_u P^\dagger \left[\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \rho_u e^{i\frac{\pi}{4}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + z_u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right] P, \tag{2}$$

$$M_d = k_d \left[\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \rho_d e^{i\frac{\pi}{4}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + z_d \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \right]. \tag{3}$$

Here, ρ_f, z_f are real parameters, and φ_ν is a phase parameter which appears in the neutrino mass matrix. P is a diagonal phase matrix defined by

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$$P = \begin{pmatrix} e^{i\phi} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{4}$$

The mass matrices (1)–(3) consist of three terms. The first term is a democratic mass matrix, while the second term is also semi democratic only in the second and third generations. The third term is a small correction term. All these terms are symmetric (up to the small phase ϕ for M_u) and have the universal 2–3 symmetry [3,12] (up to the small z_ν and z_d corrections).

Since we are interested only in the mass ratios and mixings, we neglect the overall constant k_f in the following discussions. Then, this model has only eight free parameters to explain sixteen observables of four up and down quark mass ratios, three quark mixing angles and one CP violation phase, two neutrino mass ratios, three lepton mixing angles and three CP violation phases (one Dirac and two Majorana phases).

From a theoretical point of view, let us give some physical implications on our models. Phenomenological mass matrix model like this are frequently situated as the preliminary step towards the final systematic interpretation of mass matrices like GUT. Let us consider some relation of the above mass matrices with the most predictive GUT model, the minimal SO(10) model which is composed of two Higgs Yukawa couplings with 10 and $\overline{126}$-plets Higgs fields [13,14].

$$\begin{aligned} &M_u = c_{10}M_{10} + c_{126}M_{126}, \quad M_d = M_{10} + M_{126} \\ &M_D = c_{10}M_{10} - 3c_{126}M_{126}, \quad M_e = M_{10} - 3M_{126} \\ &M_L = c_L M_{126}, \quad M_R = c_R M_{126}\,. \end{aligned} \tag{5}$$

Here M_D, M_L, and M_R denote the mass matrices of Dirac neutrino, left-handed Majorana, and right-handed Majorana neutrino, respectively with complex constants c_i. They are all symmetric mass matrices because of the property of 10 and $\overline{126}$-plets representations. From the renormalizability, we may add 120-plet Higgs but it gives antisymmetric matrix. In this case mass matrices are considered as Hermitian with real Yukawa coupling of 10-and $\overline{126}$-plets Higgs and pure imaginary Yukawa coupling in 120-plet Higgs. So it is very important whether the mass matrices are symmetric or not. As is well known, $m_b = m_\tau$ is well valid at $\Lambda_{GUT} \approx 10^{16}$ GeV if we consider the SUSY threshold correction [15]. This indicates that on the (3, 3) component of mass matrices, M_{10} dominates over M_{126}. Of course Eq. (5) works at Λ_{GUT} scale and some deviations occur especially in quark masses and mixings and lepton masses at low energy scale Λ_{EW}. The renormalization group equation (RGE) effect crucially depends on the Higgs field vevs at the intermediate energy scales. It was recently found after elaborate scan that $\Lambda_R = \Lambda_{GUT}$ still remains valid within 2σ [16], where Λ_R is the seesaw scale (M_R scale). This is very important since $\Lambda_R \ll \Lambda_{GUT}$ spoils the gauge coupling unification at Λ_{GUT}. Recently Luo and Xing discussed $\mu - \tau$ symmetry in neutrino mass matrix and its observed small breaking by the RGE from $\Lambda_R = 10^{14}$ GeV [17]. Also, starting from a degenerate neutrino mass at Λ_R, Babu, Ma, and Valle obtained a $\mu - \tau$ symmetry up to phase at Λ_{EW} by RGE [18].

It should be emphasized that RGE has quite different aspects in SUSY or Non-SUSY. In SUSY, for instance, 4-point effective operator of type I seesaw suffers only loop corrections of wave functions of Higgs and leptons (nonrenormalization theorem). Such RGE effects for lepton come mainly from gauge loop due to smallness of Yukawa coupling, which is flavor blind and MNS and mass ratios remains constant. As for quark mass ratios and CKM, they suffer some changes in the third family. As was said, our model in this paper is purely phenomenological but very predictive, and gives some hints on the above mentioned relations between the phenomenological models and GUT.

The Majorana neutrino mass matrix M_ν is diagonalized by unitary matrix U as follows

$$M_\nu = U \begin{pmatrix} m_{\nu 1} & 0 & 0 \\ 0 & m_{\nu 2} & 0 \\ 0 & 0 & m_{\nu 3} \end{pmatrix} U^T. \tag{6}$$

Here $m_{\nu i}$ are neutrino masses. The U is the MNS lepton mixing matrix itself since we assume that the mass matrix for the charged leptons is diagonal in the present model. The MNS lepton mixing matrix U is represented in the standard form,

$$\begin{aligned} U_{std} &= \mathrm{diag}(e^{i\alpha_1^e}, e^{i\alpha_2^e}, e^{i\alpha_3})U \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}^\ell} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}^\ell} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}^\ell} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}^\ell} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}^\ell} & c_{13}c_{23} \end{pmatrix} \\ &\quad \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\beta} & 0 \\ 0 & 0 & e^{i\gamma} \end{pmatrix}. \end{aligned} \tag{7}$$

Here, $e^{i\alpha_i^e}$ comes from the rephasing in the charged lepton fields to make the choice of phase convention, and $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$ with θ_{ij} being lepton mixing angles. The δ_{CP}^ℓ is the Dirac CP violating phase and β and γ are the Majorana CP violating phases.

In the present model, the MNS mixing parameters ($\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, and $\sin^2 2\theta_{13}$) and the neutrino mass squared difference ratio ($R_\nu \equiv \Delta m_{21}^2/\Delta m_{32}^2$) are functions of only three free parameters ρ_ν, φ_ν, and z_ν defined in (1). The observed values of the MNS mixing parameters and R_ν are [19]

$$\begin{aligned} &\sin^2 2\theta_{12} = 0.846 \pm 0.021, \\ &\sin^2 2\theta_{13} = 0.093 \pm 0.008, \\ &\sin^2 2\theta_{23} = 0.999^{+0.001}_{-0.018}, \end{aligned} \tag{8}$$

$$R_\nu \equiv \frac{\Delta m_{21}^2}{\Delta m_{32}^2} = \frac{m_{\nu 2}^2 - m_{\nu 1}^2}{m_{\nu 3}^2 - m_{\nu 2}^2} = (3.09 \pm 0.015) \times 10^{-2}. \tag{9}$$

By fine tuning three parameters ρ_ν, φ_ν, and z_ν, our mass matrix model (1) well reproduces all the observed three MNS mixing angles and the neutrino mass squared difference ratio, (8) and (9), for the case of normal neutrino mass hierarchy if we take values of ρ_ν, φ_ν, and z_ν as

$$\rho_\nu = 4.06, \quad \varphi_\nu = 170.86^\circ, \quad z_\nu = -0.42. \tag{10}$$

In Fig. 1, we draw the contour curves of the observed values in (8) and (9) of the MNS mixing parameters ($\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, and $\sin^2 2\theta_{13}$) and the neutrino mass squared difference ratio R_ν in the (φ_ν, ρ_ν) parameter plane by fixing z_ν. We find that the parameter set around $(\varphi_\nu, \rho_\nu) = (170.86, 4.06)$ and $z_\nu = -0.42$ indicated by a star ($\star$) in Fig. 1 is consistent with all the observed values.

We now present the predictions in the neutrino sector of our model by taking the values for the free parameters (10) as follows:

$$\sin^2 2\theta_{12} = 0.867, \tag{11}$$

$$\sin^2 2\theta_{13} = 0.0901, \tag{12}$$

$$\sin^2 2\theta_{23} = 0.9998, \tag{13}$$

$$R_\nu = 0.0293, \tag{14}$$

$$\delta_{CP}^\ell = -94.4^\circ, \tag{15}$$

which are consistent with the observed values (8) and (9). It should be noted that the present model predict $\delta_{CP}^\ell \simeq -\frac{\pi}{2}$ for the