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A nonabelian particle–vortex duality

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ABSTRACT

We define a nonabelian particle–vortex duality as a 3-dimensional analogue of the usual 2-dimensional worldsheet nonabelian T-duality. The transformation is defined in the presence of a global $SU(2)$ symmetry and, although derived from a string theoretic setting, we formulate it generally. We then apply it to so-called "semilocal strings" in an $SU(2)_G \times U(1)_L$ gauge theory, originally discovered in the context of cosmic string physics.

1. Introduction

Beginning with the remarkable correspondence between the sine-Gordon and massive Thirring models [1], dualities have played a crucial role in the modern understanding of quantum field theories. Indeed, they have been an indispensable tool in the understanding of both strongly coupled systems as well as various nonperturbative problems. This was certainly the case, for instance, for Seiberg and Witten's landmark study of $(3+1)$-dimensional, $\mathcal{N}=2$ supersymmetric gauge theory [2,3], where electric–magnetic duality (a generalized form of the usual electric–magnetic duality of Maxwell electrodynamics) that exchanges particles with monopoles, was essential in fully solving the low energy theory. In that $(3+1)$-dimensional case, even though an explicit path integral transformation exists only for the abelian case, the duality is understood as being essentially non-abelian in the sense of acting on the full non-abelian theory.

One duality which has received considerably less attention occurs in $(2+1)$-dimensional gauge theories and exchanges particles with topological solitons, specifically vortices [4]. One possible reason for the dearth of literature on the subject could be that its utility lies primarily in condensed matter systems which, being usually non-relativistic are much less susceptible to the powerful relativistic methods employed in high energy theory. Another is likely the fact that the duality was generally less well-defined than its $(3+1)$-dimensional counterpart. To the best of our knowledge, *particle–vortex duality* has, until now, only been defined in the context of abelian gauge theories, exhibiting Neilsen–Olesen-like vortices. In [5], this duality was defined as a path integral transformation in a manifestly symmetric way, and embedded into a planar $\mathcal{N}=6$ Chern–Simons-matter theory commonly known as the ABJM model, which is itself known to be dual to the type IIA superstring on an $AdS_4 \times \mathbb{CP}^3$ background [6]. In this context, the particle–vortex duality of the boundary field theory was shown to correspond to an electric–magnetic duality in the bulk. As a final point in [5], it was speculated that, based on the structure of the embedding into the ABJM model, it should be possible to define a nonabelian version that would act on the whole non-abelian ABJM model.

In this letter, we show that it is indeed the case that we can define a version of particle–vortex duality that acts on a non-abelian theory, at least in a certain restricted sense. Key to our argument are the recent advances in the study of 2-dimensional *non-abelian T-duality* acting on the string worldsheet in string theory [7] (see also [8–10] for the action of the nonabelian T-duality in supergravity). By generalizing the procedure to $(2+1)$-dimensions, we obtain a non-abelian version of particle–vortex duality that acts on gauge theories with a global $SU(2)$, as well as a local symmetry. Recognizing that this is precisely the set-up for the "semi-local" vortices found in [14] (see also [15,16]) in the context of cosmic strings in the case of a local $U(1)$ symmetry, we explicitly exhibit the action of the nonabelian particle–vortex transformation on these solutions.

The letter is organized as follows. In section 2 we revisit non-abelian T-duality and its relation to the abelian T-duality, extending it in section 3 to three spacetime dimensions, consequently defining a non-abelian particle–vortex duality on a general theory which we illustrate with a simple example of a semilocal vortex in

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section 4. This article should be viewed as a proof-of-principle of a phenomenon with potential application from condensed matter to cosmology, with a longer companion paper to follow in which we will elaborate further on the duality and provide more substantial examples [17].

2. Nonabelian T-duality

In string theory, abelian T-duality is a symmetry that acts on a compact dimension as an inversion of its radius, $R \to \alpha'/R$. First noted at the level of the string spectrum, it was proven to be a symmetry of the perturbative string path integral in [18], where it was defined as a duality transformation of the worldsheet action. Specifically, one writes a constrained first order form for the worldsheet action for the compact direction, with a Lagrange multiplier implementing the constraint that mixed second derivatives of the compact coordinate commute. Then, if instead of eliminating the Lagrange multiplier the original coordinate is integrated out, one obtains a T-dual theory in which the Lagrange multiplier plays the role of a new coordinate. This formulation is very similar in spirit to the abelian particle–vortex duality transformation at the level of the path integral [5].

Initially carried out with commuting abelian isometries, a natural next step was to "nonabelianize" the transformation. This was first accomplished in [7] with the transformation acting on three coordinates transforming under a (global) $SU(2)$ symmetry, obtaining what became known as *non-abelian T-duality*. In this section we review the procedure.

Consider the string background with metric and B-field

$$ds^2 = G_{\mu\nu}dx^\mu dx^\nu + 2G_{\mu i}dx^\mu L^i + g_{ij}L^iL^j$$
$$B = B_{\mu\nu}dx^\mu \wedge dx^\nu + B_{\mu i}dx^\mu \wedge L^i + \frac{1}{2}b_{ij}L^i \wedge L^j, \quad (1)$$

and constant dilaton $\phi = \phi_0$. Here,

$$L_1 = \frac{1}{\sqrt{2}}(-\sin\psi d\theta + \cos\psi \sin\theta d\phi),$$
$$L_2 = \frac{1}{\sqrt{2}}(\cos\psi d\theta + \sin\psi \sin\theta d\phi),$$
$$L_3 = \frac{1}{\sqrt{2}}(d\psi + \cos\theta d\phi)\ , \quad (2)$$

are $SU(2)$ left-invariant 1-forms for the Euler angles (θ, ϕ, ψ), such that $dL^i = \frac{1}{2}f^i{}_{jk}L^j \wedge L^k$. The angles have the range $0 \le \theta \le \pi$, $0 \le \phi \le 2\pi$, $0 \le \psi \le 4\pi$, and the $SU(2)$ transformations act as

$$\delta\theta = \epsilon_1 \sin\phi + \epsilon_2 \cos\phi,$$
$$\delta\phi = \cot\theta(\epsilon_1 \cos\phi - \epsilon_2 \sin\theta) + \epsilon_3,$$
$$\delta\psi = \frac{1}{\sin\theta}(-\epsilon_1 \cos\phi + \epsilon_2 \sin\phi). \quad (3)$$

Using the normalized Pauli matrices $t^i = \tau^i/\sqrt{2}$, that satisfy $\mathrm{Tr}\,(t^it^j) = \delta^{ij}$, and the group element $g = e^{\frac{i\phi\tau_3}{2}}e^{\frac{i\theta\tau_2}{2}}e^{\frac{i\psi\tau_3}{2}}$, understood here as a field $g(\tau, \sigma)$ on the string worldsheet, the 1-forms can be rewritten more conveniently as $L^i_\pm = -i\mathrm{Tr}\,(t^ig^{-1}\partial_\pm g)$. Note that while g is complex, the L_i are all real. Then, with

$$Q_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu}, \quad Q_{\mu i} = G_{\mu i} + B_{\mu i}$$
$$Q_{i\mu} = G_{i\mu} + B_{i\mu}, \quad E_{ij} = g_{ij} + b_{ij}\ , \quad (4)$$

the string worldsheet action in this background takes the globally $SU(2)$-invariant form

$$S = \int d^2\sigma \left[Q_{\mu\nu}\partial_+X^\mu\partial_-X^\nu + Q_{\mu i}\partial_+X^\mu L^i_- + Q_{i\mu}L^i_+\partial_-X^\nu + E_{ij}L^i_+L^j_- \right]. \quad (5)$$

One can make this invariance local by introducing an $SU(2)$ gauge field A and replacing derivatives with covariant derivatives, $\partial_\pm g \to D_\pm g = \partial_\pm g - A_\pm g$, which, in turn, replaces $L^i_\pm$ with $\tilde{L}^i_\pm = -i\mathrm{Tr}\,[t^ig^{-1}D_\pm g]$. Since we don't want to add a new degree of freedom (the gauge field A), we need to impose its triviality as a constraint. A good way of doing that is by requiring the field strength to vanish and enforcing this in the action through a Lagrange multiplier term $-i\mathrm{Tr}\,[vF_{+-}] = -i\epsilon^{\mu\nu}\mathrm{Tr}\,[vF_{\mu\nu}]$, where $v = v_i$ is an $SU(2)$ adjoint (a triplet) and the field strength $F_{+-} = \partial_+A_- - \partial_-A_+ - [A_+, A_-]$. In this way we obtain a first order action that acts as a *master action* for the T-duality. Integrating out the Lagrange multiplier v leads to $F_{+-} = 0$ which, in the absence of any topological issues, leads to a trivial A, equivalent to $A = 0$, recovering the original theory.

If instead, we integrate out the gauge field A and gauge fix the $SU(2)$ symmetry, we get $A_\pm$ in terms of v, and on substituting into the master action, obtain the T-dual action. Explicitly, we first partially integrate the Lagrange multiplier term to

$$-i\int \mathrm{Tr}\,[vF_{+-}] = \int \{\mathrm{Tr}\,[+i(\partial_+v)A_- - i(\partial_-v)A_+] - A_+fA_-\}\ , \quad (6)$$

where $A_+fA_- \equiv A^i_+f_{ij}A^j_-$ and $f_{ij} \equiv f_{ij}{}^kv_k$. Then, gauge fixing the $SU(2)$ to $g = 1$, replaces $L^i_\pm$ by $i\mathrm{Tr}\,[t^iA_\pm] = iA^i_\pm$, in the master action, giving

$$S = \int d^2\sigma \left[Q_{\mu\nu}\partial_+X^\mu\partial_-X^\nu + Q_{\mu i}\partial_+X^\mu(+iA^i_-) + Q_{i\mu}\partial_-X^\mu(+iA^i_+) + E_{ij}(iA^i_+)(iA^j_-) + i\partial_+v_iA^i_- - i\partial_-v_iA^i_+ - A^i_+f_{ij}A^j_- \right]. \quad (7)$$

After varying this with respect to A_+ and A_- and solving the resulting equations of motion, we obtain

$$A^i_- = -iM^{-1}_{ij}(\partial_-v_j - Q_{j\mu}\partial_-X^\mu)$$
$$A^i_+ = +iM^{-1}_{ji}(\partial_+v_j + Q_{\mu j}\partial_+X^\mu)\ , \quad (8)$$

where $M_{ij} = E_{ij} + f_{ij}$. Finally, substituting $A_\pm$ back in the master action, produces the T-dual action

$$S_{\text{dual}} = \int d^2\sigma [Q_{\mu\nu}\partial_+X^\mu\partial_-X^\nu + (\partial_+v_i + Q_{\mu i}\partial_+X^\mu) \times M^{-1}_{ij}(\partial_-v_j - Q_{j\mu}\partial_-X^\mu)]. \quad (9)$$

At the quantum level, i.e. considering the one-loop determinant, the T-duality also modifies the dilaton to

$$\Phi(x, v) = \Phi(x) - \frac{1}{2}\ln(\det M). \quad (10)$$

In general, implementing nonabelian T-duality, even in $(1+1)$-dimensions is highly nontrivial. In addition to well-known global issues [11], there are also unresolved questions about the range of the dual coordinates [12]. A full discussion of these issues in our $(2+1)$-dimensional setting is beyond the scope of this article and is left for future work.