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Thermodynamic geometry of holographic superconductors

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ABSTRACT

We obtain the thermodynamic geometry of a $(2+1)$ dimensional strongly coupled quantum field theory at a finite temperature in a holographic setup, through the gauge/gravity correspondence. The bulk dual gravitational theory is described by a $(3+1)$ dimensional charged AdS black hole in the presence of a massive charged scalar field. The holographic free energy of the $(2+1)$ dimensional strongly coupled boundary field theory is computed analytically through the bulk boundary correspondence. The thermodynamic metric and the corresponding scalar curvature are then obtained from the holographic free energy. The thermodynamic scalar curvature characterizes the superconducting phase transition of the boundary field theory.

1. Introduction

The gauge theory/gravity correspondence has been one of the most significant advances in the study of the physics of fundamental forces. This holographically relates a weakly coupled $(d+1)$ dimensional bulk classical theory of gravity coupled to matter fields in an Anti-de-Sitter (AdS) spacetime to a strongly coupled d-dimensional quantum field theory on its conformal boundary [1–4]. Apart from diverse other applications this holographic duality may be utilized to study strongly coupled quantum field theories describing condensed matter systems. In this context, it was first shown by Gubser [5] that for a charged AdS black hole minimally coupled to a complex scalar field it allows the condensation of the scalar field near the black hole horizon resulting in *scalar hair* at a certain critical temperature. From the holographic dictionary this corresponds to a scalar operator that is dual to the bulk charged scalar field, acquiring a non-zero vacuum expectation value in the strongly coupled boundary field theory. The formation of such a charged condensate describes a superconducting phase transition in the strongly coupled boundary quantum field theory that spontaneously breaks the global $U(1)$ symmetry and is referred to as a *holographic superconductor* [6–9]. Subsequently there was a surge of interest in the investigation of the condensate formation, transport and spectral properties for such holographic superconductors in various dimensions both in the probe limit and including the back reaction [10–15]. Furthermore in [16–20] the authors have studied the thermodynamic properties and the critical phenomena of such holographic superconductors and showed that the critical exponents indicate a mean field behavior corresponding to a second order phase transition.

In a distinct context over the last decade there has been important progress in associating an intrinsic Riemannian geometrical structure with equilibrium thermodynamic systems through the studies of Weinhold [21,22] and Ruppeiner [23]. Such a framework of *thermodynamic geometry* associates a Riemannian metric with an Euclidean signature in the equilibrium state space of any thermodynamic system which is based on the thermodynamic fluctuations. In a Gaussian approximation the probability distribution of such fluctuations was related to the positive definite invariant line element defined by this geometry. It was shown that the thermodynamic Riemannian scalar curvature encodes the microscopic interactions of the underlying statistical system. Specifically in [23], it was shown through standard scaling and hyperscaling arguments that the thermodynamic scalar curvature is proportional to the *correlation volume* of the system and hence diverges at a critical point of second order phase transition. This geometrical framework was used to characterize phase transitions and critical phenomena for diverse thermodynamic systems [23]. Application of this framework to study the thermodynamics and phase transition for AdS black holes have yielded interesting insights [24–35]. Naturally the direct connection between the thermodynamic scalar curvature and the microscopic correlation length makes this framework suitable to study the phase transitions in systems lacking a precise and complete microscopic statistical description like black holes or

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strongly coupled condensed matter systems. For an alternative geometrical approach to study the thermodynamics of black holes see the recent articles in [36–39].

In this article we propose to investigate the phase transition and critical phenomena for strongly coupled holographic superconductors in a grand canonical ensemble using the framework of thermodynamic geometry. Quite obviously a direct computation of the thermodynamic geometry for such strongly coupled finite temperature field theories would be intractable. However the gauge/gravity correspondence provides a holographic approach to the problem through the weakly coupled dual bulk gravitational theory. To this end we analytically compute the holographic free energy for the strongly coupled boundary quantum field theory at a finite temperature from the dual bulk charged AdS black hole in presence of charged scalar fields. For this we utilize an analytic method to implement the bulk to boundary correspondence through a saddle point approximation and in the probe limit as described in [10,13]. We emphasize here that our analytic approach is distinct from the conventional analytic and numerical approach for the computation of the holographic free energy [18,20].

The holographic free energy may then be used as the thermodynamic potential to compute the thermodynamic metric and the corresponding thermodynamic scalar curvature for the strongly coupled boundary field theory at a finite temperature through standard techniques of Riemannian geometry. The study of the thermodynamic scalar curvature as a function of the temperature then exhibits a divergence at the critical transition temperature for the superconducting phase transition in the boundary field theory for different values of the mass of the bulk charged scalar field. As mentioned earlier such a divergence indicates a critical point of second order thermal phase transition. The critical temperature for this divergence matches well with the critical temperature obtained from the conventional analytical and numerical techniques based on the condensate formation. There has been no previously such attempt to characterize the phase structure of such a strongly coupled field theory at a finite temperature using the framework of thermodynamic geometry in a holographic approach. We emphasize here that our analytical approach using a geometrical framework based on microscopic fluctuations to study the phase transition and critical phenomena is more elegant and accurate than the conventional approach based on the superconducting condensate formation. This is indicated by the slight difference in the critical temperatures arrived at through the two distinct techniques mentioned here.

This article is organized as follows, in Section 2 we briefly describe the gravitational dual of a holographic superconductor and describe the superconducting phase of the $(2+1)$ dimensional boundary field theory. Furthermore in the same section we present the computation of the holographic free energy of the strongly coupled $(2+1)$ dimensional boundary field theory. In Section 3 we obtain the thermodynamic metric using the holographic free energy and compute the corresponding thermodynamic scalar curvature for the $(2+1)$ dimensional boundary field theory and study its behavior with respect to the temperature. In Section 4 we present a summary of our results and discuss future open problems.

2. The gravity dual of a holographic superconductor

The minimal model for obtaining a holographic superconductor requires a $U(1)$ gauge field and a charged complex scalar field in an AdS black hole background [6]. The bulk action corresponding to the gravitational dual may be given as

$$S=\int d^4x\sqrt{-g}\Big[\frac{1}{2\kappa^2}(R+\frac{6}{L^2})-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

$$-\frac{1}{2}|\nabla\Psi-iqA\Psi|^2-\frac{1}{2}m^2|\Psi|^2\Big] \tag{1}$$

where, $\kappa^2=4\pi G_4$ is related to the gravitational constant in the bulk and L is the AdS radius which we set to unity for further analysis. Here, Ψ is the complex scalar field which is charged under the bulk Maxwell field A_μ. The constants q and m correspond respectively to the charge and the mass of the bulk scalar field Ψ. Here, we work in a weak gravity (or probe) limit, $q\to\infty$ in which gravity decouples from the Abelian–Higgs sector (the scalar and the gauge field). In this limit, we consider the background to be given by a planar Schwarzschild black hole in the AdS_4 bulk with the metric

$$ds^2=\frac{1}{z^2}\left(-f(z)dt^2+\frac{dz^2}{f(z)}+dx^2+dy^2\right), \tag{2}$$

$$f(z)=1-\frac{z^3}{z_h^3},\quad z_h=M^{-1/3}. \tag{3}$$

Here, M stands for the mass of the black hole and the points $z=z_h$, $z\to 0$ respectively correspond to the horizon and boundary of asymptotically Anti-de Sitter space–time. The Hawking temperature of the black hole is given as

$$T_h=\frac{|f'(z_h)|}{4\pi}=\frac{3}{4\pi z_h}. \tag{4}$$

Assuming the ansatz $A_\mu=(\phi(z),0,0,0)$ and $\Psi=\psi(z)$ for the bulk fields [6], the equations of motion for the gauge field and the charged complex scalar field in the background (2) may be expressed as follows[1]

$$\psi''+\left(-\frac{2}{z}+\frac{f'}{f}\right)\psi'+\left(\frac{\phi^2}{f^2}-\frac{m^2}{z^2f}\right)\psi=0, \tag{5}$$

$$\phi''-\frac{2\psi^2}{z^2f}\phi=0, \tag{6}$$

where prime denotes derivative with respect to z. An exact solution to equations (5) and (6) is clearly $\psi=0$ and $\phi=\mu-\rho\ z$, which corresponds to the normal phase of the strongly coupled $(2+1)$ dimensional boundary field theory at finite temperature with ρ and μ as the charge density and the chemical potential respectively.

2.1. Superconducting phase

In this section, we study the superconducting phase of the $(2+1)$ dimensional strongly coupled boundary field theory in the probe limit. It was observed in [5], that a bulk charged AdS black hole develops an instability which leads to the formation of scalar hair near the horizon at low temperatures. This phase is described by the bulk solution, $\psi\neq 0$ of the equations of motion (5) and (6). In the boundary field theory, this corresponds to a superconducting phase transition with a charged scalar operator $\mathcal{O}$ dual to ψ acquiring a non-zero vacuum expectation value at the critical temperature.

From the equations of motion (5) and (6), we observe that for a nontrivial solution we need to determine the two independent functions $(\psi(z),\phi(z))$. For this, suitable boundary conditions must be imposed at the conformal boundary $z\to 0$ and at the black hole

[1] To be exact, we consider $\Psi=\psi(z)e^{i\alpha}$ and make a gauge transformation $A_\mu\to A_\mu+\nabla_\mu\alpha$, which renders the equations of motion free from the phase α.