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Modulated natural inflation

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ABSTRACT

We discuss some model-independent implications of embedding (aligned) axionic inflation in string theory. As a consequence of string theoretic duality symmetries the pure cosine potentials of natural inflation are replaced by modular functions. This leads to "wiggles" in the inflationary potential that modify the predictions with respect to CMB-observations. In particular, the scalar power spectrum deviates from the standard power law form. As a by-product one can show that trans-Planckian excursions of the aligned effective axion are compatible with the weak gravity conjecture.

1. Introduction

Natural (axionic) inflation [1] is one of the best-motivated scenario to describe the inflationary expansion of the early universe. The flatness of the potential is guaranteed by a shift symmetry that is perturbatively exact, only broken by non-perturbative (instantonic) effects. The mechanism can accommodate sizeable primordial tensor modes that require trans-Planckian excursions of the inflaton field. While in the simplest form of axionic inflation these large-field excursions are problematic, a satisfactory solution can be found through a helical motion of the inflaton as suggested in the schemes of axion alignment [2] or axion monodromy [3].

Axions are abundant in string theory constructions that could provide a consistent ultra-violet completion of natural inflation. The embedding in string theory will have some rather model-independent implications for the inflationary potential. The non-perturbative effects responsible for the breakdown of the shift symmetry typically come from instantons or gaugino condensates that in many cases can be described by modular functions [4–10]. Instead of a pure cosine-potential we thus expect a more complicated picture including higher harmonics as subleading instanton effects. This leads to "wiggles" in the potential that modify the predictions of the simplest scheme. In case of a single axion this can be shown to lead to an upper limit on the axion decay constant [11] (as a result of the duality symmetries of string theory).

In the present paper we analyze the framework of aligned natural inflation [2] including modulated potential and higher harmonics. We show that (in contrast to the single field case) a certain amount of trans-Planckian excursion is allowed. This modulated version of aligned axionic inflation induces corrections to CMB-observables (in particular the scalar spectral index) which resolve the (mild) tension of natural inflation with the latest Planck [12] and BICEP2/Keck Array [13] data. As a by-product one can show that modulated natural inflation (including subleading higher harmonics), automatically satisfies the mild version of the weak gravity conjecture [14,15]. This version is expected to hold in string theory for the aligned axion case even in the case of an effective trans-Planckian decay constant.

2. Instantons and modular functions

In supergravity, the axion φ is part of a complex field $T = \chi + i\varphi$, where χ denotes the saxion. At the perturbative level, axions possess a continuous shift symmetry and hence correspond to flat directions in field space. Non-perturbative (instanton) terms generate a periodic potential for the axions, which in the simplest case takes the form

$$V = \Lambda^4 \left(1 - \cos\left[\frac{\varphi}{f}\right]\right) , \tag{1}$$

where f denotes the axion decay constant.

In many instances, this just represents the leading potential. Indeed, the non-perturbative superpotential often contains a series of higher harmonics. This is well known for toroidal string compactifications where these contributions can be calculated

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from first principles [4–7]. Also in more general string theory setups, higher harmonics populate the non-perturbative superpotential [8–10]. The occurrence of these subleading instanton effects in the form of η- and ϑ-functions has already been considered for inflation model building [16–19]. As an illustrative example, we consider couplings in toroidal string compactifications. Couplings y between chiral matter fields $\phi_\alpha, \ldots, \phi_\beta$ are generated by non-perturbative effects

$$W \supset y\, \phi_\alpha \cdots \phi_\beta \propto \prod_{i=1}^{3} \eta(T_i)^{2n_i} \times \phi_\alpha \cdots \phi_\beta\,, \tag{2}$$

where the n_i are determined by localization properties of the fields $\phi_\alpha, \ldots, \phi_\beta$ (see e.g. [20–23] for the case of heterotic orbifolds), and $\eta(T)$ denotes the Dedekind η-function

$$\eta(T) = e^{-\pi T/12} \times \prod_{k=1}^{\infty} \left(1 - e^{-2k\pi T}\right). \tag{3}$$

After the matter fields eventually receive a vacuum expectation value, a periodic potential for the axionic components of the T_i arises. The approximation $\eta(T) \simeq e^{-\pi T/12}$ leads to the standard cosine potential. But as we shall discuss in more detail later, the subleading instantons contained in η induce "wiggles" on the potential.

3. Axion potentials

Let us now turn in more detail to the axion potential including the higher harmonics. We will consider a class of models which have a supersymmetric ground state. They contain an additional chiral superfield ψ known as the stabilizer. The superpotential reads

$$W = \psi \left(A\, \eta(T)^{2n} - B\right), \tag{4}$$

where A and B are, in general, functions of chiral fields. We set the latter to their vacuum expectation values and treat A, B as effective constants. This superpotential is motivated from heterotic orbifolds, where T is identified with a Kähler modulus [17]. The corresponding Kähler potential

$$K = -\log(T + \bar{T}) + |\psi|^2 \tag{5}$$

is shift-symmetric.[1] The model has a supersymmetric Minkowski minimum at $T = \eta^{-1}[(B/A)^{1/2n}] \equiv T_0$. We set the saxion to the minimum and $\psi = 0$. The potential for the canonically normalized axion $\varphi = \mathrm{Im}(T)/(\sqrt{2}T_0)$ then reads

$$\begin{aligned} V = {} & \Lambda^4 e^{-S_1} \left(1 - \cos\left[\frac{\varphi}{f}\right]\right) \\ & - 2n\, \Lambda^4 e^{-S_2} \left(\cos\left[\frac{\varphi}{f_{\text{mod}}}\right] - \cos\left[\frac{\varphi}{f} + \frac{\varphi}{f_{\text{mod}}}\right]\right) \\ & + \ldots\,, \end{aligned} \tag{6}$$

with $\Lambda^4 = AB/T_0$. The decay constants are given as

$$f = \frac{3\sqrt{2}}{n\pi T_0}\,, \qquad f_{\text{mod}} = \frac{1}{2\sqrt{2}\pi T_0} = \frac{n}{12} f\,. \tag{7}$$

The instanton actions

$$S_1 = \frac{n\pi}{6} T_0\,, \qquad S_2 = \left(\frac{n\pi}{6} + 2\pi\right) T_0 \tag{8}$$

correspond to the leading and next-to-leading term in the expansion of the Dedekind η-function. The dots include further, more subleading, terms from the η-function expansion.

We thus find that higher harmonics in η show up with growing frequency and (exponentially) decreasing amplitude in the potential. Another interesting observation is that the amplitude of the wiggles on the potential is controlled by the leading potential. We may write

$$V = \Lambda^4 e^{-S_1} \left(1 - \cos\left[\frac{\varphi}{f}\right]\right) \times F(\varphi) + \ldots\,, \tag{9}$$

where we defined

$$\begin{aligned} F(\varphi) &= 1 - \delta\, \frac{\sin\left[\frac{\varphi}{f_{\text{mod}}} + \frac{\varphi}{2f}\right]}{\sin\left[\frac{\varphi}{2f}\right]} \\ &\simeq 1 - \delta \cos\left[\frac{\varphi}{f_{\text{mod}}}\right] \end{aligned} \tag{10}$$

with $\delta = 2n\, e^{-2\pi T_0}$. The last equality holds in the vicinity of the maximum of the potential which is mainly relevant in this work.[2]

The single-axion model is not suitable for inflation as it requires a trans-Planckian axion decay constant. With growing f the wiggles on the potential caused by the higher harmonics become more pronounced. For $f \gtrsim 1$ they are no longer suppressed and spoil the potential.[3] This is in agreement with the general arguments presented in [11].

4. Aligned natural inflation with modulations

4.1. Alignment mechanism

Let us now generalize the above considerations to the case of two axions and two stabilizers. We generalize the superpotential from [24] by the inclusion of higher harmonics

$$\begin{aligned} W = {} & \psi_1 \left(A_1\, \eta(T_1)^{2n_1} \eta(T_2)^{2n_2} - B_1\right) \\ & + \psi_2 \left(A_2\, \eta(T_1)^{2m_1} \eta(T_2)^{2m_2} - B_2\right), \end{aligned} \tag{11}$$

with two Kähler moduli T_i and two stabilizers ψ_i. The Kähler potential contains two copies of (5). The supersymmetric minimum is located at $\psi_i = 0$ and $T_i = T_{i,0}$ with $T_{i,0}$ such that in (11) the terms in brackets vanish. For the moment we neglect all subleading terms in the expansion of the η-function which effectively then results in the model discussed in [24]. Further, we assume that the saxions stay at their minima. This allows us to define the canonically normalized axions as $\varphi_i = \mathrm{Im}(T_i)/(\sqrt{2}T_{i,0})$. The axion potential reads

$$\begin{aligned} V = {} & \Lambda_a^4 e^{-S_a} \left(1 - \cos\left[\frac{\varphi_1}{f_1} + \frac{\varphi_2}{f_2}\right]\right) \\ & + \Lambda_b^4 e^{-S_b} \left(1 - \cos\left[\frac{\varphi_1}{g_1} + \frac{\varphi_2}{g_2}\right]\right), \end{aligned} \tag{12}$$

where we have defined $\Lambda_a^4 = A_1 B_1/(2T_{1,0}T_{2,0})$ and $\Lambda_b^4 = A_2 B_2/(2T_{1,0}T_{2,0})$. The instanton actions read

[1] The Kähler metric of ψ may in general depend on T. As the dependence does not affect our results we have neglected it here.

[2] We will be mainly concerned with modulations at field values φ_*, where the scales observed in the CMB cross the horizon. In natural inflation, the approximation holds at φ_* if we assume $f < 10$.

[3] Throughout this paper we work in units where $M_P = 1$.