Physics Letters B 738 (2014) 166–171

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Continuous description of fluctuating eccentricities

Jean-Paul Blaizot [a,*], Wojciech Broniowski [a,b,c], Jean-Yves Ollitrault [a]

[a] Institut de Physique Théorique, CNRS/URA 2306, F-91191 Gif-sur-Yvette, France
[b] H. Niewodniczański Institute of Nuclear Physics, Polish Academy of Sciences, PL-31342 Kraków, Poland
[c] Institute of Physics, Jan Kochanowski University, PL-25406 Kielce, Poland

ARTICLE INFO

Article history:
Received 25 May 2014
Received in revised form 10 September 2014
Accepted 10 September 2014
Available online 17 September 2014
Editor: A. Ringwald

Keywords:
Relativistic heavy-ion collisions
Harmonic flow
Correlations

ABSTRACT

We consider the initial energy density in the transverse plane of a high energy nucleus–nucleus collision as a random field $\rho(\boldsymbol{x})$, whose probability distribution $P[\rho]$, the only ingredient of the present description, encodes all possible sources of fluctuations. We argue that it is a local Gaussian, with a short-range 2-point function, and that the fluctuations relevant for the calculation of the eccentricities that drive the anisotropic flow have small relative amplitudes. In fact, this 2-point function, together with the average density, contains all the information needed to calculate the eccentricities and their variances, and we derive general model independent expressions for these quantities. The short wavelength fluctuations are shown to play no role in these calculations, except for a renormalization of the short range part of the 2-point function. As an illustration, we compare to a commonly used model of independent sources, and recover the known results of this model.

1. Introduction

The fluctuations of the initial energy density (to be denoted $\rho(\boldsymbol{x})$ throughout this paper) in the transverse plane of a heavy ion collision play an essential role in the dynamics of these collisions. They leave observable traces in particle distributions after the hydrodynamical evolution [1]. They are for instance responsible for elliptic flow fluctuations [2,3] triangular flow [4–7] and higher harmonics [8,9], directed flow near midrapidity [9–12], and may also explain [13,14] observed transverse momentum fluctuations [15–18]. Considerable experimental and theoretical efforts are presently devoted to pin down the details of these fluctuations [19–22] and their various correlations [5,23,24].

It is then a natural question to try and specify the nature of the information that one can extract from measurements of various features of anisotropic flows. The initial energy density fluctuations are of several origins. The most prominent ones are usually attributed to the motion of individual nucleons in the nuclear wave-functions, and treated by Glauber Monte Carlo calculations [25–28]. In addition, there are sub-nucleonic fluctuations, that reflect the partonic structure of the colliding objects [29]. In most approaches, such sub-nucleonic fluctuations are added on top of the geometrical ones, using various "recipes" [30,31]. There is considerable ambiguity in the whole procedure: sources, with various locations [32], strengths [30], spatial extents, shapes, etc., are added by hand to an already crude description of the nuclear wave-functions. It would certainly be desirable to use a description where all irrelevant details do not stand prominently.

We find it useful then to address the question from another angle, with the goal of obtaining general, model independent, statements about the fluctuations. To achieve this goal, we regard the energy density $\rho(\boldsymbol{x})$ in the transverse plane as a random field, and try to characterize the underlying probability distribution, $P[\rho]$ for finding a given $\rho(\boldsymbol{x})$ in a particular event. This probability distribution is the only ingredient of the description, and it encodes all sources of fluctuations, irrespective of their natures. We conjecture that this distribution is a local Gaussian with a short-range 2-point function. That is, we argue that the fluctuations of the density at different points in the transverse plane are essentially uncorrelated. Corrections are to be expected in regions where the nuclear density is low, and these corrections will be qualitatively discussed. Furthermore, we also argue that short wavelength fluctuations are irrelevant for the calculations of the eccentricities that drive the anisotropic flows, except for a small renormalization of the short range 2-point function.

We start, in the next section, by deriving general expressions for the eccentricities and their variances, in terms of the average density and the 2-point function of the probability distribution. The calculation exploits the fact that the relevant fluctuations have a small amplitude, relative to the average density. We then provide a simple ansatz for the 2-point function, which is dominated by a short range contribution. We compare results obtained with this

* Corresponding author.

http://dx.doi.org/10.1016/j.physletb.2014.09.028
0370-2693/

ansatz with those obtained with a model of independent sources, and we recover known analytic formulas expressing the eccentricities as products of geometrical factors by an overall measure of the strength of the fluctuations. We then discuss why the Gaussian distribution provides a simple, and presumably realistic, form for the probability distribution.

2. Expressions of fluctuation observables in terms of the two-point function

We characterize event classes by the impact parameter $\boldsymbol{b}$. Even though not directly accessible experimentally, the impact parameter is well defined in a high energy collision. For simplicity, in most of this paper, we restrict ourselves to the case of central collisions, i.e. $\boldsymbol{b}=0$, except for a remark on the general case at the end of this section. We write the energy density in a given event class as $\rho(\boldsymbol{x})=\langle\rho(\boldsymbol{x})\rangle+\delta\rho(\boldsymbol{x})$, where $\langle\rho(\boldsymbol{x})\rangle$ is the average energy density and $\delta\rho(\boldsymbol{x})$ is referred to as the fluctuation. The probability that a given $\rho(\boldsymbol{x})$ occurs in the event class considered is denoted by $P[\rho]$.

The observables that we wish to calculate characterize the shape of the fluctuating density $\rho(\boldsymbol{x})$, in terms of its moments, commonly referred to as eccentricities [10]. These are defined by

$$\boldsymbol{e}_n\equiv\int_z z^n\rho(z). \tag{1}$$

Note that $\boldsymbol{e}_n$ is a vector in the transverse plane (i.e., the plane transverse to the collision axis). In the right hand side of Eq. (1) we use the complex notation to represent vectors in the plane. That is, we allow for a slight abuse of notation and denote indifferently a vector $\boldsymbol{r}$ by its components x, y, or by the complex number $z=x+iy$. Thus the density, denoted indifferently by $\rho(\boldsymbol{r})$ or $\rho(z)$, is a real function of x and y. Similarly, we use the short hand $\int_z=\int\mathrm{d}x\mathrm{d}y$ for the integration over the transverse plane.

The zeroth and first moments are special and require specific definitions:

$$\boldsymbol{e}_0=\int_z |z|^2\rho(z),\qquad \boldsymbol{e}_1=\int_z z^2\bar{z}\rho(z), \tag{2}$$

with $\bar{z}$ denoting the complex conjugate of z. The zeroth moment $\boldsymbol{e}_0$ is the mean squared radius of the density, while $\boldsymbol{e}_1$ is a measure of the dipole moment of the distribution [10]. The particular weight $z^2\bar{z}$ in the integral defining $\boldsymbol{e}_1$, instead of the more natural one, z, is due to the fact that in a centered coordinate system, to be defined shortly, the dipole moment vanishes (Eq. (3) below).

By definition, we call “centered” a coordinate system where

$$\int_z z\rho(z)=0. \tag{3}$$

It is only in such a system that the definitions (1) and (2) above are valid. In a fixed coordinate system, however, the fluctuating density would be centered around a random point z_0, distinct form the origin,

$$z_0=\frac{\int_z z\rho(z)}{\int_z\rho(z)}, \tag{4}$$

and the definitions above need to be modified accordingly:

$$\boldsymbol{e}_0=\int_z |z-z_0|^2\rho(z),\qquad \boldsymbol{e}_1=\int_z (z-z_0)^2(\bar{z}-\bar{z}_0)\rho(z),$$
$$\boldsymbol{e}_n=\int_z (z-z_0)^n\rho(z). \tag{5}$$

Because z_0 is a functional of ρ, Eq. (4), the averages of the eccentricities are in general difficult to evaluate. However, simple expressions can be obtained in the regime of small fluctuations, which is the case of practical interest. Indeed the calculation of the eccentricities $\boldsymbol{e}_n$, with n small, involves only the lowest (small $\boldsymbol{k}$) Fourier coefficients $\delta\rho_{\boldsymbol{k}}$ of the fluctuation $\delta\rho(\boldsymbol{r})=\int_{\boldsymbol{k}}\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{r}}\,\delta\rho_{\boldsymbol{k}}$ [10]. This automatically eliminates the rare fluctuations where $\delta\rho(\boldsymbol{r})$ can be locally large (spikes). We return to this issue in the next section.

We then assume that, for the long wavelength fluctuations, $\delta\rho(z)\ll\langle\rho(z)\rangle$, and choose the coordinate system such that $\langle\rho(z)\rangle$ is centered; in particular, at vanishing impact parameter, $\langle\rho(z)\rangle$ has azimuthal symmetry. The center of mass of $\rho(z)$ is still given by Eq. (4) with $\rho(z)$ in the numerator replaced by $\delta\rho(z)$: it follows therefore that z_0 differs from the origin of the coordinate system by a small amount, of order $\delta\rho/\langle\rho\rangle$. A simple calculation then yields, to linear order in the fluctuation,

$$\boldsymbol{e}_0=\int_z |z|^2\big[\langle\rho(z)\rangle+\delta\rho(z)\big],\qquad \boldsymbol{e}_1=\int_z \big[z^2\bar{z}-2\langle r^2\rangle z\big]\delta\rho(z),$$
$$\boldsymbol{e}_n=\int_z z^n\delta\rho(z), \tag{6}$$

where we have set

$$\langle r^2\rangle\equiv\frac{\int_z|z|^2\langle\rho(z)\rangle}{\int_z\langle\rho(z)\rangle}, \tag{7}$$

and we have used symmetries of $\langle\rho(z)\rangle$ to eliminate some terms.

The anisotropic flow coefficients $\boldsymbol{v}_n$ that are experimentally measured, are not directly related to the $\boldsymbol{e}_n$'s, but are rather proportional to the dimensionless ratios [2,10] defined, in a centered system, by

$$\boldsymbol{\varepsilon}_n\equiv\frac{\int_z z^n\rho(z)}{\int_z|z|^n\rho(z)},\qquad \boldsymbol{\varepsilon}_1=\frac{\int_z z^2\bar{z}\rho(z)}{\int_z|z|^3\rho(z)}. \tag{8}$$

It has been shown indeed that the relation $\boldsymbol{v}_n\propto\boldsymbol{\varepsilon}_n$, is well satisfied in ideal hydrodynamics [33–35], and even better so in viscous hydrodynamics [36]. Note that with the sign convention chosen in Eq. (8) (which differs from that in Ref. [10]) the response coefficients $v_n/\boldsymbol{\varepsilon}_n$ are *negative*. Similarly, we define $\boldsymbol{\varepsilon}_0$ by dividing $\boldsymbol{e}_0$ by the total energy:

$$\boldsymbol{\varepsilon}_0=\frac{\int_z|z|^2\rho(z)}{\int_z\rho(z)}. \tag{9}$$

This (dimensionful) quantity represents the mean square radius of the distribution in an individual event. It is distinct from (7) which involves the average density.

Expanding the scaled moments (8), (9) in powers of the fluctuation, we obtain, to leading order,

$$\boldsymbol{\varepsilon}_0=\langle r^2\rangle+\frac{\int_z\delta\rho(z)(|z|^2-\langle r^2\rangle)}{\int_z\langle\rho(z)\rangle}, \tag{10}$$

and

$$\boldsymbol{\varepsilon}_n=\frac{\int_z z^n\,\delta\rho(z)}{\int_z|z|^n\langle\rho(z)\rangle},\qquad \boldsymbol{\varepsilon}_1=\frac{\int_z[z^2\bar{z}-2z\langle r^2\rangle]\,\delta\rho(z)}{\int_z|z|^3\langle\rho(z)\rangle}. \tag{11}$$

Note that, at this order, only $\boldsymbol{\varepsilon}_0$ contains a contribution unrelated to fluctuations, all eccentricities $\boldsymbol{\varepsilon}_n$ with $n\geq 1$, being entirely due to fluctuations for central collisions (the numerators of Eq. (11) are proportional to $\delta\rho$, the contributions of $\langle\rho\rangle$ being zero for symmetry reasons).