Physics Letters B 738 (2014) 187–190

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Functional renormalization group approach to neutron matter

Matthias Drews [a,b,*], Wolfram Weise [a,b]

[a] Physik Department, Technische Universität München, D-85747 Garching, Germany
[b] ECT*, Villa Tambosi, I-38123 Villazzano, Trento, Italy

ARTICLE INFO

Article history:
Received 3 April 2014
Received in revised form 2 September 2014
Accepted 23 September 2014
Available online 26 September 2014
Editor: J.-P. Blaizot

Keywords:
Functional renormalization group
Neutron matter
Neutron stars

ABSTRACT

The chiral nucleon-meson model, previously applied to systems with equal number of neutrons and protons, is extended to asymmetric nuclear matter. Fluctuations are included in the framework of the functional renormalization group. The equation of state for pure neutron matter is studied and compared to recent advanced many-body calculations. The chiral condensate in neutron matter is computed as a function of baryon density. It is found that, once fluctuations are incorporated, the chiral restoration transition for pure neutron matter is shifted to high densities, much beyond three times the density of normal nuclear matter.

1. Introduction

In recent years our understanding of neutron matter has been sharpened significantly. Empirical data as well as theoretical progress set increasingly strong constraints for the equation of state (EoS) at high baryon densities. The observation of two-solar mass neutron stars [1,2] implies that the EoS must be sufficiently stiff in order to support such dense systems against gravitational collapse.

At the same time different realistic calculations of neutron matter based on purely hadronic degrees of freedom are seen to be converging to a consistent picture of the energy per particle as a function of neutron density. Approaches such as chiral Fermi liquid theory [3], chiral effective field theory (ChEFT, [4–6]), or quantum Monte Carlo (QMC) calculations [7,8] all agree with each other within their ranges of applicability. Whereas compact stars with a considerable "exotic" composition, such as a substantial quark core, seem to provide not enough pressure to support a two-solar mass neutron star unless additional strongly repulsive forces are invoked, conventional hadronic matter is consistent with all available mass-radius constraints [9].

In recent publications [10,11], a successful chiral nucleon-meson model for symmetric nuclear matter, previously introduced in [12], was studied beyond mean-field approximation. Fluctuations were treated within the framework of the functional renormalization group (FRG). The importance of a proper handling of fluctuations around the nuclear liquid-gas phase transition was demonstrated. Moreover, no sign of chiral restoration was found for temperatures below about 100 MeV and densities up to about three times nuclear saturation density, $n_0 = 0.16\ \mathrm{fm}^{-3}$.

In the present letter we extend this model to asymmetric nuclear matter. The equation of state for pure neutron matter is computed and compared with state-of-the-art many-body calculations. As in symmetric nuclear matter, fluctuations tend to stabilize the hadronic phase characterized by spontaneously broken chiral symmetry and shift the chiral restoration transition to densities much larger than those anticipated in mean-field approximation. This result is of relevance for chiral approaches to strongly interacting, highly compressed baryonic matter, indicating that nucleon and meson (rather than quark) degrees of freedom are still active at densities several times that of normal nuclear matter.

2. Chiral nucleon-meson model and fluctuations

The chiral nucleon-meson model is designed to describe nuclear matter and its thermodynamics around the liquid-gas phase transition. The relevant degrees of freedom are protons and neutrons forming an isospin doublet nucleon field $\psi = (\psi_p, \psi_n)^T$. The nucleons are coupled to boson fields: a chiral four-component field $(\sigma, \boldsymbol{\pi})$ transforming under the chiral group $SO(4) \cong SU(2)_L \times SU(2)_R$, an isoscalar–vector field ω_μ and an isovector–vector field $\boldsymbol{\rho}_\mu$. Note that these ω and ρ fields are not to be identified with the known omega and rho mesons. They are introduced here to act as background mean fields representing the effects of

* Corresponding author at: Physik Department, Technische Universität München, D-85747 Garching, Germany.
E-mail addresses: matthias.drews@ph.tum.de (M. Drews), weise@tum.de (W. Weise).

http://dx.doi.org/10.1016/j.physletb.2014.09.051

short-distance interactions between nucleons, averaged over the baryonic medium. The ρ field appears as an additional degree of freedom in isospin-asymmetric matter, as compared to symmetric nuclear matter where its expectation value vanishes due to isospin symmetry. The Lagrangian of the extended nucleon-meson model reads

$$\begin{aligned}\mathcal{L} &= \bar\psi i\gamma_\mu\partial^\mu\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} \\ &\quad - \bar\psi\big[g(\sigma + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}) + \gamma_\mu\big(g_\omega\omega^\mu + g_\rho\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu\big)\big]\psi \\ &\quad - \frac{1}{4}F^{(\omega)}_{\mu\nu}F^{(\omega)\mu\nu} - \frac{1}{4}\boldsymbol{F}^{(\rho)}_{\mu\nu}\cdot\boldsymbol{F}^{(\rho)\mu\nu} \\ &\quad + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu - \mathcal{U}(\sigma,\boldsymbol{\pi}). \qquad (1)\end{aligned}$$

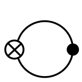
Here $\boldsymbol{\tau}$ are the isospin Pauli-matrices, and $F^{(\omega)}_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$, $\boldsymbol{F}^{(\rho)}_{\mu\nu} = \partial_\mu\boldsymbol{\rho}_\nu - \partial_\nu\boldsymbol{\rho}_\mu - g_\rho\,\boldsymbol{\rho}_\mu\times\boldsymbol{\rho}_\nu$ (only the three-component in isospin space of the time component of $\boldsymbol{\rho}_\mu$ will be involved in the further discussions, so the non-abelian part of $\boldsymbol{F}^{(\rho)}_{\mu\nu}$ is actually not relevant). The potential $\mathcal{U}(\sigma,\boldsymbol{\pi})$ has a piece, $\mathcal{U}_0(\chi)$, that depends only on the chirally invariant square $\chi = \frac{1}{2}(\sigma^2+\boldsymbol{\pi}^2)$, as well as an explicit symmetry breaking term:

$$\mathcal{U}(\sigma,\boldsymbol{\pi}) = \mathcal{U}_0(\chi) - m_\pi^2 f_\pi(\sigma - f_\pi), \qquad (2)$$

with the pion mass $m_\pi = 135$ MeV and the pion decay constant $f_\pi = 93$ MeV.

As demonstrated in [11], fluctuations beyond the mean-field approximation can be included using the functional renormalization group approach. A proper treatment of fluctuations turned out to be crucial in order to make contact with results from in-medium chiral perturbation theory calculations of symmetric nuclear matter [5], emphasizing in particular the role of two-pion exchange dynamics and three-body forces in the nuclear medium. One therefore expects that a full treatment of fluctuations with FRG methods is also important for asymmetric nuclear matter, given the pronounced isospin dependence induced by the fluctuating pion field through multiple pion exchange processes.

The effective action Γ_k based on the Lagrangian (1) depends on a renormalization scale k and interpolates between a microscopic action, $\Gamma_{k=\Lambda}$, defined at an ultraviolet renormalization scale Λ, and the full quantum effective action, $\Gamma_{\mathrm{eff}} = \Gamma_{k=0}$. As the scale k is lowered, the renormalization group flow of Γ_k is determined by Wetterich's equation [13],

$$k\frac{\partial\Gamma_k}{\partial k} =$$  $$= \frac{1}{2}\,\mathrm{Tr}\,\frac{k\frac{\partial R_k}{\partial k}}{\Gamma_k^{(2)} + R_k}, \qquad (3)$$

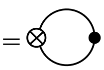
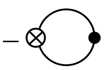
where $R_k = (k^2 - \boldsymbol{p}^2)\theta(k^2 - \boldsymbol{p}^2)$ is a regulator function and $\Gamma_k^{(2)} = \frac{\delta^2\Gamma_k}{\delta\phi^2}$ is the full inverse propagator. In leading order of the derivative expansion, $\Gamma_k = \int d^4x\,(\frac{1}{2}\partial_\mu\phi^\dagger\partial^\mu\phi + U_k)$, where ϕ symbolizes all appearing fields and U_k is the scale-dependent effective potential. The flow equation reduces now to an equation for U_k. In the spirit of Ref. [14] the flow of the difference

$$\bar U_k(T,\mu_n,\mu_p) = U_k(T,\mu_n,\mu_p) - U_k(0,\mu_c,\mu_c) \qquad (4)$$

is computed, with the effective potential $U_k(T,\mu_n,\mu_p)$ taken at given values of temperature T and of neutron/proton chemical potentials, μ_n and μ_p, subtracting $U_k(0,\mu_c,\mu_c)$ at the liquid-gas transition for symmetric matter at zero temperature. The critical chemical potential $\mu_c = 923$ MeV at vanishing temperature is the difference between nucleon mass and binding energy. The subtraction at $\mu = \mu_c$ is motivated by the fact that at this point, nuclear physics information can be optimally used to constrain the effective potential. The regime $0 \le \mu < \mu_c$ corresponds to a single physical state, the vacuum, with constants m_π and f_π unchanged by the FRG evolution [11]. A more detailed discussion will be presented in a forthcoming publication [15].

The k-dependence of $\bar U_k$ is given by the simplified flow equation

$$\frac{V}{T}\frac{k\partial\bar U_k}{\partial k}(T,\mu_n,\mu_p)$$

$$=$$  $$\Big|_{T,\mu_n,\mu_p} -$$  $$\Big|_{\substack{T=0\\ \mu_n=\mu_p=\mu_c}}. \qquad (5)$$

The loops symbolize the full propagators of both fermions (nucleons) and bosons (pions and sigma) with inclusion of the regulator. The heavy vector bosons ω_μ and $\boldsymbol{\rho}_\mu$ are treated as non-fluctuating mean fields. Their Compton wavelengths are supposed to be small compared to the distance scales characteristic of the Fermi momenta under consideration. Rotational invariance implies that the spatial components of the vector mean fields vanish. The only components that can acquire non-zero expectation values are ω_0 and ρ_0^3. Their effect is a shift of neutron and proton chemical potentials according to:

$$\mu_{n,p}^{\mathrm{eff}} = \mu_{n,p} - g_\omega\omega_0 \pm g_\rho\rho_0^3. \qquad (6)$$

The scalar boson σ and the pions $\boldsymbol{\pi}$ are light compared to the energy scales we are interested in and so they are allowed to fluctuate. Similarly, the nucleons are kept in the flow equations, thus incorporating soft nucleon-hole excitations around the Fermi surface. Under these conditions, the flow equations for the present model become:

$$\frac{\partial\bar U_k(T,\mu_n,\mu_p)}{\partial k} = f_k(T,\mu_n,\mu_p) - f_k(0,\mu_c,\mu_c), \qquad (7)$$

with

$$\begin{aligned}f_k(T,\mu_n,\mu_p) &= \frac{k^4}{12\pi^2}\Bigg\{3\cdot\frac{1+2n_{\mathrm{B}}(E_\pi)}{E_\pi} + \frac{1+2n_{\mathrm{B}}(E_\sigma)}{E_\sigma} \\ &\quad - 4\sum_{i=n,p}\frac{1-\sum_{r=\pm1}n_{\mathrm{F}}(E_{\mathrm{N}} - r\mu_i^{\mathrm{eff}}(k))}{E_{\mathrm{N}}}\Bigg\}. \qquad (8)\end{aligned}$$

Here,

$$\begin{aligned}&E_\pi^2 = k^2 + U_k'(\chi), \qquad E_\sigma^2 = k^2 + U_k'(\chi) + 2\chi U_k''(\chi), \\ &U_k'(\chi) = \frac{\partial U_k(\chi)}{\partial\chi}, \qquad E_{\mathrm{N}}^2 = k^2 + 2g^2\chi, \\ &\mu_{n,p}^{\mathrm{eff}}(k) = \mu_{n,p} - g_\omega\omega_0(k) \pm g_\rho\rho_0^3(k), \\ &n_{\mathrm{B}}(E) = \frac{1}{\mathrm{e}^{E/T}-1}, \quad \text{and} \quad n_{\mathrm{F}}(E) = \frac{1}{\mathrm{e}^{E/T}+1}. \qquad (9)\end{aligned}$$

The k-dependent mean fields $\omega_0(k)$ and $\rho_0^3(k)$ are defined at the minima of U_k for each scale k. These fields are thus eliminated as external parameters, simplifying the numerical effort. Their values at k are given by the solutions of the following equations which supplement the FRG equation (7):

$$\begin{aligned}g_\omega\omega_0(k) &= \sum_{r=\pm1}\frac{g_\omega^2}{3\pi^2 m_\omega^2}\int_k^\Lambda dp\,\frac{p^4}{E_{\mathrm{N}}} \\ &\quad \times\frac{\partial}{\partial\mu}\big[n_{\mathrm{F}}\big(E_{\mathrm{N}} - r\mu_p^{\mathrm{eff}}(k)\big) + n_{\mathrm{F}}\big(E_{\mathrm{N}} - r\mu_n^{\mathrm{eff}}(k)\big)\big],\end{aligned}$$