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Conductivity tensor in a holographic quantum Hall ferromagnet

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ABSTRACT

The Hall and longitudinal conductivities of a recently studied holographic model of a quantum Hall ferromagnet are computed using the Karch–O'Bannon technique. In addition, the low temperature entropy of the model is determined. The holographic model has a phase transition as the Landau level filling fraction is increased from zero to one. We argue that this phase transition allows the longitudinal conductivity to have features qualitatively similar to those of two dimensional electron gases in the integer quantum Hall regime. The argument also applies to the low temperature limit of the entropy. The Hall conductivity is found to have an interesting structure. Even though it does not exhibit Hall plateaux, it has a flattened dependence on the filling fraction with a jump, analogous to the interpolation between Hall plateaux, at the phase transition.

Quantum Hall ferromagnetism is an interesting example of dynamical symmetry breaking. It was predicted and observed in two dimensional electron gases formed by semiconductor heterojunctions [1–5] and it has more recently been observed in graphene in the integer quantum Hall regime [6–10]. When many-body interactions are weak, this ferromagnetism has a simple mechanism [3–5,11]. For example, an electron with two spin states and negligible Zeeman interaction has two-fold degenerate Landau levels. When a 2-fold degenerate level is precisely half-filled, that is, at filling fraction $\nu = 1$, the electrons can minimize their Coulomb exchange energy by occupying those states which have only one of the two spin labels. The result is spontaneous breaking of spin symmetry and splitting of the degeneracy of the Landau level by the formation of a charge-gapped incompressible integer quantum Hall state at $\nu = 1$. A similar mechanism is thought to work for any integer filling fraction in a Landau level with higher degeneracy. Graphene has an emergent SU(4) symmetry which would result in four-fold degenerate Landau levels. This degeneracy is seen to be completely resolved in sufficiently strong magnetic fields. Evidence that the mechanism is dynamical symmetry breaking is seen in the magnitude of the energy gaps, which are too large to be accounted for by residual non-symmetric interactions and which are characteristic of the scale of the Coulomb interaction, which is very strong in graphene [7]. This raises the question as to whether quantum Hall ferromagnetism can be understood at strong coupling.[1]

Recently, a holographic model where quantum Hall ferromagnetism persists in the strong coupling limit has been developed [21,22]. The model is a D3-probe-D5 brane system which is dual to a super-conformal defect field theory with N_5 complex fundamental representation hypermultiplets (where N_5 is the number of D5 branes) occupying a $2+1$ dimensional subspace of $3+1$ dimensional space–time. The system is Lorentz invariant, which can be regarded as analogous to the emergent Lorentz symmetry of graphene [23]. The $3+1$-dimensional bulk contains $\mathcal{N} = 4$ supersymmetric Yang–Mills theory with gauge group SU(N). This theory is readily studied in the large N planar limit and the probe limit where $N_5 \ll N$. The conformal field theory has a tuneable dimensionless coupling constant, the 't Hooft coupling $\lambda = g^2_{YM} N$ of the $\mathcal{N} = 4$ Yang–Mills theory.

One can introduce a non-zero temperature and a U(1) charge density and constant external magnetic field for the hypermultiplets. These deformations break supersymmetry. Moreover, in the

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[1] Some work in this direction considers systematic re-summations of perturbation theory which have been studied in the closely related framework known as magnetic catalysis of chiral symmetry breaking [12–18]. It suggests that magnetic catalysis, which is indistinguishable from quantum Hall ferromagnetism in this particular system, can still occur when many-body interactions are appreciable. Spontaneous symmetry breaking in a magnetic field in the charge neutral case is already well known for the holographic D3–D5 brane system [19,20].

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limit of weak coupling, $\lambda \ll 1$, as discussed in [21], the low energy states of this system are fractional fillings of a $2N_5$-fold degenerate, charge neutral, fermionic Landau level.[2] The weak coupling argument for quantum Hall ferromagnetism can be applied and one would expect that the $2N_5$-fold degeneracy is lifted and that incompressible, charge-gapped states appear at filling fractions $\nu = 0, \pm 1, \dots, \pm N_5$. Analysis of the strong coupling limit using the string theory dual, the D3-probe-D5 brane system, shows that at least some of these states with smaller values of ν are also there at strong coupling. In the strong coupling states, the D5 branes blow up to form a D7 brane. The D7 brane is capable of having incompressible integer Hall states at non-zero values of the U(1) charge density. For large values of N_5, the phase diagram of this model was discussed in reference [22].

In this paper, we shall compute the conductivity and the low temperature limit of the entropy of the strongly coupled states that are found in the D3–D5 model at finite temperature T, density ρ and in a magnetic field B. We concentrate on an interval of filling fractions between the integer quantum Hall states, $0 \le \nu \le 1$. Our main aim is to explore the consequences of the phase transition from the D5 to the D7 brane, which was found in references [21, 22], for the electronic transport properties of the system. At the phase transition, which for the values of f that we consider here occurs at a critical value of filling fraction $\nu_c \sim 0.3$–0.5, the stack of N_5 D5 branes, which are stable when $\nu < \nu_c$ blows up to a single D7 brane which is the preferred state when $\nu > \nu_c$. The two phases are distinguished by their symmetry breaking patterns, $U(N_5) \times SO(3) \times SO(3) \to U(N_5) \times SO(2) \times SO(3)$ for the D5 branes and $U(N_5) \times SO(3) \times SO(3) \to U(1) \times SO(3) \times SO(3)$ for the D7 brane. The D5 brane longitudinal conductivity, which we can find analytically in the limit where the parameter $f = \frac{2\pi N_5}{\sqrt{\lambda}}$ is large, that is where $N_5 \gg \frac{\sqrt{\lambda}}{2\pi}$, is

$$\sigma_{xx}^{D5} = \frac{\frac{\pi\sqrt{\lambda}}{2B}T^2}{1 + (\frac{\pi\sqrt{\lambda}}{2B}T^2)^2} \cdot \frac{N\nu}{2\pi}. \tag{1}$$

This expression is a rather featureless linear function of the density, in particular, exhibiting no trace of the higher Landau level or the insulating behaviour which should occur at integer quantum Hall states. This is remedied by the phase transition. If we realize that, for large values of f, when $\nu = 0.5$, the D5 branes are replaced by a D7 brane, the D7 brane conductivity should take over there. In the large f limit,

$$\sigma_{xx}^{D7} = \frac{\frac{\pi\sqrt{\lambda}}{2B}T^2}{1 + (\frac{\pi\sqrt{\lambda}}{2B}T^2)^2} \cdot \frac{N(1-\nu)}{2\pi}, \tag{2}$$

a decreasing function of ν which reverts to an insulating state precisely when $\nu = 1$. The result for σ_{xx} is depicted in the centre column of Figs. 1 and 2 (for two different temperatures). What we find for large values of f (bottom row) is qualitatively like the longitudinal conductivity that would be expected to appear between integer Hall plateaux. The first and second entries of the second columns in Figs. 1 and 2 show the behaviour for smaller values of f, where the conductivity is discontinuous at the phase transition.

The low temperature entropy exhibits similar behaviour. If we first go to weak coupling and compute the zero temperature entropy of the many-electron state coming from the degeneracy, $\binom{NBV/2\pi}{NBV\nu/2\pi}$, of a partially filled Landau level,

$$s_{\lambda\to 0} \approx B\frac{N\nu}{2\pi}\ln\frac{1}{\nu} + B\frac{N(1-\nu)}{2\pi}\ln\frac{1}{1-\nu}.$$

Here, we have assumed that interactions have created the Hall ferromagnetic state, but are not strong enough to appreciably resolve the degeneracy of the partial fillings of the Landau level. To compare, we shall compute the low temperature entropy of the D5 brane (up to order T^3). The result is identical to the one reported in [24]

$$s^{D5} = \frac{\sqrt{\lambda}}{2}B\frac{N\nu}{2\pi} = \frac{\sqrt{\lambda}}{2}\rho. \tag{3}$$

Aside from the factor of $\frac{\sqrt{\lambda}}{2}$, which normally occurs in front of the entropy of a probe brane (see reference [24] for a discussion), this entropy increases linearly with the filling fraction. Now, again, we realize that at a critical ν, the D7 brane takes over. Our computation of the D7 brane entropy in the large f regime gives

$$s^{D7} = \frac{\sqrt{\lambda}}{2}B\frac{N(1-\nu)}{2\pi}. \tag{4}$$

Interestingly, since $\nu_c = 1/2$ in the large f limit, this restores the $\nu \to 1-\nu$ symmetry of the weak coupling limit. The plot of low temperature entropy versus ν for a few values of f are displayed in Fig. 3. As in the case of the longitudinal conductivity, for finite f, they exhibit discontinuities at the phase transition.

The Hall conductivity does not exhibit integer Hall plateaux. Of course, in the translationally invariant system which we are considering here, the physics of impurity driven localization which is normally responsible for Hall plateaux is absent. Moreover, in a Lorentz covariant system, there is an argument that the zero temperature Hall conductivity is identical to its classical value, $\sigma_{xy} = \frac{N\nu}{2\pi}$.[3] Our computation of the Hall conductivity at finite temperature nevertheless reveals an interesting dependence on ν. For example, in the large f limit, the Hall conductivities become

$$\sigma_{xy}^{D5} = \frac{N\nu}{2\pi} - \frac{N\nu}{2\pi} \cdot \frac{(\frac{\pi\sqrt{\lambda}}{2B}T^2)^2}{1 + (\frac{\pi\sqrt{\lambda}}{2B}T^2)^2}, \tag{5}$$

$$\sigma_{xy}^{D7} = \frac{N\nu}{2\pi} + \frac{N(1-\nu)}{2\pi} \cdot \frac{(\frac{\pi\sqrt{\lambda}}{2B}T^2)^2}{1 + (\frac{\pi\sqrt{\lambda}}{2B}T^2)^2}. \tag{6}$$

At the zero temperature limit, the second terms in (5) and (6) vanish and the Hall conductivity is identical to the classical Hall value, $\lim_{T\to 0}\sigma_{xy} = \frac{N\nu}{2\pi}$, as expected. At finite temperature, the thermal correction decreases the conductivity for the D5 and increases it for the D7 brane providing a jump at the phase transition and a flattening of the slope of the σ_{xy} versus ν curve. If we could take the extreme high temperature limit, when $T^2 \gg 2B/\pi\sqrt{\lambda}$, in fact, $\sigma_{xy}^{D5} \to 0$ and $\sigma_{xy}^{D7} \to 1$ and we would have perfect Hall plateaux with the Hall step occurring at the phase transition. It is tantalizing to speculate that there is a strong coupling mechanism at play which, combined with temperature, gives a tendency toward plateau formation. However, in this system, we cannot take the

[2] This counting of the degeneracy assumes that candidate ground states must be colour singlets, otherwise there would be a further factor of N, the number of colour states of the fundamental representation fermion, in the degeneracy. With charge density ρ and magnetic field B (we always assume $B > 0$), we define the filling fraction as $\nu = \frac{2\pi\rho}{NB}$ as if the charge comes in quanta of N and one Landau level is completely filled when $\rho = N\frac{B}{2\pi}$ and $\nu = 1$.

[3] The charge density of a partially filled Landau level is $\rho = \frac{N\nu}{2\pi}B$. We can create a constant current $J_i = \rho v_i$ by going to a reference frame with velocity v_i. The accompanying boost of the magnetic field creates a transverse electric field $E_i = -\epsilon_{ij}v_j B$ and we have $j_i = \frac{N\nu}{2\pi}\epsilon_{ij}E_j$ giving $\sigma_{xy} = \frac{N\nu}{2\pi}$.