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Flavour symmetry breaking in the kaon parton distribution amplitude

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ABSTRACT

We compute the kaon's valence-quark (twist-two parton) distribution amplitude (PDA) by projecting its Poincaré-covariant Bethe–Salpeter wave-function onto the light-front. At a scale $\zeta = 2$ GeV, the PDA is a broad, concave and asymmetric function, whose peak is shifted 12–16% away from its position in QCD's conformal limit. These features are a clear expression of SU(3)-flavour-symmetry breaking. They show that the heavier quark in the kaon carries more of the bound-state's momentum than the lighter quark and also that emergent phenomena in QCD modulate the magnitude of flavour-symmetry breaking: it is markedly smaller than one might expect based on the difference between light-quark current masses. Our results add to a body of evidence which indicates that at any energy scale accessible with existing or foreseeable facilities, a reliable guide to the interpretation of experiment requires the use of such nonperturbatively broadened PDAs in leading-order, leading-twist formulae for hard exclusive processes instead of the asymptotic PDA associated with QCD's conformal limit. We illustrate this via the ratio of kaon and pion electromagnetic form factors: using our nonperturbative PDAs in the appropriate formulae, $F_K/F_\pi = 1.23$ at spacelike-$Q^2 = 17$ GeV^2, which compares satisfactorily with the value of 0.92(5) inferred in e^+e^- annihilation at $s = 17$ GeV^2.

1. Introduction

Kaons are strong-interaction bound-states defined by their valence-quark content: a $\bar{u}$- or $\bar{d}$-quark combined with the s-quark, or the opposite antiparticle-particle combination. The current-mass of the u/d-valence-quark is truly light but that of the s-quark has a value commensurate with $\Lambda_{\rm QCD}$, QCD's dynamically-generated mass-scale. As we shall describe, this marked imbalance between current-masses provides at least two compelling reasons for studying kaons. However, given that the s-quark is neither light nor heavy, elucidating the impact of the imbalance is challenging because it requires the use of nonperturbative techniques within QCD.

The first thing one would like to explore originates in the observation that with the introduction of the quark model as a classification scheme for the hadron spectrum [1,2] it became common to assume, in the absence of reliable dynamical information to the contrary, that hadron wave functions and interaction currents exhibit SU(2) ⊗ SU(3) spin-flavour symmetry. That assumption has implications for numerous observables, including the hadron spectrum itself and a host of other static and dynamical properties. Moreover, in an asymptotically free gauge field theory with N_c colours, this symmetry is exact on $1/N_c \simeq 0$ [3]. Kaons therefore provide the simplest system in which the accuracy of these assumptions and predictions can be tested.

The second aspect convolves the first challenge with the fact that, as strong interaction bound states whose decay is mediated only by the weak interaction, so that they have a relatively long lifetime, kaons have been instrumental in establishing the foundation and properties of the Standard Model; notably, the physics of CP violation. In this connection the nonleptonic decays of B mesons are crucial because, e.g., the transitions $B^\pm \to (\pi K)^\pm$ and $B^\pm \to \pi^\pm\pi^0$ provide access to the imaginary part of the CKM matrix element V_{ub}: $\gamma = {\rm Arg}(V_{ub}^*)$ [4]. Factorisation theorems have

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been derived and are applicable to such decays [5]. However, the formulae involve a certain class of so-called "non-factorisable" corrections because the parton distribution amplitudes (PDAs) of strange mesons are not symmetric with respect to quark and antiquark momenta. Therefore, any derived estimate of γ is only as accurate as the evaluation of both the difference between K and π PDAs and also their respective differences from the asymptotic distribution, $\varphi^{\rm asy}(u) = 6u(1-u)$. Amplitudes of twist-two and -three are involved. With this motivation, we focus on the twist-two amplitudes herein.

Historically, the difficulty with placing constraints on this sort of nonfactorisable contribution is that methods such as lattice gauge theory, QCD sum rules and large-N_c provide little information about the QCD dynamics relevant to hadronic B-decays. We therefore employ QCD's Dyson–Schwinger equations, whose value in the computation of valence-quark distribution amplitudes has recently been established [6–10].

One of the key features to emerge from Refs. [6–10] is the crucial role played by dynamical chiral symmetry breaking (DCSB) in shaping PDAs. DCSB is a remarkable emergent feature of the Standard Model. It plays a critical role in forming the bulk of the visible matter in the Universe [11] and is expressed in numerous aspects of the spectrum and interactions of hadrons; e.g., the large splitting between parity partners [12–14] and the existence and location of a zero in some hadron elastic and transition form factors [15,16]. The impact of DCSB is expressed with particular force in properties of light pseudoscalar mesons. Indeed, their very existence as the lightest hadrons is grounded in DCSB.

2. Computing the kaon twist-two PDA

The kaon's valence-quark distribution amplitude may be obtained via

$$f_K \varphi_K(u) = N_c \,{\rm tr}\, Z_2 \int_{dq}^{\Lambda} \delta(n\cdot q_\eta - u n\cdot P)\gamma_5\gamma\cdot n \chi_K^P(q_\eta, q_{\bar\eta}), \tag{1}$$

where: $N_c = 3$; f_K is the kaon's leptonic decay constant; the trace is over spinor indices; $\int_{dq}^{\Lambda}$ is a Poincaré-invariant regularisation of the four-dimensional integral, with Λ the ultraviolet regularisation mass-scale; $Z_2(\zeta,\Lambda)$, with ζ the renormalisation scale, is the quark wave-function renormalisation constant computed using a mass-independent renormalisation scheme [17]; n is a light-like four-vector, $n^2 = 0$; P is the kaon's four-momentum, $P^2 = -m_K^2$ and $n\cdot P = -m_K$, with m_K being the kaon's mass; and ($q_{\eta\bar\eta} = [q_\eta + q_{\bar\eta}]/2$)

$$\chi_K^P(q_\eta, q_{\bar\eta}) = S_s(q_\eta)\Gamma_K(q_{\eta\bar\eta}; P)S_u(q_{\bar\eta}), \tag{2}$$

is the kaon's Poincaré-covariant Bethe–Salpeter wave-function, with Γ_K the Bethe–Salpeter amplitude, $S_{s,u}$ the dressed s- and u-quark propagators, which take the form

$$S_{f=s,u}(q) = -i\gamma\cdot p\sigma_V^f(q^2) + \sigma_S^f(q^2) \tag{3a}$$
$$= Z_f(q^2)/[i\gamma\cdot p + M_f(p^2)], \tag{3b}$$

and $q_\eta = q + \eta P$, $q_{\bar\eta} = q - (1-\eta)P$, $\eta \in [0,1]$. Owing to Poincaré covariance, no observable can legitimately depend on η; i.e., the definition of the relative momentum.

With χ_K^P in hand, it is straightforward to generalise the procedure explained and employed in Ref. [6], and thereby obtain $\varphi_K(u)$ from Eq. (1). One first computes the moments

$$\langle u_\Delta^m\rangle = \int_0^1 du\,(2u-1)^m \varphi_K(u), \tag{4}$$

which, using Eq. (1), can be obtained via

$$f_K(n\cdot P)^{m+1}\langle u_\Delta^m\rangle = N_c\,{\rm tr}\, Z_2 \int_{dq}^{\Lambda} (2n\cdot q_\eta - n\cdot P)^m \gamma_5\gamma\cdot n \chi_\pi^P(q_\eta, q_{\bar\eta}). \tag{5}$$

Notably, beginning with an accurate form of χ_K^P, arbitrarily many moments can be computed so that $\varphi_K(u)$ can reliably be reconstructed using the method we now describe.

Since the kaon is composed from valence-quarks with unequal current-masses, then $\varphi_K(u) \neq \varphi_K(1-u)$ and all moments produced by Eq. (5) are nonzero. (The asymmetry disappears with the difference between current-quark masses: with mass degeneracy, the odd-m moments vanish, as occurs, e.g., for the π-, ρ- and ϕ-mesons [6,18].) It follows that one may write

$$\varphi_K(u) = \varphi_K^E(u) + \varphi_K^O(u), \tag{6a}$$
$$\varphi_K^{E,O}(u) = (1/2)\big[\varphi_K(u) \pm \varphi_K(1-u)\big]. \tag{6b}$$

In this form, the nonzero moments of $\varphi_K^E(u)$ reproduce all the m-even moments of φ_K and the nonzero moments of $\varphi_K^O(u)$ are the m-odd moments of φ_K.

Consider now that Gegenbauer polynomials of order α, $\{C_n^\alpha(2u-1)\mid n = 0,\dots,\infty\}$, are a complete orthonormal set on $u \in [0,1]$ with respect to the measure $[u(1-u)]^{\alpha_-}$, $\alpha_- = \alpha - 1/2$. They therefore enable reconstruction of any function defined on $u \in [0,1]$ that vanishes at the endpoints; and hence, with complete generality and to a level of accuracy defined by the summation upper bounds,

$$\varphi_K^{E,O}(u) \approx {}_m\varphi_K^{E,O}(u), \tag{7}$$

where

$${}_m\varphi_K^E(u) = N_{\bar\alpha}\big[u(1-u)\big]^{\bar\alpha_-} \sum_{j=0,2,4,\dots}^{\bar j_{\max}} a_j^{\bar\alpha} C_j^{\bar\alpha}(2u-1), \tag{8a}$$

$${}_m\varphi_K^O(u) = N_{\hat\alpha}\big[u(1-u)\big]^{\hat\alpha_-} \sum_{j=1,3,\dots}^{\hat j_{\max}+1} a_j^{\hat\alpha} C_j^{\hat\alpha}(2u-1), \tag{8b}$$

$N_\alpha = \Gamma(2\alpha+1)/[\Gamma(\alpha+1/2)]^2$ and $a_0^{\bar\alpha} = 1$. In general, $\bar\alpha \neq \hat\alpha$ because $\varphi_K^E(u)$ and $\varphi_K^O(u)$ are orthogonal components of $\varphi_K(u)$.

At this point, from a given set of $2m_{\max}$ moments computed via Eq. (5), the even and odd component-PDAs are determined independently by separately minimising

$$\varepsilon_m^E = \sum_{l=2,4,\dots,2m_{\max}} \big|\langle u_\Delta^l\rangle_m^E/\langle u_\Delta^l\rangle - 1\big|, \tag{9a}$$

$$\varepsilon_m^O = \sum_{l=1,3,\dots,2m_{\max}-1} \big|\langle u_\Delta^l\rangle_m^O/\langle u_\Delta^l\rangle - 1\big|, \tag{9b}$$

over the sets $\{\bar\alpha, a_2, a_4, \dots, a_{j_{\max}}\}$, $\{\hat\alpha, a_1, a_3, \dots, a_{j_{\max}+1}\}$, where

$$\langle u_\Delta^l\rangle_m^{E,O} = \int_0^1 du\,(2u-1)^l\, {}_m\varphi_K^{E,O}(u). \tag{10}$$

This procedure acknowledges that at all empirically accessible scales the pointwise profile of PDAs is determined by nonperturbative dynamics [6–10,19]; and hence they should be reconstructed from moments by using Gegenbauer polynomials of order α, with the order α determined by the moments themselves, not fixed beforehand. In the case of π-, ρ- and ϕ-mesons, this procedure converges rapidly: $j_{\max} = 2$ is sufficient [6,18].