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Off-diagonal ekpyrotic scenarios and equivalence of modified, massive and/or Einstein gravity

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ABSTRACT

Using our anholonomic frame deformation method, we show how generic off-diagonal cosmological solutions depending, in general, on all spacetime coordinates and undergoing a phase of ultra-slow contraction can be constructed in massive gravity. In this paper, there are found and studied new classes of locally anisotropic and (in)homogeneous cosmological metrics with open and closed spatial geometries. The late time acceleration is present due to effective cosmological terms induced by nonlinear off-diagonal interactions and graviton mass. The off-diagonal cosmological metrics and related Stückelberg fields are constructed in explicit form up to nonholonomic frame transforms of the Friedmann–Lamaître–Robertson–Walker (FLRW) coordinates. We show that the solutions include matter, graviton mass and other effective sources modeling nonlinear gravitational and matter fields interactions in modified and/or massive gravity, with polarization of physical constants and deformations of metrics, which may explain certain dark energy and dark matter effects. There are stated and analyzed the conditions when such configurations mimic interesting solutions in general relativity and modifications and recast the general Painlevé–Gullstrand and FLRW metrics. Finally, we elaborate on a reconstruction procedure for a subclass of off-diagonal cosmological solutions which describe cyclic and ekpyrotic universes, with an emphasis on open issues and observable signatures.

The idea that graviton may have a nontrivial mass was proposed by Fierz and Pauli work [1] (for recent reviews and related $f(R)$ modifications, see [2]). The key steps in elaborating a modern version of a ghost free (bimetric) massive gravity theory were made in a series of papers: The so-called vDVZ discontinuity problem was solved using the Vainshtein mechanism [3] (avoiding discontinuity by going beyond the linear theory), or following more recent approaches based on DGP model [4]. But none solution was found for another problem with ghosts because at nonlinear order in massive gravity appears a sixth scalar degree of freedom as a ghost (see the Boulware and Deser paper and similar issues related to the effective field theory approach in Refs. [5]). That stoppled for almost two decades the research on formulating a consistent theory of massive gravity.

Recently, a substantial progress was made when de Rham and co-authors have shown how to eliminate the scalar mode and Hassan and Rosen established a complete proof for a class of bigravity/bimetric gravity theories, see [6]. The second metric describes an effective exotic matter related to massive gravitons and does not suffer from ghost instability to all orders in a perturbation theory and away from the decoupling limit.

The possibility that the graviton has a nonzero mass $\mathring{\mu}$ results not only in fundamental theoretical implications but give rise to straightforward phenomenological consequences. For instance, a gravitational potential of Yukawa form $\sim e^{-\mathring{\mu} r}/r$ results in decay of gravitational interactions at scales $r \geq \mathring{\mu}^{-1}$ and this can result in the accelerated expansion of the Universe. This way, a theory of massive gravity provides alternatives to dark energy and, via effective polarizations of fundamental physical constants (in result of generic off-diagonal nonlinear interactions), may explain certain dark matter effects. Recently, various cosmological models derived for ghost free (modified) massive gravity and bigravity theories have been elaborated and studied intensively (see, for instance, Refs. [7,2,8]).

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The goal of this work is to construct generic off-diagonal cosmological solutions in massive gravity theory, MGT, and state the conditions when such configurations are modeled equivalently in general relativity (GR). We shall develop and apply in massive gravity theory the so-called anholonomic frame deformation method, AFDM [9]. As a first step, we consider off-diagonal deformations of a "prime" cosmological solution taken in general Painlevé–Gullstrand (PG) form, when the Friedman–Lamaître–Robertson–Walker (FLRW) can be recast for well-defined geometric conditions. At the second step, the "target" metrics will be generated to possess one Killing symmetry (or other none Killing symmetries) and depend on timelike and certain (all) spacelike coordinates. In general, such off-diagonal solutions are with local anisotropy and inhomogeneities for effective cosmological constants and polarizations of other physical constants and coefficients of cosmological metrics which can be modeled both in MGT and GR. Finally (the third step), we shall emphasize and speculate on importance of off-diagonal nonlinear gravitational interactions for elaborating cosmological scenarios with anisotropic polarization of vacuum and/or de Sitter like configurations.

We study modified massive gravity theories determined on a pseudo-Riemannian spacetime $\mathbf{V}$ with physical metric $\mathbf{g}=\{\mathbf{g}_{\mu\nu}\}$ and certain fiducial metric as we shall explain below. The action for our model is

$$S=\frac{1}{16\pi}\int \delta u^4\sqrt{|\mathbf{g}_{\alpha\beta}|}[\widehat{f}(\widehat{R})-\frac{\mathring{\mu}^2}{4}\mathcal{U}(\mathbf{g}_{\mu\nu},\mathbf{K}_{\alpha\beta})+{}^mL] \quad (1)$$

$$=\frac{1}{16\pi}\int \delta u^4\sqrt{|\mathbf{g}_{\alpha\beta}|}[f(R)+{}^mL]. \quad (2)$$

In this formula, $\widehat{R}$ is the scalar curvature for an auxiliary (canonical) connection $\widehat{\mathbf{D}}$ uniquely determined by two conditions 1) it is metric compatible, $\widehat{\mathbf{D}}\mathbf{g}=0$, and 2) its h- and v-torsions are zero (but there are nonzero h–v components of torsion $\widehat{\mathcal{T}}$ completely determined by $\mathbf{g}$) for a conventional splitting $\mathbf{N}:\ T\mathbf{V}=h\mathbf{V}\oplus v\mathbf{V}$, see details in [10].[2] The "priority" of the connection $\widehat{\mathbf{D}}$ is that it allows to decouple the field equations in various gravity theories and construct exact solutions in very general forms. We shall work with generic off-diagonal metrics and generalized connections depending on all spacetime coordinates when, for instance, of type $\widehat{\mathbf{D}}=\nabla+\widehat{\mathbf{Z}}[\widehat{\mathcal{T}}]$. Such distortion relations from the Levi–Civita (LC) connection ∇ are uniquely determined by a distorting tensor $\widehat{\mathbf{Z}}$ completely defined by $\widehat{\mathcal{T}}$ and (as a consequence for such models) by $(\mathbf{g},\mathbf{N})$. Having constructed certain general classes of solutions (for instance, locally anisotropic and/or inhomogeneous cosmological ones), we can impose additional nonholonomic (non-integrable constraints) when $\widehat{\mathbf{D}}_{|\widehat{\mathcal{T}}=0}\to\nabla$ and $\widehat{R}\to R$, where R is the scalar curvature of ∇, and it is possible to extract exact solutions in GR.

The theories with actions of type (1) generalize the so-called modified $f(R)$ gravity, see reviews and original results in [2], and the ghost-free massive gravity (by de Rham, Gabadadze and Tolley, dRGT) [6]. We use the units when $\hbar=c=1$ and the Planck mass M_{Pl} is defined via $M_{Pl}^2=1/8\pi G$ with 4-d Newton constant G. We write δu^4 instead of d^4u because there are used N-elongated differentials (see below the formulas (11)) and consider $\mathring{\mu}=const$ as the mass of graviton. For LC-configurations, we can fix (as particular cases) conditions of type

$$\widehat{f}(\widehat{R})-\frac{\mathring{\mu}^2}{4}\mathcal{U}(\mathbf{g}_{\mu\nu},\mathbf{K}_{\alpha\beta})=f(\widehat{R}),$$
$$\text{or}\ \widehat{f}(\widehat{R})=f(R),\ \text{or}\ \widehat{f}(\widehat{R})=R, \quad (3)$$

which depend on the type of models we elaborate and what classes of solutions we wont to construct. The first one is necessary if we wont to encode massive gravity effects into a MGT with a generalized connection and corresponding Ricci scalar $\widehat{R}$ which allows us to decouple the gravitational field equations and generate off-diagonal solutions. The second condition is necessary for extracting MGT models with Levi–Civita conditions. We can also consider the third type classes of solutions when theories with both f- and linear connection modifications are effectively modeled as certain off-diagonal solutions in GR. It will be possible to find solutions in explicit form if we fix the coefficients $\{N_i^a\}$ of $\mathbf{N}$ and local frames for $\widehat{\mathbf{D}}$ when $\widehat{R}=const$ and $\partial_\alpha\widehat{f}(\widehat{R})=(\partial_{\widehat{R}}\widehat{f})\times\partial_\alpha\widehat{R}=0$ but, in general, $\partial_\alpha f(R)\neq 0$. The equations of motion for such modified massive gravity theory can be written[3]

$$(\partial_{\widehat{R}}\widehat{f})\widehat{\mathbf{R}}_{\mu\nu}-\frac{1}{2}\widehat{f}(\widehat{R})\mathbf{g}_{\mu\nu}+\mathring{\mu}^2\mathbf{X}_{\mu\nu}=M_{Pl}^{-2}\mathbf{T}_{\mu\nu}, \quad (4)$$

where M_{Pl} is the Plank mass, $\widehat{\mathbf{R}}_{\mu\nu}$ is the Einstein tensor for a pseudo-Riemannian metric $\mathbf{g}_{\mu\nu}$ and $\widehat{\mathbf{D}}$, $\mathbf{T}_{\mu\nu}$ is the standard matter energy-momentum tensor. For $\widehat{\mathbf{D}}\to\nabla$, we get $\widehat{\mathbf{R}}_{\mu\nu}\to R_{\mu\nu}$ with a standard Ricci tensor $R_{\mu\nu}$ for ∇. The effective energy-momentum tensor $\mathbf{X}_{\mu\nu}$ is defined in a "sophisticate" form by the potential of graviton $\mathcal{U}=\mathcal{U}_2+\alpha_3\mathcal{U}_3+\alpha_4\mathcal{U}_4$, where α_3 and α_4 are free parameters. The values $\mathcal{U}_2$, $\mathcal{U}_3$ and $\mathcal{U}_4$ are certain polynomials on traces of some other polynomials of a matrix $\mathcal{K}_\mu^\nu=\delta_\mu^\nu-\left(\sqrt{g^{-1}\Sigma}\right)_\mu^\nu$ for a tensor determined by four Stückelberg fields $\phi^{\underline{\mu}}$ as

$$\Sigma_{\mu\nu}=\partial_\mu\phi^{\underline{\mu}}\partial_\nu\phi^{\underline{\nu}}\eta_{\underline{\mu\nu}}, \quad (5)$$

when $\eta_{\underline{\mu\nu}}=(1,1,1,-1)$. Following a series of arguments presented in [8], when the parameter choice $\alpha_3=(\alpha-1)/3,\alpha_4=(\alpha^2-\alpha+1)/12$ is useful for avoiding potential ghost instabilities, we can fix

$$\mathbf{X}_{\mu\nu}=\alpha^{-1}\mathbf{g}_{\mu\nu}. \quad (6)$$

De Sitter solutions for an effective cosmological constant are possible, for instance, for ansatz of PG type,

$$ds^2=U^2(r,t)[dr+\epsilon\sqrt{f(r,t)}dt]^2+\widetilde{\alpha}^2r^2(d\theta^2+\sin^2\theta d\varphi^2)$$
$$-V^2(r,t)dt^2. \quad (7)$$

In above formula, there are used spherical coordinates labeled in the form $u^\beta=(x^1=r,x^2=\theta,y^3=\varphi,y^4=t)$, the function f takes non-negative values and the constant $\widetilde{\alpha}=\alpha/(\alpha+1)$ and $\epsilon=\pm1$. For such bimetric configurations, the Stückelberg fields are parameterized in the unitary gauge as $\phi^{\underline{4}}=t$ and $\phi^{\underline{1}}=r\widehat{n}^{\underline{1}}$, $\phi^{\underline{2}}=r\widehat{n}^{\underline{2}}$, $\phi^{\underline{3}}=r\widehat{n}^{\underline{3}}$, where a three dimensional (3-d) unit vector is defined as $\widehat{n}=(\widehat{n}^{\underline{1}}=\sin\theta\cos\varphi,\widehat{n}^{\underline{2}}=\sin\theta\sin\varphi,\widehat{n}^{\underline{3}}=\cos\theta)$. Any PG metric of type (7) defines solutions both in GR and in MGT. It allows us to extract the de Sitter solution, in the absence of matter, and to obtain standard cosmological equations with FLRW metric, for a perfect fluid source

$$T_{\mu\nu}=[\rho(t)+p(t)]u_\mu u_\nu+p(t)g_{\mu\nu}, \quad (8)$$

[2] We consider a conventional $2+2$ splitting when coordinates are labeled in the form $u^\alpha=(x^i,y^a)$, or $u=(x,y)$, with indices $i,j,k...=1,2$ and $a,b,\ldots=3,4$. There will be used boldface symbols in order to emphasize that certain geometric/physical objects and/or formulas are written with respect to N-adapted bases (11). There will be considered also left up/low indices as labels for some geometric/physical objects.

[3] See details on action and variational methods in [6]; we shall follow some conventions from [8]; the Einstein summation rule on repeating indices will be applied if the contrary is not stated.