Physics Letters B 752 (2016) 84–88

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Resonant primordial gravitational waves amplification

Chunshan Lin *, Misao Sasaki

Yukawa Institute for Theoretical Physics, Kyoto University, Japan

ARTICLE INFO

Article history:
Received 22 August 2015
Received in revised form 28 September 2015
Accepted 8 November 2015
Available online 11 November 2015
Editor: S. Dodelson

ABSTRACT

We propose a mechanism to evade the Lyth bound in models of inflation. We minimally extend the conventional single-field inflation model in general relativity (GR) to a theory with non-vanishing graviton mass in the very early universe. The modification primarily affects the tensor perturbation, while the scalar and vector perturbations are the same as the ones in GR with a single scalar field at least at the level of linear perturbation theory. During the reheating stage, the graviton mass oscillates coherently and leads to resonant amplification of the primordial tensor perturbation. After reheating the graviton mass vanishes and we recover GR.

1. Introduction

Inflation [1] is the leading paradigm of very early universe cosmology, but its physical origin is still mysterious. The generation of primordial gravitational waves is a generic prediction of the inflationary universe. It leads to B mode polarization in the CMB, and provides an important window to the physics of very early universe. It was reported that the primordial tensor-to-scalar ratio is $r < 0.11$ (95% CL), based on *Planck* full sky survey [2]. Several next-generation satellite missions (CMBPol, COrE and LiteBIRD) as well as the ground based experiments (AdvACT, CLASS, Keck/BICEP3, Simons Array, SPT-3G) and balloons (EBEX, Spider), are aimed at measuring primordial gravitational waves down to $r \sim 10^{-3}$. See Ref. [3] for a recent updated forecast on these future experiments.

According to the Lyth bound [4], the tensor-to-scalar ratio is proportional to the variation of the inflaton field during inflation, i.e. $\Delta\phi/M_p \simeq \int dN\sqrt{r/8}$. The threshold $\Delta\phi = M_p$ then corresponds to $r = 2 \times 10^{-3}$, assumed that tensor power spectrum is nearly scale-invariant. The sizeable amplitude of the primordial gravitational waves requires a super-Planckian excursion of the inflaton, i.e. $\Delta\phi > M_p$.

In quantum field theory, the naturalness principle tells us that the variation of a field ϕ over the distance greater than the cutoff scale is generally regarded as being out of the validity of the theory. In a gravitational system, we take the Planck mass as the UV cutoff scale, because gravity strongly couples to the matter sector and the graviton–graviton scattering violates unitarity above this scale. Thus the inflationary prediction may not be reliable in the case of a super-Planckian excursion. Therefore, the detection of the primordial tensor perturbation with its amplitude larger than the threshold value $r = 2 \times 10^{-3}$ has a profound impact on our understanding of fundamental physics. It implies that either quantum field theory or gravity may be modified in the very early universe.

In this letter, by means of modifying gravity, we propose a new mechanism to evade the Lyth bound. We consider a minimal extension of GR with a non-vanishing graviton mass term in the very early universe. Specifically we propose a model in which the graviton mass is proportional to the inflaton during reheating. Then the coherent oscillation of the inflaton induces that of graviton mass and gives rise to resonant amplification of the primordial tensor perturbation. This is a broad parametric resonance which includes all long wavelength modes, given the graviton mass is much greater than the Hubble constant during reheating. After reheating, the graviton mass vanishes as the inflaton decays and we recover GR.

2. A massive gravity theory

The theoretical and observational consistency of massive gravity has been a longstanding problem, the pioneering attempt could be traced back to Fierz and Pauli's work in 1939 [5]. However, Fierz–Pauli's theory and its non-linear completion, the so-called dRGT massive gravity [6], suffer from many pathologies [7–11]. The origin of these pathologies is probably the Poincare symmetry of the *Stückelberg* scalar field configuration.

Away from the Poincare symmetry, a broad class of massive gravity theories have been discussed in the literature [12–18]. In

* Corresponding author.
E-mail address: chunshan.lin@ipmu.jp (C. Lin).

http://dx.doi.org/10.1016/j.physletb.2015.11.021
0370-2693/

this letter, we consider a massive gravity theory with the internal symmetry [12,19]

$$\varphi^i \to \Lambda^i_j \varphi^j, \qquad \varphi^i \to \varphi^i + \Xi^i\left(\varphi^0\right), \tag{1}$$

where Λ^i_j is the $SO(3)$ rotational operator, $\Xi^i\left(\varphi^0\right)$ are three arbitrary functions of their argument, φ^i and φ^0 are four *Stückelberg* scalars with non-trivial VEVs,

$$\varphi^0 = f(t), \quad \varphi^i = x^i, \;\; i = 1, 2, 3. \tag{2}$$

These non-trivial VEVs give a non-vanishing graviton mass, due to the presence of preferred space–time frame. At the first derivative level, there are two combinations of the *Stückelberg* fields that respect this symmetry,

$$\begin{aligned} X &= g^{\mu\nu}\partial_\mu\varphi^0\partial_\nu\varphi^0, \\ Z^{ij} &= g^{\mu\nu}\partial_\mu\varphi^i\partial_\nu\varphi^j - \frac{g^{\mu\nu}\partial_\mu\varphi^0\partial_\nu\varphi^i \cdot g^{\lambda\rho}\partial_\lambda\varphi^0\partial_\rho\varphi^j}{X}. \end{aligned} \tag{3}$$

Note that in the language of ADM formalism, in the unitary gauge we have $X = N^{-2}$ and $Z^{ij} = h^{ij}$, where N is lapse and h^{ij} is the spatial metric. We will see below that the Z^{ij} term gives rise to a non-vanishing mass to gravitational waves. The graviton mass term could be written as a generic scalar function of the above two ingredients.

Due to the internal symmetry $\varphi^i \to \varphi^i + \Xi^i\left(\varphi^0\right)$, there are only 3 dynamical degrees of freedom (DOF) in our theory, i.e. 2 tensor modes, and 1 scalar mode. In the language of ADM formalism or the $(3+1)$-decomposition of space–time, we find that these two ingredients in Eq. (3) are free from the shift N^i and thus the associated Hamiltonian of gravity is linear in N^i. This implies that 3 momentum constraints and the associated secondary constraints eliminate 3 DOF in h_{ij}, and the number of residual DOF is thus 3 [15].

Now we apply this massive gravity theory to the early universe. To minimize our model, we identify the time-like *Stückelberg* scalar with the inflaton scalar field ϕ, i.e. $\varphi^0 = \phi$. By doing this, we achieve a minimal model of massive gravity, in which only the tensor modes receive a modification, while the scalar and vector modes remain the same as the ones in the single scalar model in GR.

To be specific, we consider the following action with enhanced global symmetry $\varphi^i \to \text{constant} \cdot \varphi^i$,

$$\begin{aligned} S = \int d^4x\sqrt{-g}\Bigg[&\frac{M_p^2}{2}\mathcal{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) \\ &- \frac{9}{8}M_p^2 m_g^2(\phi)\,\frac{\bar{\delta}Z^{ij}\bar{\delta}Z^{ij}}{Z^2}\Bigg], \end{aligned} \tag{4}$$

where $V(\phi)$ is the inflaton potential, the numerical factor 9/8 is inserted for later convenience, and $\bar{\delta}Z^{ij}$ is a traceless tensor defined by [18]

$$\bar{\delta}Z^{ij} \equiv Z^{ij} - 3\frac{Z^{ik}Z^{kj}}{Z}, \tag{5}$$

where Z^{ij} is defined by Eq. (3) with φ^0 replaced by ϕ, $Z \equiv Z^{ij}\delta_{ij}$, and the summation over repeated indices is understood. Note that the 2nd line of Eq. (4) is the graviton mass term, which does not contribute to the background energy momentum tensor. Its non-trivial contribution starts from the quadratic action in perturbations.

The scalar functional dependence of graviton mass $m_g^2(\phi)$ is still left undecided due to our ignorance of underlying fundamental theory. However, from the particle physics perspective, particle masses can be neglected at high energies, so that physics would become scale invariant and weakly coupled. We could reasonably expect that on the Hubble scale, i.e. one of the typical scales during inflation, physics is still weakly coupled. We thus assume the following scalar dependence

$$m_g^2(\phi) = \frac{\lambda\phi^2}{1 + (\phi/\phi_*)^4}, \tag{6}$$

where ϕ_* is the inflaton field value at the end of inflation. Without loss of generality, we assume $\phi = 0$ is the minimum of the potential at which the inflaton settles down after reheating. During inflation, $\phi \gg \phi_*$ and graviton becomes massless on the scale that we are interested in, different polarizations of graviton are thus weakly coupled.

As usual, we consider a flat FLRW background,

$$ds^2 = -dt^2 + a^2 d\boldsymbol{x}^2. \tag{7}$$

Due to the $SO(3)$ rotational symmetry of the 3-space, we can decompose the metric perturbation into scalar, vector, and tensor modes. These modes are completely decoupled at linear order. We define the metric perturbation variables as

$$\begin{aligned} g_{00} &= -(1 + 2\alpha), \\ g_{0i} &= a(t)\left(S_i + \partial_i\beta\right), \\ g_{ij} &= a^2(t)\left[\delta_{ij} + 2\psi\delta_{ij} + \partial_i\partial_j E + \frac{1}{2}(\partial_i F_j + \partial_j F_i) + \gamma_{ij}\right], \end{aligned} \tag{8}$$

where α, β, ψ and E are scalars, S_i and F_i are vectors, and γ_{ij} is tensor. The vector modes satisfy the transverse condition, $\partial_i S^i = \partial_i F^i = 0$, and the tensor modes satisfy the transverse and traceless condition, $\gamma^i_i = \partial_i\gamma^{ij} = 0$.

3. Tensor perturbation

The action for the tensor perturbation reads

$$S_T^{(2)} = \frac{M_p^2}{8}\int dt d^3x a^3\left[\dot{\gamma}_{ij}\dot{\gamma}^{ij} - \left(\frac{k^2}{a^2} + m_g^2\right)\gamma_{ij}\gamma^{ij}\right]. \tag{9}$$

We see that the graviton receives a mass correction. We quantize the tensor mode as

$$\gamma_{ij}(x) = \sum_{s=\pm}\int d^3k\left[a_{\boldsymbol{k}} e_{ij}(\boldsymbol{k}, s)\gamma_k e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + h.c.\right], \tag{10}$$

where $a_{\boldsymbol{k}}$ is the annihilation operator and $e_{ij}(\boldsymbol{k}, s)$ is the transverse and traceless polarization tensor which we normalize as

$$e_{ij}(k, s)e^{ij}(k, s') = \delta_{ss'}. \tag{11}$$

The equation of motion for the tensor modes reads

$$\ddot{\gamma}_k + 3H\dot{\gamma}_k + \left(\frac{k^2}{a^2} + m_g^2\right)\gamma_k = 0. \tag{12}$$

During inflation, the universe undergoes a superluminal expansion with a nearly constant Hubble parameter. The vacuum fluctuations are stretched and frozen on super-horizon scales. At this stage, the graviton mass is

$$m_g^2 \simeq \lambda\phi_*^2(\phi_*/\phi_I)^2, \quad \text{because} \quad \phi_*^4 \ll \phi_I^4, \tag{13}$$

where the subscript "I" is for "inflation". The inflaton fluctuation and metric perturbations are mixed on the scale $\epsilon^{1/2}H_I$ during inflation [20]. In our framework, it is natural to expect that inflation and massive gravity as new physics appear on the same scale.