Physics Letters B 752 (2016) 13–17

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

A relativistic acoustic metric for planar black holes

Sabine Hossenfelder

Nordita, KTH Royal Institute of Technology and Stockholm University, Roslagstullsbacken 23, SE-106 91 Stockholm, Sweden

ARTICLE INFO

Article history:
Received 15 August 2015
Received in revised form 9 November 2015
Accepted 10 November 2015
Available online 14 November 2015
Editor: A. Ringwald

ABSTRACT

We demonstrate here that the metric of a planar black hole in asymptotic anti-de Sitter space can, on a slice of dimension $3+1$, be reproduced as a relativistic acoustic metric. This completes an earlier calculation in which the non-relativistic limit was used, and also serves to obtain a concrete form of the Lagrangian.

1. Introduction

The AdS/CFT duality [1–3] identifies a gravitational theory in asymptotic anti-de-Sitter (AdS) space with a strongly coupled conformal field theory (CFT) on the boundary of the same space. This identification can be used to translate a non-perturbative computation from a strongly coupled condensed matter system to semi-classical gravity, and thereby make it conceptually easier to treat. This approach has shown much promise to deal with systems such as the quark gluon plasma and strange metals [4–6], whose behavior is very difficult to calculate by other methods.

In a previous paper [7], we pointed out the possibility to use the AdS/CFT duality to arrive at a new type of duality which connects a strongly coupled condensed matter system with a weakly coupled one. This can be done by combining AdS/CFT with analog gravity. Analog gravity [8,9] is a way to assign an effective metric to certain types of weakly coupled condensed matter systems. It can be shown that in these systems perturbations propagate in the matter background according to an equation of motion identical to that of particles traveling in curved space. Gravitational analogs are known to exist for the Schwarzschild black hole [10–12] and expanding Friedmann–Robertson–Walker space-times that mimic inflation in the early universe [13–17]. Research in this area is presently very active, and theory and experiments both are being rapidly developed [18,19].

The effective metric is obtained from the behavior of the background field, and completely specified by the degrees of freedom of the condensed matter system. If this effective metric is also a gravitational dual of a strongly coupled system, then this leads to a relation between the two condensed matter systems that give rise to the same metric. The connection between analog gravity and the AdS/CFT duality was also explored in [20–24].

Not all metrics can be obtained as effective analog metrics from condensed matter systems. It must both be possible to bring the metric into a specific form, which amounts to a certain gauge condition, and it must then be demonstrated that the condensed matter system needed to obtain this metric does fulfill the equations of motion. In the previous paper [7] it was demonstrated that the type of metrics used to model strongly coupled systems via the AdS/CFT duality can indeed also be obtained as effective metrics of a weakly coupled condensed matter system.

While intriguing evidence, this finding by itself does not suffice to show that there is a new duality between weakly and strongly coupled condensed matter systems, because the identification of the metric used in the AdS/CFT correspondence as an effective metric does only demonstrate that the semi-classical limits are identical. It does however show that this necessary condition is fulfilled and thus represents a first step on the way to a more general proof.

In the present paper, we will look at the next step, which is to demonstrate that there exists a relativistic completion of the system used in [7]. In [7] it was found that the non-relativistic limit is good towards the boundary of AdS-space, but not close by the horizon. This is unfortunate because the near-horizon region is the most interesting part, so we will here derive a relativistic acoustic metric and recover the non-relativistic limit. We will find that this also tells us more about the form of the Lagrangian of the analog gravity system than could be extracted from the non-relativistic limit.

Throughout this paper we use units in which the speed of light and $\hbar = 1$. The constant c denotes the speed of sound and *not* the speed of light. The metric signature is $(-1, 1, 1, 1)$. Small Greek indices run from zero to three. By non-relativistic we refer to the limit in which the four-velocity is $\ll c$.

E-mail address: hossi@nordita.org.

http://dx.doi.org/10.1016/j.physletb.2015.11.026
0370-2693/

2. Framing the question

The most instructive way to obtain the effective analog metric of a fluid is to use the Lagrangian approach in a mean-field approximation and then derive the equations of motion for perturbations around the mean [7,9,17]. It can be shown then that the perturbations obey a wave-equation that is identical to the wave-equation in a curved background whose metric is the effective analog metric. The form of the metric one obtains in this way depends on the Lagrangian, and we will be dealing here specifically with an effective metric known as the 'acoustic metric' because it determines the propagation of sound waves. There are other types of analog metrics for different systems, for example the optic metric [9,25–27], but these will not be discussed here.

It must be emphasized that the key finding of analog gravity is not trivial. While the assumption of Lorentz-invariance of the original Lagrangian already tells us that the resulting equation of motion for the perturbation must respect this symmetry too, this alone does not single out a wave-equation in curved space. The important property of the resulting equation is that it separates the background from the perturbations in just the right way so that the degrees of freedom of the background can be collected in something that takes the form of a metric tensor, or that the derivatives can be reformulated as covariant derivatives in curved space respectively. Just by looking at all the terms that are possible when one requires that indices are contracted suitably, there could be combinations between the background field and the perturbations that do not lend themselves to the description in terms of an effective metric.

Indeed, it is interesting to observe that Lorentz-invariance in the equations governing the propagation of the excitations can emerge even if the equations of the background themselves are in the non-relativistic limit [28]. The Lorentz-group that is relevant here is that in which the limiting velocity is the speed of sound, and not the speed of light. At high energies, this emergent Lorentz-invariance can be violated, a phenomenon that has been used to study the robustness under UV-corrections of quantum field theory in curved background [29].

Concretely, we take a Lagrangian for a real scalar field θ of the form

$$\mathcal{L} = \mathcal{L}(\chi(\partial\theta) - V)\,, \tag{1}$$

which depends on a kinetic energy term

$$\chi = \eta^{\mu\nu}\left(\partial_\nu\theta\right)\left(\partial_\mu\theta\right) \tag{2}$$

and some additional potential V that may include a mass term (more about the potential later). η is the metric in the laboratory that hosts the analog gravity system. We assume that this space-time in the laboratory is flat, i.e. that its curvature tensor vanishes. However, since we are free to choose a coordinate system, the metric might not have the Minkowski-form.

We then make a perturbation around a background field that is assumed to fulfill the equations of motion, $\theta = \theta_0 + \varepsilon\theta_1$. The inverse of the analog metric in terms of derivatives with respect to the field then takes the rather simple form [7,9,17,30]

$$\sqrt{-g}g^{\mu\nu} = -\sqrt{-\eta}\frac{\partial^2\mathcal{L}}{\partial(\partial_\nu\theta_0)\partial(\partial_\mu\theta_0)}\,. \tag{3}$$

Here and in the following, the lower index 0 refers to quantities describing the background field (at zeroth order). This metric then has to be rewritten into the hydrodynamic variables of the background fluid, and be inverted.

One can identify the pressure p_0, the density ρ_0 and the fluid-velocity u_ν by comparing the stress-energy derived from the Lagrangian (1) to the familiar stress-energy tensor of a fluid, from which one finds

$$u_\nu = \frac{\partial_\nu\theta_0}{\sqrt{\chi}}\,,\; p_0 = \mathcal{L}\,,\; \rho_0 = 2\chi\frac{\partial\mathcal{L}}{\partial\chi} - \mathcal{L}\,. \tag{4}$$

To match this to the notation of [31], it is $\chi = (\rho_0 + p_0)^2/n_0^2$, where n_0 is the particle-density of the fluid and χ is the specific enthalpy. The four-velocity is normalized to minus one

$$\eta^{\mu\nu}u_\mu u_\nu = -1\,. \tag{5}$$

With these variables one then gets the acoustic metric

$$g_{\mu\nu} = \left(\frac{\rho_0 + p_0}{c\chi}\right)\left(\eta_{\mu\nu} + \left(1 - c^2\right)u_\mu u_\nu\right)\,, \tag{6}$$

where c is the speed of sound and defined by

$$\frac{1}{c^2} = \frac{\partial\rho_0}{\partial p_0} = \frac{2\chi\,\partial^2\mathcal{L}/\partial\chi^2 + \partial\mathcal{L}/\partial\chi}{\partial\mathcal{L}/\partial\chi}\,. \tag{7}$$

What we aim to show here is that there exists a scalar field Lagrangian of the general form (1) that gives rise to an acoustic metric which describes the planar black hole in asymptotic $4+1$ dimensional AdS that reads

$$\mathrm{d}s^2 = -\frac{L^2}{z^2}\gamma(z)\mathrm{d}\tilde{t}^2 + \frac{L^2}{z^2}\gamma(z)^{-1}\mathrm{d}z^2 + \frac{L^2}{z^2}\sum_{i=1}^{3}\mathrm{d}x^i\mathrm{d}x^i\,, \tag{8}$$

where [5]

$$\gamma(z) = 1 - \left(\frac{z}{z_0}\right)^4. \tag{9}$$

We have introduced the tilde for the coordinate $\tilde{t}$ for later convenience. The length scale L is the AdS radius and inversely related to the cosmological constant. The analog gravitational system will have to reproduce the metric (8) on a space-like slice of dimension $3+1$ perpendicular to the horizon. Since the metric (8) is translationally invariant into the directions parallel to the horizon, this just means that for the effective metric the sum in the last term runs only over 1 and 2.

3. Gauging the metric

We now have to find a transformation that brings the metric (8) into the form (6). As noted in [7], this procedure generally isn't unique and one can spend a lot of time changing coordinate systems in AdS space just to then realize that the resulting acoustic metric cannot be derived from any Lagrangian. For this reason we will stay as close as possible to the transformation that was found to work previously and use the coordinate transformation $\tilde{t} \to t = \tilde{t} - f(z)$ also used earlier, but now add the rescaling $z \to \tilde{z} = g^{-1}(z)$. This transformation does not change the $1/z^2$ prefactor of the AdS metric, except that now z is implicitly a function of $\tilde{z}$. We therefore can read off

$$\frac{\partial\mathcal{L}}{\partial\chi} = \frac{cL^2}{z^2} \tag{10}$$

directly by comparing (6) with (8).

Further comparing the components of the metric in the new coordinates with (6) one obtains