Physics Letters B 752 (2016) 278–284

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Consistent Pauli reduction on group manifolds

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ARTICLE INFO

Article history:
Received 19 November 2015
Accepted 20 November 2015
Available online 28 November 2015
Editor: M. Cvetič

ABSTRACT

We prove an old conjecture by Duff, Nilsson, Pope and Warner asserting that the NS–NS sector of supergravity (and more general the bosonic string) allows for a consistent Pauli reduction on any d-dimensional group manifold G, keeping the full set of gauge bosons of the $G \times G$ isometry group of the bi-invariant metric on G. The main tool of the construction is a particular generalised Scherk–Schwarz reduction ansatz in double field theory which we explicitly construct in terms of the group's Killing vectors. Examples include the consistent reduction from ten dimensions on $S^3 \times S^3$ and on similar product spaces. The construction is another example of globally geometric non-toroidal compactifications inducing non-geometric fluxes.

1. Introduction

Although the idea of Kaluza–Klein theories originated in the 1920s [1,2], it was with the advent of higher-dimensional supergravities and string theory that the need for developing schemes for obtaining lower-dimensional theories by means of dimensional reduction became compelling. The original idea of Kaluza [1], subsequently developed by Klein [2], was straightforwardly extended from a circle reduction to a reduction on a d-dimensional torus. By this means, for example, four-dimensional ungauged $\mathcal{N} = 8$ supergravity was constructed, by reducing eleven-dimensional supergravity on a 7-torus [3,4]. A key feature in this, and most other, dimensional reductions is that one truncates the infinite "Kaluza Klein towers" of lower-dimensional fields that result from the generalised Fourier expansions of the higher-dimensional fields to just a finite subset, typically, but not always, just the massless fields.

Since the reduction is being applied to a highly non-linear theory, the question then arises as to whether the truncation to a finite subset of the fields is a consistent one. One way to formulate the question is whether in the full lower-dimensional theory, prior to the truncation, the equations of motion of the fields to be truncated are satisfied when one sets these fields to zero. The potential danger is that non-linear products of the fields that are being retained might act as sources for the fields that are to be truncated.

In the case of a circle or toroidal reduction, the consistency of the truncation is guaranteed by a simple group-theoretic argument. The fields that are retained are all the singlets under the $U(1)^d$ isometry of the d-torus, while all the fields that are set to zero are non-singlets (i.e. they are charged under the $U(1)$ factors). It is evident, by charge conservation, that no powers of neutral fields can act as sources for charged fields, and so the consistency is guaranteed.

A more general class of dimensional reductions was described by DeWitt in 1963 [5]. In these, one takes the internal d-dimensional space to be a compact group manifold G, equipped with its bi-invariant metric. The isometry group of this metric is $G_L \times G_R$, where G_L denotes the left action of the group G and G_R denotes the right action. If all the towers of lower-dimensional fields were retained in a reduction on the group manifold G, then the massless sector would include the Yang–Mills gauge bosons of the isometry group, $G_L \times G_R$. However, in the DeWitt reduction only the gauge bosons of G_R (or, equivalently and alternatively, G_L) are retained. To be precise, the lower-dimensional fields that are retained in the truncation are all those that are singlets under G_L. There is now again a simple group-theoretic argument that demonstrates the consistency of the DeWitt reduction: The fields that are being truncated are all those that are non-singlets under G_L. It is evident that no non-linear powers of the G_L-singlets that are retained can act as sources for the fields that are being set to zero, and so the truncation must be consistent.

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http://dx.doi.org/10.1016/j.physletb.2015.11.062
0370-2693/

A much more subtle situation arises if one tries to make more general kinds of dimensional reduction that are not of the toroidal or DeWitt type. One of the earliest, and most important, examples is the 7-sphere reduction of eleven-dimensional supergravity. The massless sector of the reduced four-dimensional theory contains the fields of maximal $\mathcal{N}=8$ gauged $SO(8)$ supergravity [6, 7], but there is no obvious reason why it should be consistent to set the massive towers of fields to zero. In particular, one can easily see that if a generic theory is reduced on S^7 (or indeed any other sphere), then a quadratic product formed from the $SO(8)$ gauge bosons will act as a source for certain massive spin-2 fields. This sets off a chain reaction that then requires an infinity of four-dimensional fields to be retained. A first indication that something remarkable might be occurring in the case of eleven-dimensional supergravity and the S^7 reduction was found in [8], where it was shown that a conspiracy between contributions in the reduction ansatz for the eleven-dimensional metric and the 4-form field strength resulted in an exact cancellation of the potentially-troublesome source term for the massive spin-2 fields that was mentioned above. Subsequent work by de Wit and Nicolai in the 1980s [9], with more recent refinements [10–12], has established that the truncation to the massless $\mathcal{N}=8$ gauged $SO(8)$ supergravity is indeed consistent. There are a few other examples of supergravity sphere reductions that also admit analogous remarkable consistent truncations.

Dimensional reductions on a d-dimensional internal manifold M_d with isometry group G that admit a consistent truncation to a finite set of fields that includes all the gauge bosons of the Yang–Mills group G were called Pauli reductions in [13]. (The idea of such reductions was first proposed, but not successfully implemented, by Pauli in 1953 [14–16].) It was also observed in [13] that in addition to the necessary condition for consistency that was first seen in [8], which was essentially the absence of a cubic coupling of two gauge bosons to the massive spin-2 modes in the untruncated lower-dimensional theory, a rather different necessary condition of group-theoretic origin could also be given. Namely, one can consider first the (trivially consistent) truncation of the theory when reduced instead on the torus T^d. The resulting lower-dimensional theory will have a (non-compact) group S of global symmetries, with a maximal compact subgroup K. If the higher-dimensional theory were to admit a consistent Pauli reduction on the manifold M_d then it must be possible to obtain that theory, with its Yang–Mills gauge group G, by gauging the theory obtained instead in the T^d reduction. (Conversely, by scaling the size of the M_d reduction manifold to infinity, the gauged theory should limit to the ungauged one.) This will only be possible if the isometry group G of the manifold M_d is a subgroup of the maximal compact subgroup K of the global symmetry group S of the T^d reduction.

A generic theory will not satisfy the above necessary condition for admitting a consistent Pauli reduction. For example, pure Einstein gravity in $(n+d)$ dimensions gives rise, after reduction on T^d, to an n-dimensional theory with $S=GL(d,\mathbb{R})$ global symmetry, whose maximal compact subgroup is $K=SO(d)$. By contrast, the isometry group of the d-sphere is $G=SO(d+1)$, which is thus not contained within K. The situation is very different if we consider certain supergravity theories, such as eleven-dimensional supergravity. If it is reduced on T^7 the resulting four-dimensional ungauged theory has an enhanced $E_{7(7)}$ global symmetry, for which the maximal compact subgroup is $K=SU(8)$. This is large enough to contain the $G=SO(8)$ isometry group of the 7-sphere, and thus this necessary condition for consistency of the truncation in the S^7 reduction is satisfied.

It is evident from the above discussion that if an $(n+d)$-dimensional theory is to admit a consistent Pauli reduction on S^d, in which all the Yang–Mills gauge bosons of the isometry group $SO(d+1)$ are retained, then the theory must have some special features that lead to its T^d reduction yielding a massless truncation with some appropriate enhancement of the generic $GL(d,\mathbb{R})$ global symmetry group. Similarly, one may be able to rule out other putative consistent Pauli reductions by analogous arguments.

This brings us to the topic of the present paper. It was observed in [17] that in a reduction of the $(n+d)$-dimensional bosonic string on a group manifold G of dimension d, the potentially dangerous trilinear coupling of a massive spin-2 mode to bilinears built from the Yang–Mills gauge bosons of $G_L\times G_R$ was in fact absent. On that basis, it was conjectured in [17] that there exists a consistent Pauli reduction of the $(n+d)$-dimensional bosonic string on a group manifold G of dimension d, yielding a theory in n dimensions containing the metric, the Yang–Mills gauge bosons of $G_L\times G_R$, and d^2+1 scalar fields which parameterise $\mathbb{R}\times SO(d,d)/(SO(d)\times SO(d))$. Further support for the conjectured consistency was provided in [13], where it was observed that the $K=SO(d)\times SO(d)$ maximal compact subgroup of the enhanced $O(d,d)$ global symmetry of the T^d reduction of the bosonic string is large enough to contain the $G_L\times G_R$ gauge group as a subgroup.

In this paper, we shall present a complete and constructive proof of the consistency of the Pauli reduction of the bosonic string on the group manifold G. Our construction makes use of the recent developments realising non-toroidal compactifications of supergravity via generalised Scherk–Schwarz-type reductions [18] on an extended spacetime within duality covariant reformulations of the higher-dimensional supergravity theories [19–28]. In this language, consistency of a truncation ansatz translates into a set of differential equations to be satisfied by the group-valued Scherk–Schwarz twist matrix U encoding all dependence on the internal coordinates. Most recently, this has been put to work in the framework of exceptional field theory in order to derive the full Kaluza–Klein truncation of IIB supergravity on a 5-sphere to massless $\mathcal{N}=8$ supergravity in five dimensions [29,30]. In this paper, we explicitly construct the $SO(d,d)$ valued twist matrix describing the Pauli reduction of the bosonic string on a group manifold G in terms of the Killing vectors of the group manifold. We show that it satisfies the relevant consistency equations, thereby establishing consistency of the truncation. From the Scherk–Schwarz reduction formulas we then read off the explicit truncation ansätze for all fields of the bosonic string. We find agreement with the linearised ansatz proposed in [17] and for the metric we confirm the non-linear reduction ansatz conjectured in [13].

Our solution for the twist matrix straightforwardly generalises to the case when G is a non-compact group. In this case, the construction describes the consistent reduction of the bosonic string on an internal manifold M_d whose isometry group is given by the maximally compact subgroup $K_L\times K_R\subset G_L\times G_R$. The truncation retains not only the gauge bosons of the isometry group, but the gauge group of the lower-dimensional theory enhances to the full non-compact $G_L\times G_R$. At the scalar origin, the gauge group is broken down to its compact part. This is a standard scenario in supergravity with non-compact gauge groups: for the known sphere reductions the analogous generalisations describe the compactification on hyperboloids $H^{p,q}$ and lower-dimensional theories with $SO(p,q)$ gauge groups [31,32,26,33].

The paper is organised as follows. In section 2 we briefly review the $O(d,d)$ covariant formulation of the low-energy effective action of the $(n+d)$-dimensional bosonic string. In section 3 we review how this framework allows the reformulation of consistent truncations of the original theory as generalised Scherk–Schwarz reductions on the extended space–time. We spell out the consistency equations for the Scherk–Schwarz twist matrix and construct an explicit solution in terms of the Killing vectors of the bi-invariant metric on a d-dimensional group manifold G. For