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Minimal theory of massive gravity

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ABSTRACT

We propose a new theory of massive gravity with only two propagating degrees of freedom. While the homogeneous and isotropic background cosmology and the tensor linear perturbations around it are described by exactly the same equations as those in the de Rham–Gabadadze–Tolley (dRGT) massive gravity, the scalar and vector gravitational degrees of freedom are absent in the new theory at the fully nonlinear level. Hence the new theory provides a stable nonlinear completion of the self-accelerating cosmological solution that was originally found in the dRGT theory. The cosmological solution in the other branch, often called the normal branch, is also rendered stable in the new theory and, for the first time, makes it possible to realize an effective equation-of-state parameter different from (either larger or smaller than) −1 without introducing any extra degrees of freedom.

1. Introduction

Since the seminal work by Fierz and Pauli [1], especially in the recent years, much theoretical effort in cosmology has been put in order to develop theories of massive gravity [2,3]. These theories were indeed able to introduce, at non-linear level, the desired five modes necessary to describe a massive graviton in a Lorentz invariant way. In other words, these theories are free from the so called Boulware–Deser ghost [4], which had been thought to plague any theories of massive gravity. Together with this first success, much work came in order to see whether these same theories could be viable. Unfortunately, these theories suffer from instability on some key backgrounds, such as the Friedmann–Lemaître–Robertson–Walker (FLRW) universe [5]. In this regard several attempts have been analyzed to find stable cosmological solutions in massive gravity: 1) abandoning the homogeneity and/or the isotropy of cosmological models; 2) changing the theory by introducing new fields interacting with gravity in a way as to save the theory. It proved difficult, even with these attempts, to find a theory of massive gravity with a theoretically consistent and experimentally viable cosmology.

In this letter, we present a new theory of massive gravity which modifies general relativity in a minimal way. We will perform this by looking for a theory with only two tensor modes, which are massive. This will make FLRW backgrounds (including de Sitter) stable and viable as in the standard cosmology. Indeed the tensor modes of the gravity sector will be massive, whereas there are no scalar and vector propagating modes in the gravity sector. In order to achieve this goal we will not impose the Lorentz symmetry, so that a massive graviton does not need to have five degrees of freedom any longer.

2. Precursor theory

In order to define the theory we will make use of the lapse N, shift N^i, and the three-dimensional vielbein $e^I{}_j$ as basic variables. We can then introduce the three-dimensional metric as

$$\gamma_{ij} \doteq \delta_{IJ}\, e^I{}_i\, e^J{}_j\,. \tag{1}$$

Hereafter, $I, J \in \{1,2,3\}$ so as i and j. Out of the variables introduced so far, we can build a four-dimensional vielbein as

$$\left\| e^{\mathcal{A}}{}_\mu \right\| = \begin{pmatrix} N & \vec{0}^T \\ e^I{}_i N^i & e^I{}_j \end{pmatrix}, \tag{2}$$

and a four-dimensional metric as

$$g_{\mu\nu} \doteq \eta_{\mathcal{AB}}\, e^{\mathcal{A}}{}_\mu\, e^{\mathcal{B}}{}_\nu\,, \tag{3}$$

where $\eta_{\mathcal{AB}}$ is the Minkowski metric tensor, so that

$$g_{00} = -N^2 + \gamma_{ij} N^i N^j\,,$$
$$g_{0i} = \gamma_{ij} N^j = g_{i0}\,, \quad g_{ij} = \gamma_{ij}\,, \tag{4}$$

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corresponding to the metric tensor written in the ADM variables. We also introduce a non-dynamical four-dimensional vielbein built out of a non-dynamical lapse M, a non-dynamical shift M^i, and a non-dynamical three-dimensional vielbein $E^I{}_j$, as follows:

$$\left\| E^{\mathcal{A}}{}_\mu \right\| \doteq \begin{pmatrix} M & \vec{0}^T \\ E^I{}_i M^i & E^I{}_j \end{pmatrix}. \tag{5}$$

The four-dimensional vielbein of the form (2), often called the ADM vielbein, has 13 independent components, as opposed to 16 independent components of a completely general vielbein in four dimensions. The missing 3 components are the boost parameters that would transform the vielbein of the form (2) to a general vielbein. Therefore, by choosing the form (2) for the vielbein, we introduce a preferred frame and thus explicitly break the local Lorentz symmetry.

We now introduce a precursor action, which will then be used as the starting point to define the theory:

$$\begin{aligned} S_{\rm pre} = {} & \frac{M_{\rm P}^2}{2}\int d^4x\sqrt{-g}\,\mathcal{R}[g_{\mu\nu}] \\ & + \frac{M_{\rm P}^2}{2}m^2\int d^4x\left[\frac{c_0}{24}\epsilon_{\mathcal{ABCD}}\epsilon^{\alpha\beta\gamma\delta}E^{\mathcal{A}}{}_\alpha E^{\mathcal{B}}{}_\beta E^{\mathcal{C}}{}_\gamma E^{\mathcal{D}}{}_\delta \right.\\ & + \frac{c_1}{6}\epsilon_{\mathcal{ABCD}}\epsilon^{\alpha\beta\gamma\delta}E^{\mathcal{A}}{}_\alpha E^{\mathcal{B}}{}_\beta E^{\mathcal{C}}{}_\gamma e^{\mathcal{D}}{}_\delta \\ & + \frac{c_2}{4}\epsilon_{\mathcal{ABCD}}\epsilon^{\alpha\beta\gamma\delta}E^{\mathcal{A}}{}_\alpha E^{\mathcal{B}}{}_\beta e^{\mathcal{C}}{}_\gamma e^{\mathcal{D}}{}_\delta \\ & + \frac{c_3}{6}\epsilon_{\mathcal{ABCD}}\epsilon^{\alpha\beta\gamma\delta}E^{\mathcal{A}}{}_\alpha e^{\mathcal{B}}{}_\beta e^{\mathcal{C}}{}_\gamma e^{\mathcal{D}}{}_\delta \\ & \left. + \frac{c_4}{24}\epsilon_{\mathcal{ABCD}}\epsilon^{\alpha\beta\gamma\delta}e^{\mathcal{A}}{}_\alpha e^{\mathcal{B}}{}_\beta e^{\mathcal{C}}{}_\gamma e^{\mathcal{D}}{}_\delta\right], \end{aligned} \tag{6}$$

where $\mathcal{R}[g_{\mu\nu}]$ is the four-dimensional Ricci scalar for the metric $g_{\mu\nu}$ and the Levi-Civita symbol is normalized as $\epsilon_{0123} = 1 = -\epsilon^{0123}$. The precursor action would be exactly the same as that for the dRGT massive gravity if $e^{\mathcal{A}}{}_\alpha$ were a general, i.e. totally unconstrained, vielbein in four dimensions. At the level of the definition of the precursor theory, the only difference from the dRGT theory is thus that the four-dimensional vielbein is restricted to the form (2).

Having given the action for the precursor theory, it is straightforward to write down its Hamiltonian. The precursor Hamiltonian turns out to be linear in N and N^i and independent of their time derivatives. One can thus safely consider N and N^i as Lagrange multipliers, and the phase space to be considered here then consists of $9\times 2 = 18$ variables, $e^L{}_k$ and their conjugate momenta $\Pi^k{}_L$. The coefficients of N and N^i define primary constraints, that we denote as $-\mathcal{R}_0$ and $-\mathcal{R}_i$, respectively. The rank of the 4×4 matrix made of Poisson brackets among them is two, leading to two secondary constraints, which we denote as $\tilde{\mathcal{C}}_\tau$ $(\tau = 1, 2)$. Combined with other six (three primary and three secondary) constraints, that we name as $\mathcal{P}^{[MN]}$ and $Y^{[MN]}$, associated with a symmetry condition on the vielbein $e^L{}_k$, it is deduced that the physical phase space is $18 - 6 - 6 = 6$ dimensional and that the number of physical degrees of freedom in the precursor theory is three at the fully nonlinear level.

3. Minimal theory

While the precursor theory itself is interesting, we further proceed to remove one more degree of freedom to define a theory with only two degrees of freedom, that we call the minimal theory of massive gravity. From now on, we will fix the units so that $M_{\rm P}^2 = 2$. Also, we neglect the entirely non-dynamical part proportional to c_0.

The minimal theory is defined in the Hamiltonian language by imposing four constraints, which we denote as $\mathcal{C}_0$ and $\mathcal{C}_i$ and are defined in (9) below, on the precursor theory. Only two combinations among these four constraints are new since the other two independent combinations are $\tilde{\mathcal{C}}_\tau \approx 0$ $(\tau = 1, 2)$, that already exist in the precursor theory. Hence the Hamiltonian of the minimal theory is

$$\begin{aligned} H = \int d^3x[& -N\mathcal{R}_0 - N^i\mathcal{R}_i + m^2M\mathcal{H}_1 \\ & + \lambda\mathcal{C}_0 + \lambda^i\mathcal{C}_i + \alpha_{MN}\mathcal{P}^{[MN]} + \beta_{MN}Y^{[MN]}], \end{aligned} \tag{7}$$

where N, N^i, λ, λ^i, α_{MN} (antisymmetric) and β_{MN} (antisymmetric) are 14 Lagrange multipliers. This is a constrained version of the precursor Hamiltonian, because we have added two additional constraints. As a consequence, on the constrained surface the Hamiltonian density reduces only to $H \approx H_1 \doteq \int d^3x m^2 M\mathcal{H}_1$. Each constraint has a specific meaning. The following terms are all derived from the precursor theory,

$$\begin{aligned} \mathcal{R}_0 &= \mathcal{R}_0^{\rm GR} - m^2\mathcal{H}_0\,, \\ \mathcal{R}_0^{\rm GR} &= \sqrt{\gamma}\,R[\gamma] - \frac{1}{\sqrt{\gamma}}\left(\gamma_{nl}\gamma_{mk} - \frac{1}{2}\gamma_{nm}\gamma_{kl}\right)\pi^{nm}\pi^{kl}\,, \\ \mathcal{R}_i &= \mathcal{R}_i^{\rm GR} = 2\gamma_{ik}\mathcal{D}_j\pi^{kj}\,, \\ \mathcal{H}_0 &= \sqrt{\tilde{\gamma}}(c_1 + c_2\,Y_I{}^I) + \sqrt{\gamma}(c_3\,X_I{}^I + c_4)\,, \\ \mathcal{H}_1 &= \sqrt{\tilde{\gamma}}\left[c_1 Y_I{}^I + \frac{c_2}{2}\,(Y_I{}^I Y_J{}^J - Y_I{}^J Y_J{}^I)\right] + c_3\sqrt{\gamma}\,, \end{aligned}$$

and

$$\begin{aligned} \mathcal{P}^{[MN]} &= e^M{}_j\,\Pi^j{}_I\delta^{IN} - e^N{}_j\,\Pi^j{}_I\,\delta^{IM}\,, \\ Y^{[MN]} &= \delta^{ML}\,Y_L{}^N - \delta^{NL}\,Y_L{}^M\,, \end{aligned}$$

out of which the precursor Hamiltonian is $H_{\rm pre} = \int d^3x[-N\mathcal{R}_0 - N^i\mathcal{R}_i + m^2M\mathcal{H}_1 + \tilde{\lambda}^\tau\tilde{\mathcal{C}}_\tau + \alpha_{MN}\mathcal{P}^{[MN]} + \beta_{MN}Y^{[MN]}]$. Here, $\tau = 1, 2$, $\mathcal{D}_j$ is the spatial covariant derivative compatible with γ_{ij}, $\sqrt{\gamma} = \sqrt{\det\gamma_{ij}}$, $\sqrt{\tilde{\gamma}} = \sqrt{\det\tilde{\gamma}_{ij}}$, $\tilde{\gamma}_{ij} = \delta_{IJ}E^I{}_iE^J{}_j$, $\pi^{jk} = \delta^{IJ}\Pi^j{}_Ie_J{}^k$, $\Pi^j{}_I$ is the canonical momentum conjugate to $e^I{}_j$, and

$$Y_I{}^J = E_I{}^k e^J{}_k\,, \quad \text{and} \quad X_I{}^J = e_I{}^k E^J{}_k\,, \tag{8}$$

satisfying $Y_I{}^L X_L{}^J = \delta_I^J$.

Throughout the present letter, for simplicity we adopt the unitary gauge so that M, $M^iE^I{}_i$ and $E^I{}_j$ are only functions of the coordinates. This makes $\mathcal{H}_0$ and $\mathcal{H}_1$ explicitly time-dependent. The remaining constraints, $\mathcal{C}_0$ and $\mathcal{C}_i$, are then defined as

$$\mathcal{C}_0 \doteq \{\mathcal{R}_0, H_1\} + \frac{\partial\mathcal{R}_0}{\partial t}\,, \quad \mathcal{C}_i \doteq \{\mathcal{R}_i, H_1\}\,. \tag{9}$$

The two constraints $\tilde{\mathcal{C}}_\tau \approx 0$ $(\tau = 1, 2)$ in the precursor theory can be written as linear combinations of these four constraints. Therefore, only the remaining two combinations are new. In other words, the minimal theory is defined by adding two additional constraints to the precursor theory. The set of two new constraints removes one degree of freedom from the precursor theory. Since the precursor theory has three degrees of freedom, this means that the minimal theory has only two degrees of freedom.

Rigorously speaking, what we have proved here is that there are enough number of constraints, meaning the inequality, (number of d.o.f.) ≤ 2, holds. One might in fact worry that the consistency of the additional two constraints with time evolution might lead to further secondary constraints, overconstraining the theory. This