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Waves and causality in higher dimensions

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ABSTRACT

We give a new, wave-like solution of the field equations of five-dimensional relativity. In ordinary three-dimensional space, the waves resemble de Broglie or matter waves, whose puzzling behaviour can be better understood in terms of one or more extra dimensions. Causality is appropriately defined by a null higher-dimensional interval. It may be possible to test the properties of these waves in the laboratory.

1. Introduction

Despite data from the double-slit and related experiments, the theory behind de Broglie or matter waves is not well understood [1–4]. For example, a fundamental feature is that the product of the particle velocity and the wave velocity equals the square of the speed of light, so if the former is subluminal then the latter must be superluminal. This and other puzzles can be better understood if the de Broglie waves observed in ordinary 3D space originate in five or more dimensions [5,6]. The extension of general relativity to five dimensions, as in Membrane theory and Space–Time–Matter theory, is now well established [7]. It is in agreement with extant observations and is widely regarded as a viable step towards a grand-unified theory of all the interactions of physics. In the present work we will present a new, exact solution of the field equations of five-dimensional relativity, and compare this with the approach in four-dimensional spacetime. Our conclusion will be that phenomena involving de Broglie waves may be better understood in terms of the physics of one or more extra dimensions, where causality is defined by setting the extended interval to zero. It might be possible to study such higher-dimensional waves in the laboratory.

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2. An exact 5D wave solution

The field equations of five-dimensional relativity are usually defined by the 5D Ricci tensor as $R_{AB} = 0$ ($A, B = 0$, 123, 4 for time, space and the extra coordinate). These 5D equations actually contain Einstein's 4D ones of general relativity, by an old embedding theorem of Campbell. Many exact solutions of the 5D equations are known. But only one exhibits wave-like behaviour of the type shown by experiments on matter waves, and this is restricted by having a constant extra potential [5,8–10]. It would be of special interest to find a solution which has wave-like properties and involves the extra dimension in a meaningful manner. This because the extra potential represents a scalar field, modulated by spin-0 quanta, which is believed to be of potential importance for both particle physics and cosmology [7]. In this section, we will present such a solution.

Consider the following 5D line element:

$$dS^2 = \exp\left[\frac{i}{L}(t+ql)\right]dt^2 - \exp\left[\frac{2i}{L}(t+\alpha x+ql)\right]dx^2 - \exp\left[\frac{2i}{L}(t+\beta y+ql)\right]dy^2 - \exp\left[\frac{2i}{L}(t+\gamma z+ql)\right]dz^2 - q^2\exp\left[\frac{i}{L}(t+ql)\right]dl^2. \quad (1)$$

This metric satisfies $R_{AB} = 0$, as may be verified from a tedious calculation by hand or a short run on a computer. It describes a wave in a 5D manifold consisting of ordinary spacetime (t, xyz)

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plus an extra dimension ($x^4 = l$; we use this symbol to avoid confusion with the Euclidean coordinate). The solution is typified by a constant length L, and four dimensionless constants α, β, γ and q that relate to ordinary 3D space and the extra dimension. All 5 of these constants are arbitrary from the mathematical viewpoint.

The 5D metric (1) is complex, and since this differs from the standard usage in 4D general relativity some comments may be useful. There are actually several solutions of this type in the literature (see Ref. [7] for a summary). They usually arise when the field equations possess a symmetry that allows the extra dimension to switch between spacelike and timelike, when one solution is real and the other complex. Both choices of signature are allowed in Space–Time–Matter theory, whereas a timelike extra dimension is the common choice in Membrane theory. Complex solutions in $N \geq 5$D typically have properties which cannot be described by real solutions in 4D. They are essential to studying waves and other quantum-related phenomena such as tunnelling. The focus here is on wave-like behaviour, so it is natural to consider solutions like (1) above. A discussion of the physics of 5D wave solutions may be found in connection with a previous study [5]. The criterion for the acceptability of any 5D complex solution is that the 4D properties of matter calculated from it should be real. We will find below that the solution (1) satisfies this criterion.

From the physical viewpoint, (1) has several interesting properties which we will illustrate by bringing in the speed of light c and Planck's constant h at appropriate places. Thus the frequency of the wave in (1) is c/L, and the wave-numbers in the x, y and z directions are α/L, β/L and γ/L. There are actually two dynamical modes of (1) involving the corresponding quantity q/L which is coupled to the extra coordinate $x^4 = l$. When q is part of a complex phase as in (1), the wave number is q/L, and the metric has signature $(+ - - - -)$ so the extra dimension is spacelike. When q is taken out of the complex phase via $q \to iq$ in (1), the motion in l is not wavelike but monotonic, and the metric has signature $(+ - - - +)$ so the extra dimension is timelike. Both options are allowed in 5D relativity [7]. We will concentrate on the former case, since we will find that (1) shares several properties with de Broglie waves. One characteristic property of de Broglie waves is that the product of their phase velocity v_p and group velocity v_g is equal to the square of the speed of light, where the group velocity is identified with the speed of the associated particle [1–4]. The same relation is implied by (1), as may be seen by considering the spacetime part of the wave travelling along the x-axis (say). This is described by $\exp[i(ft + k_x x)]$, where f is the frequency and k_x is the wave number. Here, as noted above, the frequency in conventional units is $f = c/L$. To fix L, we take Planck's law and apply it to the energies of the wave and its associated particle: $E = hf = hc/L = mc^2$, so $L = h/mc$, which is the Compton wavelength of the particle whose mass is m. To fix the wave number k_x, we take de Broglie's relation between the wavelength and the momentum of its associated particle, $\lambda_x = h/mv_g$, and invert it to write the wave number as $k_x = (mc/h)(v_g/c) = v_g/cL$. Combining the frequency and the wave number now gives a relation between the phase velocity of the wave and the velocity of the particle:

$$v_p = \frac{f}{k_x} = \frac{c}{L}\left(\frac{cL}{v_g}\right) = \frac{c^2}{v_g} \quad \text{so } v_p v_g = c^2. \tag{2}$$

This is the aforementioned relation for a matter wave and its associated particle. Ordinary particles observed in the laboratory have velocities $v_g < c$ so (2) necessarily implies $v_p > c$. This possibility is clearly present in (1), where the wave number along the x axis is $k_x = \alpha/L$ so the phase velocity is $f/k_x = (c/L)(\alpha/L)^{-1} = c/\alpha$, where α is arbitrary and can be less than unity.

Causality in 5D is most logically defined by the 5D null paths given by $dS^2 = 0$ [5–7]. This includes the conventional 4D paths for both photons and massive particles, given in terms of the 4D interval or proper time by $ds^2 \geq 0$. It has been known for a while that certain 5D metrics admit superluminal velocities, the simplest example being 5D Minkowski space with a timelike extra coordinate. However, such velocities are covered by the condition $dS^2 = 0$, which ensures that all events in the manifold are in causal contact.

The metric (1) shows that motion along (say) the x axis is simple harmonic in nature. If this motion were present in a mechanical system, it would be governed by a 'spring constant' $1/L^2$. The question arises of whether the waves in (1) exist in empty space, or whether they are supported by some kind of fluid. Campbell's theorem, mentioned above, helps to answer this. For it implies that any solution of the apparently empty 5D field equations $R_{AB} = 0$ can be reduced to the 4D Einstein equations $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$, where $T_{\alpha\beta}$ is an *effective* energy–momentum tensor induced by the extra dimension ([6,7]: we use geometrical units here). The precise form of $T_{\alpha\beta}$ for metric (1) can be calculated by longhand or computer, and we have done both. In such problems, the source depends on how the 4D part of the metric is embedded in the extra dimension, and for perfect fluids the density and pressure are typically proportional to $1/\Phi^2$ where $g_{44} \equiv -\Phi^2(x^\gamma, l)$ defines the scalar field. For metric (1), we find that the density and pressure are given by

$$8\pi\rho = \frac{9}{2}\frac{q^2}{L^2\Phi^2}, \qquad p = \frac{\rho}{3}. \tag{3}$$

The equation of state is that of radiation or ultra-relativistic particles. The properties of matter depend on the wave number in the extra dimension (q) and the magnitude of the scalar field (Φ). They do *not* depend on the wave-numbers of the motion in ordinary 3D space. The oscillations described by the 5D metric (1) are not therefore ordinary electromagnetic waves or conventional gravitational waves (the latter have different properties and propagate through truly empty space). But they exist in 3D space, and share characteristics with a previously-studied case whose background metric describes the Einstein vacuum [5,6]. In that case, the waves were identified as de Broglie or matter waves. These can exist in any kind of medium, so we believe that the new solution (1) also describes de Broglie waves.

The new solution (1) may be usefully compared with the 5D de Sitter solution, which has been much studied. In 4D, this solution is the basic one with vacuum energy as measured by the cosmological constant, and the field equations have the familiar form $R_{\alpha\beta} = \Lambda g_{\alpha\beta}$. In 5D, the de Sitter solution is known to be not only Ricci-flat with $R_{AB} = 0$ but also Riemann-flat with $R^A{}_{BCD} = 0$. For the new solution, we find that both relations are only satisfied if the constant which appears in the last term of (1) is exactly equal to the square of the wave number (q) associated with the extra coordinate. This implies that the existence of the solution (1) and the fact that it is flat in 5D depends, loosely speaking, on the 'size' of the extra dimension.

3. Matter waves and causality in 4D and 5D

De Broglie waves as they are understood in 4D raise questions to do with causality [2,3,6]. In this section, we wish to give a brief discussion of how matter waves are viewed in 4D spacetime, and then indicate how solutions like (1) above relate to causality in a 5D manifold.

It should be recalled that de Broglie was led to infer that particles have associated waves by essentially comparing the time and space components of the 4-vectors associated with the particle (mass, momentum) and the wave (frequency, wavelength). This