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# Classical and quantum initial conditions for Higgs inflation

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ABSTRACT

We investigate whether Higgs inflation can occur in the Standard Model starting from natural initial conditions or not. The Higgs has a non-minimal coupling to the Ricci scalar. We confine our attention to the regime where quantum Einstein gravity effects are small in order to have results that are independent of the ultraviolet completion of gravity. At the classical level we find no tuning is required to have successful Higgs inflation, provided the initial homogeneity condition is satisfied. On the other hand, at the quantum level we obtain that the renormalization for large non-minimal coupling requires an additional degree of freedom, unless a tuning of the initial values of the running parameters is made. In order to see that this effect may change the predictions we finally include such degree of freedom in the field content and show that Starobinsky's $R^2$ inflation dominates over Higgs inflation.



## 1. Introduction

Inflation [1–3] is perhaps one of the most natural ways to stretch the initial quantum vacuum fluctuations to the size of the current Hubble patch, seeding the initial perturbations for the cosmic microwave background (CMB) radiation and large scale structure in the universe [4] (for a theoretical treatment, see [5]). Since inflation dilutes all matter it is pertinent that after the end of inflation the universe is filled with the right thermal degrees of freedom, i.e. the Standard Model (SM) degrees of freedom (for a review on pre- and post-inflationary dynamics, see [6]). The most economical way to achieve this would be via the vacuum energy density stored within the SM Higgs, whose properties are now being measured at the Large Hadron Collider (LHC) [7,8]. Naturally, the decay of the Higgs would create all the SM quarks and leptons observed within the visible sector of the universe. Albeit, with just alone SM Higgs and minimal coupling to gravity, it is hard to explain the temperature anisotropy observed in the CMB radiation without invoking physics beyond the SM.[1]

However, a very interesting possibility may arise within the SM if the Higgs were to couple to gravity non-minimally – such as in the context of extended inflation [10], which has recently received particular attention after the Higgs discovery at the LHC in the context of Higgs inflation [11]. By tuning this non-minimal coupling constant, $\xi$, between the Ricci scalar of the Einstein–Hilbert term and the SM Higgs, it is possible to explain sufficient amount of e-folds of inflation and also fit other observables such as the amplitude of temperature anisotropy and the spectral tilt in the CMB data. Indeed, this is very nice and satisfactory, except that the non-minimal coupling, $\xi$, turns out to be very large (at the classical level $\xi \sim 10^4$) in order to explain the CMB observables. This effectively redefines the Planck's constant during inflation, and invites new challenges for this model, whose consequences have been debated vigorously in many papers, such as [12].

One particular consequence of such large non-minimal coupling is that there is a new scale in the theory, $\bar{M}_{\rm Pl}/\sqrt{\xi}$, lower than the standard reduced Planck mass, $\bar{M}_{\rm Pl} \approx 2.435 \times 10^{18}$ GeV. Typically inflation occurs above this scale, the Higgs field takes a vacuum expectation value (VEV) above $\bar{M}_{\rm Pl}/\sqrt{\xi}$ in order to sustain inflation sufficiently. In fact, the inflaton potential, in the Einstein frame, approaches a constant plateau for sufficiently large field values. Effectively, the inflaton becomes a flat direction, where it does not cost any energy for the field to take any VEV beyond this cut-off.

Given this constraint on the initial VEV of the inflaton and the new scale, we wish to address two particularly relevant issues concerning the Higgs inflation model [11], one on the classical front and the other on the quantum front.

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[1] Within supersymmetry it is indeed possible to invoke the flat direction composed of the Higgses to realize inflation with minimal gravitational interaction, see [9], which can explain the current CMB observations.

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**I**. Classically, a large VEV of the inflaton does not pose a big problem as long as the initial energy density stored in the inflaton system, in the Einstein frame, is below the cut-off of the theory. Since, the potential energy remains bounded below this cut-off, the question remains – what should be the classical initial condition for the kinetic energy of the inflaton?

A priori there is no reason for the inflaton to move slowly on the plateau, therefore the question we wish to settle in this paper is what should be the range of phase space allowed for a sustainable inflation to occur with almost a flat potential? The aim of this paper is to address this classical initial condition problem.[2] Here we strictly assume homogeneity of the universe from the very beginning; we do not raise the issue of initial homogeneity condition required for a successful inflation; this issue has been discussed earlier in a generic inflationary context in many classic papers (see [15,16]). In our paper, instead we look into the possibility of initial phase space for a *spatially flat* universe, and study under what pre-inflationary conditions Higgs inflation could prevail.

**II**. At quantum level, the original Higgs model poses a completely different challenge. A large $\xi$ will inevitably modify the initial action. One may argue that there will be quantum corrections to the Ricci scalar, $R$, such as a Higgs-loop correction – leading to a quadratic in curvature action, i.e. $R+\alpha R^2$ type correction, where $\alpha$ is a constant, whose magnitude we shall discuss in this paper. The analysis is based on the renormalization group equations (RGEs) of the SM parameters and the gravitational interactions. By restricting for simplicity the study to operators up to dimension 4, the RGE analysis will yield a gravitational action that will become very similar to the Starobinsky type inflationary model [17].[3]

One of the features of theories with curvature squared terms is that there are extra degrees of freedom involved in the problem, besides the SM ones and the graviton. There is another scalar mode arising from $R^2$, which will also participate during inflation. The question then arises when this new scalar degree of freedom becomes dominant dynamically, and play the role of an inflaton creating the initial density perturbations?

The aim of this paper will be to address both the classical and quantum issues.

We briefly begin our discussion with essential ingredients of Higgs inflation in Section 2, then we discuss the classical pre-inflationary initial conditions for Higgs inflation in Section 3. In this section, we discuss both analytical in Subsection 3.1, and numerical results in Subsection 3.2. In Section 4, we discuss the quantum correction to the original Higgs inflation model, i.e. we discuss the RGEs of the Planck mass in Subsection 4.1, SM parameters in Subsection 4.2, and the gravitational correction arising due to large $\xi$ in Subsection 4.3, respectively. We briefly discuss our results and consequences for inflation in Subsection 4.4, before concluding our paper.

## 2. The model

Let us define the Higgs inflation model [11]. The action is

$$S=\int d^4x\sqrt{-g}\left[\mathscr{L}_{\rm SM}-\left(\frac{\bar{M}_{\rm Pl}^2}{2}+\xi|\mathcal{H}|^2\right)R\right], \tag{1}$$

where $\mathscr{L}_{\rm SM}$ is the SM Lagrangian minimally coupled to gravity, $\xi$ is the parameter that determines the non-minimal coupling between the Higgs and the Ricci scalar $R$, and $\mathcal{H}$ is the Higgs doublet. The part of the action that depends on the metric and the Higgs doublet *only* is

$$\int d^4x\sqrt{-g}\left[|\partial\mathcal{H}|^2-V-\left(\frac{\bar{M}_{\rm Pl}^2}{2}+\xi|\mathcal{H}|^2\right)R\right], \tag{2}$$

where $V=\lambda(|\mathcal{H}|^2-v^2/2)^2$ is the Higgs potential and $v$ is the electroweak Higgs VEV. We take a sizable non-minimal coupling, $\xi>1$, because this is required by inflation as we will see.

The non-minimal coupling $-\xi|\mathcal{H}|^2R$ can be eliminated through the *conformal* transformation

$$g_{\mu\nu}\to\Omega^{-2}g_{\mu\nu},\quad \Omega^2=1+\frac{2\xi|\mathcal{H}|^2}{\bar{M}_{\rm Pl}^2}. \tag{3}$$

The original frame, where the Lagrangian has the form in (1), is called the Jordan frame, while the one where gravity is canonically normalized (obtained with the transformation above) is called the Einstein frame. Therefore, the scalar-tensor action for the metric and the physical Higgs mode $\phi\equiv\sqrt{2|\mathcal{H}|^2}$ is (after the conformal transformation)

$$S_{\rm st}=\int d^4x\sqrt{-g}\left[K\frac{(\partial\phi)^2}{2}-\frac{V}{\Omega^4}-\frac{\bar{M}_{\rm Pl}^2}{2}R\right], \tag{4}$$

where $K\equiv\left(\Omega^2+6\xi^2\phi^2/\bar{M}_{\rm Pl}^2\right)/\Omega^4$. The non-canonical Higgs kinetic term can be made canonical through the (invertible) field redefinition $\phi=\phi(\chi)$ defined by

$$\frac{d\chi}{d\phi}=\sqrt{\frac{\Omega^2+6\xi^2\phi^2/\bar{M}_{\rm Pl}^2}{\Omega^4}}, \tag{5}$$

with the conventional condition $\phi(\chi=0)=0$. One can find a closed expression of $\chi$ as a function of $\phi$:

$$\chi(\phi)=\bar{M}_{\rm Pl}\sqrt{\frac{1+6\xi}{\xi}}\sinh^{-1}\left[\frac{\sqrt{\xi(1+6\xi)}\phi}{\bar{M}_{\rm Pl}}\right]-\sqrt{6}\bar{M}_{\rm Pl}\tanh^{-1}\left[\frac{\sqrt{6}\xi\phi}{\sqrt{\bar{M}_{\rm Pl}^2+\xi(1+6\xi)\phi^2}}\right]. \tag{6}$$

Thus, $\chi$ feels a potential

$$U\equiv\frac{V}{\Omega^4}=\frac{\lambda(\phi(\chi)^2-v^2)^2}{4(1+\xi\phi(\chi)^2/\bar{M}_{\rm Pl}^2)^2}. \tag{7}$$

Let us now recall how slow-roll inflation emerges. From (5) and (7) it follows [11] that $U$ is exponentially flat when $\chi\gg\bar{M}_{\rm Pl}$, which is the key property to have inflation. Indeed, for such high field values the slow-roll parameters

$$\epsilon\equiv\frac{\bar{M}_{\rm Pl}^2}{2}\left(\frac{1}{U}\frac{dU}{d\chi}\right)^2,\quad \eta\equiv\frac{\bar{M}_{\rm Pl}^2}{U}\frac{d^2U}{d\chi^2} \tag{8}$$

are guaranteed to be small. Therefore, the region in field configurations where $\chi>\bar{M}_{\rm Pl}$ (or equivalently [11] $\phi>\bar{M}_{\rm Pl}/\sqrt{\xi}$) corresponds to inflation. We will investigate whether successful slow-roll inflation emerges also for large initial field kinetic energy in the next section. Here we simply assume that the time derivatives are small.

All the parameters of the model can be fixed through experiments and observations, including $\xi$ [11,21]. $\xi$ can be obtained by requiring that the measured power spectrum [4],

[2] Some single monomial potentials and exponential potentials exhibit a classic example of late time attractor where the inflaton field approaches a slow roll phase from large initial kinetic energy, see [13,14].

[3] In principle, large $\xi$ may also yield higher derivative corrections up to quadratic in order, see [18], and also higher curvature corrections, but in this paper, we will consider for simplicity the lowest order corrections. We will argue that the $\alpha R^2$ is necessarily generated unless one is at the critical point of Refs. [19,20] or invokes a fine-tuning on the initial values of the running parameters.