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Constraining the double gluon distribution by the single gluon distribution

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ABSTRACT

We show how to consistently construct initial conditions for the QCD evolution equations for double parton distribution functions in the pure gluon case. We use to momentum sum rule for this purpose and a specific form of the known single gluon distribution function in the MSTW parameterization. The resulting double gluon distribution satisfies exactly the momentum sum rule and is parameter free. We also study numerically its evolution with a hard scale and show the approximate factorization into product of two single gluon distributions at small values of x, whereas at large values of x the factorization is always violated in agreement with the sum rule.

1. Introduction

Multiparton interactions play an important role in the hadronic collisions at high energies. They occur when at one encounter of the initial hadrons, more than one partonic interaction occurs. They were first observed and measured at the Tevatron [1–4] and subsequently a systematic experimental study was performed at the higher energy Large Hadron Collider [5–7]. The theoretical description of such interactions within perturbative QCD is possible in the presence of the sufficiently hard scales. The computation of double parton scattering (DPS) cross sections within the collinear framework makes use of the double parton distribution functions (DPDFs) [8–35]. In the collinear leading logarithmic approximation DPDFs obey QCD evolution equations [8,9,12,13,18,36], similar to the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) evolution equations for single parton distribution functions (PDFs). The evolution equations for DPDFs conserve new sum rules which relate the double and single parton distributions once they are imposed on initial conditions to the evolution equations at some initial scale. All the attempts up till now to construct conditions which satisfy these sum rules were rather unsuccessful, see e.g. Refs. [13,18,37] with an exception of the analysis [38] for valence quarks only.

In this letter, we show how to consistently perform such a construction in a pure gluon case, using the known single PDFs in the MSTW parameterization [39] and the momentum sum rule. We find the parameter free double gluon distribution which we evolve with our numerical program. In particular, we study the buildup of its approximately factorizable form for small values of parton momentum fractions, $x_{1,2} < 0.1$. The full case with quarks and gluons is postponed to a separate publication.

2. Evolution equations and sum rules

We consider the DPDFs with equal hard scales, $Q_1 = Q_2 \equiv Q$, and the relative momentum $\mathbf{q} = 0$:

$$D_{f_1 f_2}(x_1, x_2, Q) \equiv D_{f_1 f_2}(x_1, x_2, Q, Q, \mathbf{q} = 0)\,, \tag{1}$$

where $x_{1,2} \in [0, 1]$ are parton momentum fractions, which obey the condition $x_1 + x_2 \leq 1$, and $f_{1,2}$ are parton flavors (including gluon) [17,22]. In this case, the evolution equations in the leading logarithmic approximation read

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$$\frac{\partial}{\partial \ln Q^2} D_{f_1 f_2}(x_1, x_2, Q)$$
$$= \frac{\alpha_s(Q)}{2\pi} \sum_{f'} \left\{ \int_{x_1}^{1-x_2} \frac{du}{u} \mathcal{P}_{f_1 f'}\left(\frac{x_1}{u}\right) D_{f' f_2}(u, x_2, Q) \right.$$
$$+ \int_{x_2}^{1-x_1} \frac{du}{u} \mathcal{P}_{f_2 f'}\left(\frac{x_2}{u}\right) D_{f_1 f'}(x_1, u, Q)$$
$$+ \frac{1}{x_1 + x_2} P^R_{f' \to f_1 f_2}\left(\frac{x_1}{x_1 + x_2}\right)$$
$$\left. \times D_{f'}(x_1 + x_2, Q) \right\}, \quad (2)$$

where the functions $\mathcal{P}$ on the r.h.s. are the leading order Altarelli–Parisi splitting functions (with virtual corrections for $\mathcal{P}_{ff}$ included). The third term on the r.h.s. corresponds to the splitting of one parton into two daughter partons, described by the Altarelli–Parisi splitting function for real emission, $P^R_{f' \to f_1 f_2}$. It contains the single PDF, $D_{f'}$, thus eq. (2) has to be solved together with the ordinary DGLAP equations, see e.g. Ref. [18] for more details.

The significance of the splitting terms in the evolution equations (2) for the computation of the double parton scattering cross sections was a subject of intensive debate in the literature over the last few years [17,18,21,22,26,31,32,40]. The conclusion which emerges from this discussion is that the processes which are summed up by the splitting terms and coming from both hadrons in hadron–hadron collisions should rather be classified as the single parton scattering process [26]. On the other hand, the so-called single splitting contributions, with parton splitting from one hadronic side only, are important for the double parton scattering cross sections [15,31,32,41]. From the perspective of the present paper, in which we only concentrate on the evolution of the DPDFs, the splitting terms in the evolution equations are crucial for the conservation of sum rules which are discussed below.

The sum rules which are conserved by the evolution equations (2) are the momentum and valence quark number sum rules [14]. Imposing them for initial conditions specified at some initial scale Q_0, they are guaranteed to be satisfied at any other scale Q. The momentum sum rule for the DPDFs reads

$$\sum_{f_1} \int_0^{1-x_2} dx_1\, x_1 D_{f_1 f_2}(x_1, x_2) = (1 - x_2) D_{f_2}(x_2)\,, \quad (3)$$

while the valence quark number sum rule is given by

$$\int_0^{1-x_2} dx_1 \left\{ D_{q f_2}(x_1, x_2) - D_{\bar{q} f_2}(x_1, x_2) \right\}$$
$$= (N_q - \delta_{f_2 q} + \delta_{f_2 \bar{q}}) D_{f_2}(x_2)\,, \quad (4)$$

where $q = u, d, s$ and $N_u = 2$, $N_d = 1$, $N_s = 0$ are the valence quark number for each of the quark flavors. The same relations hold true with respect to the second parton

$$\sum_{f_2} \int_0^{1-x_1} dx_2\, x_2 D_{f_1 f_2}(x_1, x_2) = (1 - x_1) D_{f_1}(x_1)\,, \quad (5)$$

$$\int_0^{1-x_1} dx_2 \left\{ D_{f_1 q}(x_1, x_2) - D_{f_1 \bar{q}}(x_1, x_2) \right\}$$
$$= (N_q - \delta_{f_1 q} + \delta_{f_1 \bar{q}}) D_{f_1}(x_1)\,. \quad (6)$$

Notice that if the DPDFs are parton exchange symmetric,

$$D_{f_1 f_2}(x_1, x_2) = D_{f_2 f_1}(x_2, x_1)\,, \quad (7)$$

the sum rules with respect to the first parton imply the sum rules with respect to the second one since the evolution equations also conserve parton exchange symmetry.

We see that the above sum rules relate the double and single parton distribution functions, which reflects the common origin of those distributions, namely, the expansion of the nucleon state in Fock light-cone components [14]. In addition, the sum rules for the single parton distributions are also satisfied – the momentum sum rule

$$\sum_f \int_0^1 dx\, x D_f(x) = 1 \quad (8)$$

and the quark valence sum rule for $q = u, d, s$

$$\int_0^1 dx \left\{ D_q(x) - D_{\bar{q}}(x) \right\} = N_q\,. \quad (9)$$

3. Mellin moment formulation

Let us perform the double Mellin transform of the DPDFs

$$\tilde{D}_{f_1 f_2}(n_1, n_2) = \int_0^1 dx_1 \int_0^1 dx_2\, (x_1)^{n_1 - 1} (x_2)^{n_2 - 1}$$
$$\times D_{f_1 f_2}(x_1, x_2) \Theta(1 - x_1 - x_2), \quad (10)$$

where $n_{1,2}$ are complex numbers and we omit the scale Q_0 in the notation from now on. The step function $\Theta(1 - x_1 - x_2)$ is inserted into the definition of the Mellin transform since this is the region over which the double parton distribution is defined. Similarly, for the single parton distribution functions, we define the Mellin moments

$$\tilde{D}_f(n) = \int_0^1 dx\, x^{n-1} D_f(x)\,, \quad (11)$$

where n is a complex number. The Mellin moments can be transformed back to the x-space using the inverse transformation for the single parton distribution,

$$D_f(x_1) = \int_C \frac{dn}{2\pi i} (x_1)^{-n} \tilde{D}_f(n)\,, \quad (12)$$

and similarly for the double parton distribution function

$$D_{f_1 f_2}(x_1, x_2) = \int_{C_1} \frac{dn_1}{2\pi i} (x_1)^{-n_1} \int_{C_2} \frac{dn_2}{2\pi i} (x_2)^{-n_2} \tilde{D}_{f_1 f_2}(n_1, n_2), \quad (13)$$

where the integration contours C_1 and C_2 lie to the right of the rightmost singularity in the complex plane of n_1 and n_2, respectively. Let us emphasize that formula (13) is only applicable to $x_{1,2} \in [0, 1]$ and $x_1 + x_2 \le 1$.