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Application of Kawaguchi Lagrangian formulation to string theory

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ABSTRACT

String-scalar duality proposed by Y. Hosotani and membrane-scalar duality by A. Sugamoto are reexamined in the context of Kawaguchi Lagrangian formulation. The characteristic feature of this formulation is the indifferent nature of fields and parameters. Therefore even the exchange of roles between fields and parameters is possible. In this manner, dualities above can be proved easily. Between Kawaguchi metrics of the dually related theories, a simple relation is found. As an example of the exchange between fermionic fields and parameters, a replacement of the role of Grassmann parameters of the 2-dimensional superspace by the 9th component of Neveu–Schwarz–Ramond (NSR) fermions is studied in superstring model. Compactification is also discussed in this model.

1. Introduction

Recently, one of the authors (RY) proposed in collaboration with Ootsuka, Ishida and Tanaka, a covariant Lagrangian formalism for field theories with the aid of Kawaguchi geometry [1]. Kawaguchi space [2], also known as areal space, is defined by a set of a manifold M and an areal metric called Kawaguchi metric K. In case of field theories, M includes fields y^i $(i = 1, \cdots, n)$ and parameters x^μ $(\mu = 0, 1, \cdots, D-1)$ of space and time, on the equal footing, and is called extended configuration space. We label them as Z^a $(a = 0, \cdots, n+D-1)$. Kawaguchi metric K is a kind of Lagrangian density, depending on the coordinates of point in M and its derivatives,

$$K = K(Z^a,\ dZ^{a_1 a_2 \cdots a_D}),\ \text{where} \tag{1}$$

$$dZ^{a_1 a_2 \cdots a_D} \equiv dZ^{a_1} \wedge dZ^{a_2} \wedge \cdots \wedge dZ^{a_D}. \tag{2}$$

Integral of K over a given "sheet" S (or a higher dimensional submanifold depending on the theory) gives an area (or volume) of the "sheet". This gives an action of field theories:

$$\begin{aligned} \text{Action} &= \int_S K \\ &= \text{the area or volume of a sub-manifold } S. \end{aligned} \tag{3}$$

To guarantee the reparametrization invariance, the homogeneity condition is imposed on the Kawaguchi metric, namely

$$K(Z,\ \lambda dZ^{a_1 a_2 \cdots a_D}) = \lambda K(Z,\ dZ^{a_1 a_2 \cdots a_D}). \tag{4}$$

In the formulation, fields and parameters are not identified. If we assign $\{y^i\}$ as fields and $\{x^\mu\}$ as parameters among $\{Z^a\}$, the fields become functions of the parameters, such as $\sigma:\ y^i = y^i(x^0, \cdots, x^{D-1})$. After the parametrization σ is fixed, we have

$$\sigma^* \left(\frac{dx^{0 \cdots \mu-1} \wedge dy^i \wedge dx^{\mu+1 \cdots D-1}}{dx^{0 \cdots D-1}} \right) = \partial_\mu y^i(x), \tag{5}$$

and the usual description of field theories appears. This operation σ^* is sometimes called pullback of a parametrization σ. However it is important to note that there are a number of different ways for the parametrization.

In [1], it is proved that every known action can be an area (a volume) of a certain subspace, so that every field theory can be reformulated à la Kawaguchi. Nambu–Goto action is the prototype of this formulation. The equal treatment of fields and parameters

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in Kawaguchi Lagrangian formulation gives much potential to reveal dualities which exist between different physical models.

Historically, Finsler introduced in the metric a derivative $\dot{x}^\mu$ in addition to the coordinates x^μ. This Finsler metric reads

$$F = F(x^\mu, \dot{x}^\mu)dt. \tag{6}$$

Physicists understand easily the Finsler metric is nothing but the Lagrangian of quantum mechanics, giving a temporal development of the dynamics in terms of t. Afterwards Kawaguchi generalizes the Finsler geometry so as to include higher order derivatives $\ddot{x}^\mu, \cdots$, and also generalize it to the case with many parameters (field theories) mentioned above.

The purpose of this paper is to derive dualities among different physical models, using the indifferent nature of fields and parameters in the Kawaguchi Lagrangian formalism.

It is known in string theory and membrane theory in four space-time dimensions, the models are dually related to a field theory with two scalars by Hosotani [3] and a single scalar field theory by Sugamoto [4], respectively. Therefore we first reexamine these dualities in the next section. The exchange of fields by parameters is clearly demonstrated.

A generalization of the dualities given in [3] and [4] is studied by Morris [5] afterwards. Baker and Fairlie [6] studied scalar field description of p-branes, by generalizing the Hamilton–Jacobi formalism of string by Nambu [7].

In section three, we display an example of exchanging fermonic fields and parameters in the superstring model, where a fermionic field (NSR field) in the 9th component ψ^9 is exchanged by a parameter θ of the superspace. Compactification is also discussed in this model.

2. String and membrane dualities

In this section, we illustrate how to see dualities in terms of Kawaguchi Lagrangian formalism, taking up two Nambu–Goto type examples. One example is introduced by Y. Hosotani [3], which gives a duality between Nambu–Goto string and scalar field theory. Another example is given by one of the authors (AS) for membrane theory [4]. We demonstrate that their dualities can be observed manifestly at action level, which is originally proved by seeing the equations of motion. Here we consider Euclidean spacetime.

2.1. String-scalar duality

The actions for strings and two scalar fields in 4-dimensional spacetime in [3] are given by

$$S_{\text{string}} = \int d\tau d\sigma \sqrt{\frac{1}{2}(V^{\mu\nu})^2}, \qquad V^{\mu\nu} = \frac{\partial(X^\mu, X^\nu)}{\partial(\tau, \sigma)}, \tag{7}$$

$$S_{\text{scalars}} = \int d^4x \sqrt{\frac{1}{2}(W_{\mu\nu})^2}, \qquad W_{\mu\nu} = \frac{\partial(\rho, \phi)}{\partial(X^\mu, X^\nu)}, \tag{8}$$

respectively, where ρ and ϕ are scalar fields on spacetime X^μ, $\mu = 0, 1, 2, 3$. τ and σ are worldsheet coordinates. Coefficients are taken arbitrary, since they are of no importance in this argument. Important fact is that one is 2-dimensional field theory and the other is 4-dimensional field theory.

To consider the duality between these two theories, we set a manifold $M = \{(\tau, \sigma, \rho, \phi, X^\mu)\}$. Kawaguchi metrics for these actions are

$$K_{\text{string}} = \sqrt{\frac{1}{2}(dX^{\mu\nu})^2} \tag{9}$$

$$K_{\text{scalars}} = \sqrt{\frac{1}{2}(dX^{\mu\nu} \wedge d\rho \wedge d\phi)^2}. \tag{10}$$

(9) and (10) have the same structure; only the difference is the degree of differential forms. Let K be

$$K(\cdots) = \sqrt{\frac{1}{2}(\cdots)^2}. \tag{11}$$

We can write

$$S_{\text{string}} = \int K\left(\frac{\partial(X^\mu, X^\nu)}{\partial(\xi^0, \xi^1)}\right) d\xi^{01}, \tag{12}$$

for arbitrary parametrization (ξ^0, ξ^1). It can be naturally extended to 4-dimensional field theory by adding extra scalar degrees of freedom as

$$S' = \int K\left(\frac{\partial(X^\mu, X^\nu)}{\partial(\xi^0, \xi^1)}\right) d\xi^{01} \wedge d\rho \wedge d\phi. \tag{13}$$

These additional ρ and ϕ are degrees of freedom that are perpendicular to the worldsheet. An identity of Jacobian gives

$$\begin{aligned} S' &= \int K\left(\frac{1}{2}\epsilon^{\mu\nu\lambda\eta}\frac{\partial(X^0, X^1, X^2, X^3)}{\partial(\xi^0, \xi^1, \rho, \phi)}\frac{\partial(\rho, \phi)}{\partial(X^\lambda, X^\eta)}\right) d\xi^{01} \wedge d\rho \wedge d\phi \\ &= \int K\left(\frac{1}{2}\epsilon^{\mu\nu\lambda\eta}\frac{\partial(\rho, \phi)}{\partial(X^\lambda, X^\eta)}\right)\frac{\partial(X^0, X^1, X^2, X^3)}{\partial(\xi^0, \xi^1, \rho, \phi)} d\xi^{01} \wedge d\rho \wedge d\phi \\ &= \int K\left(\frac{\partial(X^\mu, X^\nu, \rho, \phi)}{\partial(X^0, X^1, X^2, X^3)}\right) dX^{0123} = S_{\text{scalars}}, \end{aligned} \tag{14}$$

where $\epsilon^{\mu\nu\lambda\eta}$ is the anti-symmetric Levi–Civita symbol with $\epsilon^{0123} = 1$. From the first line to the second line of the above equation, we use the homogeneity condition of the Kawaguchi metric. The last line shows that S' is indeed a pullbacked action determined by (10) to the parameter space (X^0, X^1, X^2, X^3).

2.2. Membrane-scalar duality

Similar duality can be seen between membrane theory and scalar field theory in 4-dimension. The actions are

$$S_{\text{membrane}} = \int d\tau d\sigma d\rho \sqrt{\frac{1}{3!}(V^{\mu\nu\lambda})^2}, \quad V^{\mu\nu\lambda} = \frac{\partial(X^\mu, X^\nu, X^\lambda)}{\partial(\tau, \sigma, \rho)}, \tag{15}$$

$$S_{\text{scalar}} = \int d^4x \sqrt{\left(\frac{\partial\phi}{\partial X^\mu}\right)^2}, \tag{16}$$

with scalar field ϕ.

We consider a manifold $M = \{(\tau, \sigma, \rho, \phi, X^\mu)\}$, and Kawaguchi metrics for these actions are

$$K_{\text{membrane}} = \sqrt{\frac{1}{3!}(dX^{\mu\nu\lambda})^2} \tag{17}$$

$$K_{\text{scalar}} = \sqrt{\frac{1}{3!}(dX^{\mu\nu\lambda} \wedge d\phi)^2}. \tag{18}$$

As well as the string-scalar case, the membrane action (15) is written by

$$S_{\text{membrane}} = \int K\left(\frac{\partial(X^\mu, X^\nu, X^\lambda)}{\partial(\xi^0, \xi^1, \xi^2)}\right) d\xi^{012}, \quad K(\cdots) = \sqrt{\frac{1}{3!}(\cdots)^2}, \tag{19}$$

for arbitrary parameters (ξ^0, ξ^1, ξ^3). Then we obtain