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Entropy of an extremal electrically charged thin shell and the extremal black hole

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ABSTRACT

There is a debate as to what is the value of the entropy S of extremal black holes. There are approaches that yield zero entropy $S = 0$, while there are others that yield the Bekenstein–Hawking entropy $S = A_+/4$, in Planck units. There are still other approaches that give that S is proportional to r_+ or even that S is a generic well-behaved function of r_+. Here r_+ is the black hole horizon radius and $A_+ = 4\pi r_+^2$ is its horizon area. Using a spherically symmetric thin matter shell with extremal electric charge, we find the entropy expression for the extremal thin shell spacetime. When the shell's radius approaches its own gravitational radius, and thus turns into an extremal black hole, we encounter that the entropy is $S = S(r_+)$, i.e., the entropy of an extremal black hole is a function of r_+ alone. We speculate that the range of values for an extremal black hole is $0 \leq S(r_+) \leq A_+/4$.

1. Introduction

The entropy S and thermodynamics of black holes have been worked out first by Bekenstein [1] and Hawking and collaborators [2,3]. The Bekenstein–Hawking entropy is given by $S = A_+/4$, where $A_+ = 4\pi r_+^2$, A_+ and r_+ are the horizon area and the horizon radius, respectively, and we are putting all the natural constants equal to one, i.e., we use Planck units. York and collaborators [4–6] (see also [7,8]) have further worked out the black hole thermodynamic properties by using canonical and grand canonical thermodynamic ensembles. There are several other methods that can be used to study black hole thermodynamics, one that suits us here uses matter shells [9–11]. In this method, one studies the generic thermodynamics of the shells at any shell radius, and as one sends the shell to its own gravitational radius one recovers the $S = A_+/4$ Bekenstein–Hawking entropy. This is the quasiblack hole method, the evident power of it was displayed in [12].

A particular class of black holes is the extremal black hole class. Electrically charged black holes in general relativity, the ones we are interested here, have $m \geq Q$, and the extremal black holes are characterized by having their mass m equal to their electric charge Q, $m = Q$. The extremal black holes seem to have distinct properties. For instance, according to the Hawking temperature formula, extremal black holes have zero temperature. In addition the entropy of an extremal black hole is a subject of a wide debate as there are different reasonings that can be applied which lead to different values for the entropy. Hawking and collaborators [13] and Teitelboim [14] have given topological arguments which point to the conclusion that extremal black holes have zero entropy. Further evidence from other arguments for $S = 0$ for extremal black holes was provided in [15–17], see also [18,19]. One could also argue, naively, that since the Hawking temperature is zero, then according to one of the formulations of the third law of thermodynamics as many textbooks present it should have zero entropy.

However, there remain doubts why the Bekenstein–Hawking formula does not hold. After all, working out the entropy of non-extremal black holes and taking the extremal limit $m = Q$ yields $S = A_+/4$, see, e.g., [2,3,5,10]. In this case, the thermodynamic argument would not hold, the extremal black hole could be a system of minimum energy and degenerate ground state and such systems can have entropy even at zero temperature. Moreover, in string theory, there are arguments, other than geometrical, that make use of a direct counting of string and D-brane states in composite

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systems, which manipulate carefully the turning on of gravity and electricity adiabatically by equal amounts to maintain extremality without changing the counting of states and thus the entropy, that show that the entropy of an extremal black hole is $S = A_+/4$, as first delivered by Strominger and Vafa [20], see [21] for a review. Other methods also indicate the $S = A_+/4$ value. These are methods that use quantum field corrections to the black hole entropy [22], Hamiltonian methods [23,24], and thermodynamics ensemble methods [25–27]. There are works that show that depending on the black hole type, i.e., black holes with scalar fields, one has $S = 0$ or $S = A_+/4$ [28,29].

Given that several different calculations give $S = 0$ or $S = A_+/4$ one might open the box and speculate that other values for the entropy are possible, e.g., any value between 0 and $A_+/4$, or possibly other functions. Indeed, in a semiclassical calculation of the entropy on an extremal black hole background Ghosh and Mitra [30] pointed to an entropy value proportional to r_+ rather than A_+ (i.e., r_+^2). This was followed by some discussion for the exact possible values for the entropy of an extremal black hole [31–33].

In addition, in a setup using an extremal charged thin shell contracting reversibly and arranged to maintain extremality it was shown in [34] that any value of the entropy of the shell in passing its own gravitational radius could be achieved. In another setting, using also thin shells, it was shown that in the quasiblack hole limit, i.e., when the boundary of the matter is at its own gravitational radius, the entropy of the extremal black hole is a generic function of r_+ [35].

One feature important to note is that a calculation of the stress-energy tensor of quantum fields at the neighborhood of the event horizon excludes the possibility that an extremal black hole can be in thermal equilibrium with radiation at any temperature. An extremal black hole has zero temperature and if the temperature of the surrounding fields is nonzero then the stress-energy diverges strongly [36].

This paper is committed to the study of the entropy of extremally charged spherically symmetric thin shells of any radii, and in particular, to the understanding of the entropy of the system when the radius of the shell is its own gravitational radius, i.e., the extremal shell spacetime turns into an extremal black hole spacetime. This yields an expression for the black hole entropy. We follow the formalism of Martinez [9] developed for electric non-extremal shells in [10] and for rotating BTZ spacetimes in [11]. The thermodynamic analysis of Callen [37] is used, as it was used in [9–11]. The importance of Callen's analysis for black hole thermodynamics was first understood by York [4], see also [5,8]. Here, we restrict ourselves to spherically symmetric systems but, as the results have a rather general character, we believe that, with some minor changes, they are valid for distorted and rotating systems as well (see [11] for a rotating $(2+1)$-dimensional spacetime).

The work is organized as follows: In Section 2 we give the mechanical properties of an extremal electrically charged thin shell. Such type of matter is also called electrically counterpoised dust. In Section 3 we will analyze the first law of thermodynamics applied to such a thin shell of any radius and find the entropy of the spacetime. In Section 4 we take the shell to its own gravitational radius and find that the entropy of an extremal black hole is a generic function of r_+. We also speculate on the possible values for the entropy of an extremal black hole. In Section 5 we display another interesting shell that can be taken to its own gravitational radius and find the corresponding entropy. In Section 6 we draw our conclusions.

2. The extremal charged thin shell spacetime

The Einstein–Maxwell equations in four spacetime dimensions are given by

$$G_{\alpha\beta} = 8\pi\, T_{\alpha\beta}\,, \tag{1}$$

$$\nabla_\beta F^{\alpha\beta} = 4\pi\, J^\alpha\,. \tag{2}$$

$G_{\alpha\beta}$ is the Einstein tensor, built from the spacetime metric $g_{\alpha\beta}$ and its first and second derivatives, 8π is the coupling, and we are using units in which the velocity of light is one and the gravitational constant G is also put to one $G = 1$. $T_{\alpha\beta}$ is the energy-momentum tensor. $F_{\alpha\beta}$ is the Faraday–Maxwell tensor, J_α is the electromagnetic four-current and ∇_β denotes covariant derivative. The other Maxwell equation $\nabla_{[\gamma} F_{\alpha\beta]} = 0$, where $[\ldots]$ means antisymmetrization, is automatically satisfied for a properly defined $F_{\alpha\beta}$. Greek indices will be used for spacetime indices and run as $\alpha, \beta = 0, 1, 2, 3$, with 0 being the time index.

The concept of thin shell is associated with the presence of matter in the surface that separates two partitions of spacetime, each with its own metric. We will be considering the case of a four-dimensional spherically symmetric spacetime and a spherical thin shell at some radius R separating an inner region $\mathcal{V}_i$ with flat metric and an outer region $\mathcal{V}_o$ with an extremal Reissner–Nordström line element. Thus, for the inner region the metric is

$$ds_i^2 g^i_{\alpha\beta} dx^\alpha dx^\beta = -dt_i^2 + dr^2 + r^2\, d\Omega^2\,, \quad r \le R\,, \tag{3}$$

where $x^\alpha = (t_i, r, \theta, \phi)$ are the inner coordinates, with t_i being the inner time, and (r, θ, ϕ) polar coordinates, and $d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$. For the outer region the metric is

$$\begin{aligned} ds_o^2 &= g^o_{\alpha\beta} dx^\alpha dx^\beta \\ &= -\left(1 - \frac{m}{r}\right)^2 dt_o^2 + \frac{dr^2}{\left(1 - \frac{m}{r}\right)^2} + r^2 d\Omega^2\,, \quad r \ge R\,, \end{aligned} \tag{4}$$

where $x^\alpha = (t_o, r, \theta, \phi)$ are the outer coordinates, with t_o being the outer time, and (r, θ, ϕ) polar coordinates. In addition, m is the ADM mass, and Q is the electric charge. In the extremal case they are related by

$$m = Q\,. \tag{5}$$

On the hypersurface itself, $r = R$, the metric is that of a 2-sphere with an additional time dimension, such that the line element is

$$ds_\Sigma^2 = h_{ab} dy^a dy^b = -d\tau^2 + R^2(\tau) d\Omega^2\,, \quad r = R\,, \tag{6}$$

where we have chosen $y^a = (\tau, \theta, \phi)$ as the time and spatial coordinates on the shell. Latin indices apply for the components on the hypersurface. The time coordinate τ is the proper time for an observer located at the shell. The shell radius is given by the parametric equation $R = R(\tau)$ for an observer on the shell. We consider a static shell so that $R(\tau) = \text{constant}$. On each side of the hypersurface, the parametric equations for the time and radial coordinates are denoted by $t_i = T_i(\tau)$, $r_i = R_i(\tau)$, and $t_o = T_o(\tau)$, $r_o = R_o(\tau)$.

Imposing that the fluid in the shell is a perfect fluid with stress-energy tensor $S^a{}_b$ given by

$$S^a{}_b = (\sigma + p) u^a u_b + p h^a{}_b\,, \tag{7}$$

where u^a is the 3-velocity of a shell element, one finds through the junction conditions (see, e.g., [10])

$$\sigma = \frac{m}{4\pi R^2}\,, \tag{8}$$

$$p = 0\,. \tag{9}$$