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On the regularization of extremal three-point functions involving giant gravitons

Charlotte Kristjansen [a], Stefano Mori [a], Donovan Young [b,*]

[a] Niels Bohr Institute, Copenhagen University, Blegdamsvej 17, 2100 Copenhagen Ø, Denmark
[b] Centre for Research in String Theory, School of Physics and Astronomy, Queen Mary University of London, Mile End Road, London E1 4NS, United Kingdom

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ABSTRACT

In the AdS_5/CFT_4 set-up, extremal three-point functions involving two giant 1/2 BPS gravitons and one point-like 1/2 BPS graviton, when calculated using semi-classical string theory methods, match the corresponding three-point functions obtained in the tree-level gauge theory. The string theory computation relies on a certain regularization procedure whose justification is based on the match between gauge and string theory. We revisit the regularization procedure and reformulate it in a way which allows a generalization to the ABJM set-up where three-point functions of 1/2 BPS operators are not protected and where a match between tree-level gauge theory and semi-classical string theory is hence not expected.

After the successful application of integrability techniques to the planar spectral problem of the AdS_5/CFT_4 set-up [1], the calculation of three-point functions in the same set-up has attracted renewed attention with some recent highlights being the conjecture of an all loop formula for three-point functions of single trace operators in certain sub-sectors of $\mathcal{N}=4$ SYM [2] and the formulation of certain integrability axioms for the cubic string theory vertex [3].

We will be considering three-point functions which do not belong to the class of three-point functions considered in the above references. Our three-point functions involve giant gravitons which in the string theory language correspond to higher dimensional D- or M-branes wrapping certain submanifolds of the string theory background and which in the gauge theory picture are represented by specific linear combinations of multi-trace operators, namely Schur polynomials. Remaining in the gauge theory picture, our three-point functions will involve two Schur polynomials and one single trace operator, all of 1/2 BPS type. Furthermore, the three operators will be chosen such that $\Delta_1 = \Delta_2 + \Delta_3$, where the Δ's are the conformal dimensions of the operators. Such three-point functions are denoted as extremal three-point functions and are known to require special care in the comparison between gauge and string theory [4]. On the gauge theory side the three-point functions of interest can be calculated using techniques from zero-dimensional field theories [5] (see also [6]) and on the string theory side they can be determined by generalizing a method developed for the calculation of heavy–heavy–light correlators [7–9] from string states to membranes [5].

In the case of the $AdS_5 \times S^5$ correspondence the 1/2-BPS nature of the operators involved implies that the three-point function considered is protected and thus should take the same value whether calculated in string theory or in gauge theory. As pointed out in [10] the need for special treatment of extremal correlators in string theory is relevant here and in [11] a regularization procedure for the string theory computation which led to the desired match between gauge and string theory was presented.

The $AdS_4 \times \mathbb{CP}^3$ set-up [12] allows one to consider a similar correlator i.e. an extremal three-point function involving two 1/2 BPS giant gravitons in combination with one 1/2 BPS point-like graviton and the methods developed in [5] for the AdS_5/CFT_4 calculation can be generalized to this case as well [13]. One remaining subtle point is the choice of regularization procedure in the string theory computation. In the $AdS_4 \times \mathbb{CP}^3$ correspondence three-point functions of 1/2 BPS operators are not protected and hence in this set-up we cannot expect a match between gauge and string theory results. In particular, this means that on one hand we cannot justify our choice of regulator by a match between the gauge and string theory results but on the other hand a computation of the correlator in the weakly coupled string theory will provide us with a non-trivial prediction about the behaviour of the correlator in the dual strongly coupled field theory. Below we will revisit the regularization procedure employed for the $AdS_5 \times S^5$

* Corresponding author.
E-mail addresses: kristjan@nbi.ku.dk (C. Kristjansen), moriste@nbi.ku.dk (S. Mori), d.young@qmul.ac.uk (D. Young).

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computation and modify it in a way that allows us to generalize it to the $AdS_4 \times \mathbb{C}\mathrm{P}^3$ case. Subsequently, we carry out the string theory calculation of the extremal three-point function involving two giant 1/2 BPS gravitons and one point-like 1/2 BPS graviton in $AdS_4 \times \mathbb{C}\mathrm{P}^3$.

1. Giant graviton correlators in $AdS_5 \times S^5$ revisited

Giant gravitons in $AdS_5 \times S^5$ are D3-branes which wrap an S^3 which constitutes a subset of either AdS_5 or S^5 [14–16]. We will consider the giants for which the wrapped sphere $S^3 \subset S^5$ and which spin along a circle in the S^5 while being located at the center of AdS_5. For such giants the dual gauge theory operators are Schur polynomials built on completely anti-symmetric Young diagrams and containing a single complex scalar field that we will denote as Z [17,6]. The D3-brane action is (in units where the AdS radius has been set to 1)

$$S_{D3} = -\frac{N}{2\pi^2}\int d^4\sigma \left(\sqrt{-g} - P[C_4]\right), \tag{1}$$

where $g_{ab} = \partial_a X^M \partial_b X^N G_{MN}$, with $a, b = 0, \ldots, 3$ being the worldvolume coordinates and X^M the embedding coordinates. Working in global AdS coordinates

$$\begin{aligned} ds^2 = &-\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\widetilde{\Omega}_3^2 \\ &+ d\theta^2 + \sin^2\theta\, d\phi^2 + \cos^2\theta\, d\Omega_3^2, \end{aligned} \tag{2}$$

the four-form potential C_4 can be written as [15]

$$C_{\phi\chi_1\chi_2\chi_3} = \cos^4\theta\, \mathrm{Vol}(\Omega_3), \tag{3}$$

where the χ_i are the angles of the wrapped sphere, i.e. $d\Omega_3^2 = d\chi_1^2 + \sin^2\chi_1 d\chi_2^2 + \cos^2\chi_1 d\chi_3^2$. Using the ansatz

$$\rho = 0, \quad \sigma^0 = t, \quad \phi = \phi(t), \quad \sigma^i = \chi_i, \tag{4}$$

one can show that a D3-brane with angular momentum k is stable when it sits at

$$\cos^2\theta = k/N, \tag{5}$$

and spins at the speed of light, $\dot{\phi} = 1$ [14]. In order to determine the three-point function of two giant gravitons and a point-like graviton (i.e. a chiral primary) we should determine the variation of the Euclidean version of the D3-brane action in response to the insertion of the desired chiral primary at the boundary of AdS and subsequently evaluate these fluctuations on the Wick rotated giant graviton solution. As this procedure was described in detail in [5] we shall be brief here. Denoting the spherical harmonic representing the point-like graviton as Y_Δ (with Δ referring to its conformal dimension) we can write the variation of the D3-brane action as [5]

$$\begin{aligned} \delta S = \frac{N}{2\pi^2}\cos^2\theta \int d^4\sigma \Bigg(&\frac{2}{\Delta+1} Y_\Delta\, (\partial_t^2 - \Delta^2)\, s^\Delta \\ &+ 4\left[\Delta\cos^2\theta - \sin\theta\cos\theta\,\partial_\theta\right] s^\Delta Y_\Delta \Bigg), \end{aligned} \tag{6}$$

where s^Δ is the bulk to boundary propagator. As our spherical harmonic we choose

$$Y_\Delta = \frac{\sin^\Delta\theta\, e^{i\Delta\phi}}{2^{\Delta/2}}, \tag{7}$$

which corresponds to the single trace operator $\mathrm{Tr}\, Z^\Delta$ in the gauge theory language. With this choice for Y_Δ the first term in eq. (6) is finite and gives the following contribution to the three-point function structure constant

$$C^3_{finite} = -\sqrt{\Delta}\, \frac{k}{N}\left(1 - \frac{k}{N}\right)^{\Delta/2}, \tag{8}$$

whereas the contribution coming from the term with square brackets takes the form of a divergent integral with a pre-factor which tends to zero. In Ref. [11] it was proposed to regularize the divergent integral by replacing the simple spherical harmonic Y_Δ with the more involved one

$$Y_{\Delta+2l,\Delta} = \mathcal{N}_{\Delta,l}\sin^\Delta\theta\, e^{i\Delta\phi}\, {}_2F_1(-l, \Delta+l+2; \Delta+1; \sin^2\theta), \tag{9}$$

where $\mathcal{N}_{\Delta,l}$ is a normalization factor, and to consider the limit $l \to 0$ where $Y_{\Delta+2l,\Delta} \to Y_\Delta$, and where the contribution from the ill defined term of eq. (6) becomes finite. This procedure leads to a match between the string and gauge theory computation. The obtained match justifies the choice of the regularization procedure but does not suggest a general principle that one could build on when aiming at a generalization to giant gravitons in $AdS_4 \times \mathbb{C}\mathrm{P}^3$. One property which characterizes the spherical harmonic (9) is that it extends the simple one without making use of additional coordinates on S^5. However, this property is somewhat deceptive and is not the correct clue to an extension to the $AdS_4 \times \mathbb{C}\mathrm{P}^3$ set-up.

Here we shall formulate the regularization procedure in a slightly different manner which will allow us to generalize it to the latter set-up. For that purpose we make use of the fact that spherical harmonics on S^5 are in one-to-one correspondence with symmetric traceless $SO(6)$ tensors. In particular (leaving out normalization factors) the spherical harmonic (7), which translates into $\mathrm{Tr}\, Z^\Delta$ in the field theory, corresponds to the tensor[1]

$$C_{\underbrace{1\ldots1}_{\Delta-k}\underbrace{2\ldots2}_{k}} = i^k, \tag{10}$$

where symmetrization is understood. It is easy to show that adding more indices of type 1 and type 2 to the tensor (i.e. adding more fields of type Φ_1 and Φ_2 to the operator) does not regularize the divergent integral. However, one can regularize the integral by considering the following symmetric traceless tensor

$$C_{\underbrace{1\ldots1}_{\Delta-k}\underbrace{2\ldots2}_{k}\underbrace{3\ldots3}_{2l-n}\underbrace{4\ldots4}_{n}} = i^{k+n}, \tag{11}$$

where $n < 2l$ and subsequently taking the limit $l \to 0$. Obviously, the gauge theory operator resulting from this tensor involves the complex field $Y = \Phi_3 + i\Phi_4$ in addition to the complex field Z. It is easy to check that the spherical harmonic (9) corresponds to an operator involving all six scalar fields of $\mathcal{N} = 4$ SYM but it is not straightforward to express the corresponding C-tensor in a closed form. The tensor (11) translates into the following spherical harmonic

$$Y_{\Delta l} = \mathcal{N}_{\Delta l}\sin^\Delta(\theta)\, e^{i\Delta\phi}\cos^2(l\theta)\, \sin^2(l\,\chi_1)\, e^{2il\,\chi_2}, \tag{12}$$

where $\mathcal{N}_{\Delta l}$ is another normalization constant. Using this spherical harmonic instead of Y_Δ when evaluating the second line of (6) and subsequently taking the limit $l \to 0$ gives us the following result for the regularized contribution to the three-point function

[1] The chiral primary operators of $\mathcal{N} = 4$ SYM can be written in the form $C_I^{i_1 i_2 \ldots i_\Delta}\, \mathrm{Tr}(\Phi_{i_1}\Phi_{i_2}\ldots\Phi_{i_\Delta})$ where the Φ_i's can be any of the six real scalar fields and where C_I is a symmetric traceless tensor. We take the complex scalar field Z to be given by $Z = \Phi_1 + i\Phi_2$.