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"Hot entanglement"? – A nonequilibrium quantum field theory scrutiny

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ABSTRACT

The possibility of maintaining entanglement in a quantum system at finite, even high, temperatures – the so-called 'hot entanglement' – has obvious practical interest, but also requires closer theoretical scrutiny. Since quantum entanglement in a system evolves in time and is continuously subjected to environmental degradation, a nonequilibrium description by way of open quantum systems is called for. To identify the key issues and the contributing factors that may permit 'hot entanglement' to exist, or the lack thereof, we carry out a model study of two spatially-separated, coupled oscillators in a shared bath depicted by a finite-temperature scalar field. From the Langevin equations we derived for the normal modes and the entanglement measure constructed from the covariance matrix we examine the interplay between direct coupling, field-induced interaction and finite separation on the structure of late-time entanglement. We show that the coupling between oscillators plays a crucial role in sustaining entanglement at intermediate temperatures and over finite separations. In contrast, the field-induced interaction between the oscillators which is a non-Markovian effect becomes very ineffective at high temperature. We determine the critical temperature above which entanglement disappears to be bounded in the leading order by the inverse frequency of the center-of-mass mode of the reduced oscillator system, a result not unexpected, which rules out hot entanglement in such settings.

1. Introduction

Recently Galve et al. [1,2] pointed out the possibility of keeping quantum entanglement alive in a system at high temperatures by driving the system of two oscillators with a time-dependent interaction term. This is important in practical terms because if entanglement in a quantum open system can be maintained at high temperatures, it eases the way to how devices for quantum information processing can be conceptualized and designed. From a theoretical viewpoint understanding the basic mechanisms of obtaining this so-called 'hot entanglement' [3] is also of great interest.

Before beginning the analysis, we note the word 'hot' conveys three layers of meaning in three different contexts, referring to quantum systems A) kept in thermal *equilibrium* at all times, B) in a *nonequilibrium* condition and evolving, possibly but not necessarily, toward an equilibrium state, and C) in a *nonequilibrium steady state* at late times. In this study we derive the fully nonequilibrium dynamics of a system of two coupled quantum harmonic oscillators interacting with a common bath described by a bosonic field at finite temperature T. Thus our present work falls under Case B, which is in contrast to Case A [4,5], where a quantum system is assumed to be already in equilibrium and remains that way. We depict how entanglement of the open quantum system evolves in time and derive the critical temperature above which entanglement cannot survive. In an accompanying paper [6] we study one subcase of Case C, that of a quantum system in nonequilibrium steady state (NESS) at late times, using the framework and results obtained in [7]. The system we analyze consists of two coupled quantum harmonic oscillators each interacting with its own bath, described by a scalar field, set at two different temperatures $T_1 > T_2$ which together form the environment. Carrying out a fully systematic analysis of how quantum entanglement in open systems under different nonequilibrium conditions evolves is, in our view, a necessity before any claim of "hot entanglement" can be asserted.

In terms of methodology our present study makes use of the conceptual framework of quantum open systems [8] and the tech-

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niques of nonequilibrium quantum field theory [9]. It is a finite temperature generalization of our recent work [10] where the entanglement behavior at late times between two coupled and spatially separated oscillators interacting with a common bath modeled by a scalar field at zero temperature is analyzed in detail. That work in turn is a generalization of the paper of Lin and Hu [11] with coupling between the two oscillators added in the consideration.

2. System setup

Our system is made up of two spatially separated coupled detectors, which are entities with internal degrees of freedom (idf) $\chi_{1,2}$. The idf of each detector is described by a harmonic oscillator of mass m and bare frequency ω_b. This system is placed in a common finite-temperature bath modeled by a massless scalar field ϕ initially prepared in a thermal state at temperature β^{-1}. The system is allowed to interact with the bath initially at $t=0$. We want to track down its evolution in time, derive the entanglement dynamics between the two detectors at late times and determine the critical temperature above which entanglement no longer exists.

The action of the whole system is

$$S[\chi,\phi]=\int ds\left[\sum_{i=1}^{2}\frac{m}{2}\,\dot{\chi}_i^2(s)-\frac{m\omega_b^2}{2}\,\chi_i^2(s)\right]-\int ds\,m\sigma\,\chi_1(s)\chi_2(s)+\int d^4x\;j(x)\phi(x)+\int d^4x\,\frac{1}{2}\,\partial_\mu\phi\partial^\mu\phi\,,\qquad(1)$$

where the current $j(x)$ takes the form $j(x)=e\sum_{i=1}^{2}\chi_i(t)\,\delta^{(3)}[\mathbf{x}-\mathbf{z}_i(t)]$. The spacetime coordinate x is understood as a shorthand notation of $(t,\mathbf{x})$. The parameter σ in the action is the coupling strength between the two idfs, while e is the coupling constant between each idf and the bath. We have written down the action to allow for the detectors to move along an arbitrary yet prescribed trajectory $\mathbf{z}_i(t)$. In this work we assume they stay at rest throughout.

When the initial state of the idf has a Gaussian form, the reduced density matrix of the idf can be found exactly with the help of the influence functional formalism in the closed-time path integral framework. This enables us to obtain the full-time dynamics of the reduced system under the influence of the environment for arbitrary coupling strengths, as was done in full detail in [10]. Here, to highlight the physics behind thermal entanglement, we opt for a simpler, more physically transparent yet no less general way, by means of the Langevin equation approach, which has been shown to be totally compatible with the reduced-density-matrix description for linear systems [7]. For the current configuration, the Langevin equations of, say, χ_1 is given by

$$m\,\ddot{\chi}_1(t)+m\omega_b^2\,\chi_1(t)+m\sigma\,\chi_2(t)-e^2\int_0^t ds'\left[G_R(\mathbf{z}_1,s;\mathbf{z}_1,s')\chi_1(s')+G_R(\mathbf{z}_1,s;\mathbf{z}_2,s')\chi_2(s')\right]=\xi_1(t)\,.\qquad(2)$$

In Eq. (2), in addition to the restoring force $-m\omega_b^2\chi_1$ and the direct coupling $m\sigma\,\chi_2(t)$ with the other idf, the essential (most interesting) physics is contained in the nonlocal interactions generated by the system's interaction with its environment, and the stochastic driving force ξ_1 which recounts both the quantum and thermal noises originating from the heat bath at the location of Detector 1. It obeys the Gaussian statistics with $\langle\xi_1(t)\rangle=0$ and $\langle\xi_1(t)\xi_1(t')\rangle=e^2\,G_H(\mathbf{z}_1,t;\mathbf{z}_1,t')$, where $G_H(x,x')=\frac{1}{2}\langle\{\phi(x),\phi(x')\}\rangle$, with $\{\,,\}$ denoting symmetrization, is the Hadamard function of the scalar field. In addition, nonzero correlation of the bath between the locations of detector 1 and 2 implies $\langle\xi_1(t)\xi_2(t')\rangle=e^2\,G_H(\mathbf{z}_1,t;\mathbf{z}_2,t')$. The $\langle\cdots\rangle$ can represent the ensemble average or the quantum expectation values, depending on the context.

The nonlocal expressions in (2) containing the retarded Green function $G_R(x,x')=i\,\theta(t-t')[\phi(x),\phi(x')]$ of the scalar field, with $[\,,]$ denoting anti-symmetrization, embrace the dissipative self-force and the history-dependent non-Markovian interaction between the two idfs as the consequences of coupling between the idfs and the bath. In particular, these nonlocal expressions are independent of the initial bath state. Essentially the stochastic forcing term and the nonlocal terms in (2) capture the overall influences from the environment. The temporal Fourier transforms of these two kernel functions G_H and G_R are connected via the fluctuation–dissipation relation,

$$\overline{G}_H(\mathbf{R},\kappa)=\coth\frac{\beta\kappa}{2}\;\mathrm{Im}\,\overline{G}_R(\mathbf{R},\kappa),$$

$$\text{where } G(\mathbf{R},\tau)=\int_{-\infty}^{\infty}\frac{d\kappa}{2\pi}\,\overline{G}(\mathbf{R},\kappa)\,e^{-i\kappa\tau}.\qquad(3)$$

In certain contexts it signifies a balance between the energy transfer via noise from, and the dissipation back to, the environment. Thus the stochastic equations of motion of χ_1, χ_2 describe a set of coupled, damped, driven oscillators undergoing non-Markovian dynamics.

3. Dynamics

The set of equations of motion for χ_1, χ_2 in fact can be decoupled into the center of mass (CoM) mode $\chi_+=(\chi_1+\chi_2)/2$ and the relative mode $\chi_-=\chi_1-\chi_2$ [10],

$$\ddot{\chi}_+(t)+2\gamma\,\dot{\chi}_+(t)-2\gamma\,\frac{\theta(t-\ell)}{\ell}\,\chi_+(t-\ell)+\omega_+^2\,\chi_+(t)=\frac{1}{m}\,\xi_+(t)\,,\qquad(4)$$

$$\ddot{\chi}_-(t)+2\gamma\,\dot{\chi}_-(t)+2\gamma\,\frac{\theta(t-\ell)}{\ell}\,\chi_-(t-\ell)+\omega_-^2\,\chi_-(t)=\frac{1}{m}\,\xi_-(t)\,.\qquad(5)$$

Here the damping term and the retarded term are derived from the nonlocal expressions in (2). However, the nonlocal term that is proportional to $G_R(\mathbf{z}_1,t;\mathbf{z}_1,t')=-\dfrac{1}{2\pi}\,\theta(t-t')\,\delta'(t-t')$ is potentially divergent because

$$\int_0^t dt'\;G_R(\mathbf{z}_1,t;\mathbf{z}_1,t')\chi_1(t')=\frac{\delta(0)}{2\pi}\,\chi_1(t)-\frac{1}{2\pi}\int_0^t dt'\;\delta(t-t')\,\chi_1'(t')\,.\qquad(6)$$

This divergent expression can be absorbed into the bare frequency ω_b to form a renormalized frequency ω by

$$\omega^2=\omega_b^2+\delta\omega^2\,,\qquad\text{with}\qquad\delta\omega^2=-4\gamma\,\delta(0)\,.\qquad(7)$$