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Critical string from non-Abelian vortex in four dimensions

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ABSTRACT

In a class of non-Abelian solitonic vortex strings supported in certain $\mathcal{N}=2$ super-Yang–Mills theories we search for the vortex which can behave as a critical fundamental string. We use the Polchinski–Strominger criterion of the ultraviolet completeness. We identify an appropriate four-dimensional bulk theory: it has the $U(2)$ gauge group, the Fayet–Iliopoulos term and four flavor hypermultiplets. It supports semilocal vortices with the world-sheet theory for orientational (size) moduli described by the weighted $CP(2,2)$ model. The latter is superconformal. Its target space is six-dimensional. The overall Virasoro central charge is critical. We show that the world-sheet theory on the vortex supported in this bulk model is the *bona fide* critical string.

1. Introduction

Since 2003 a large variety of non-Abelian solitonic vortices supported in certain four-dimensional super-Yang–Mills theories were discovered [1]. Such vortices contain extra non-Abelian moduli (the so-called orientational moduli), in addition to the conventional translational moduli. The low-energy theory for the orientational moduli fields on the vortex world sheet is usually a nonlinear sigma model, typically $CP(N-1)$, with different degrees of supersymmetry. The primary purpose of the non-Abelian vortex explorations is modeling confinement and related phenomena in QCD-like theories.

These vortex strings are not similar to the critical strings of the fundamental string theory, and cannot be treated as such. The most clear-cut distinction is the fact that the world-sheet theory is not conformal. In the terminology of Ref. [2] such world-sheet theories are not ultraviolet (UV) complete: higher derivative terms are needed to make them consistent in the UV.

In this Letter we report the following finding. A semilocal $\mathcal{N}=(2,2)$ vortex with the orientational moduli described by the weighted $CP(2,2)$ model *is* UV complete, and reduces to a critical string in four dimensions.

This vortex is supported in four-dimensional $\mathcal{N}=2$ super-Yang–Mills with the $U(2)$ gauge group, the Fayet–Iliopoulos term, and four flavor hypermultiplets. The target space metric in the world-sheet theory has a block form: four-by-four block in the upper left corner (corresponding to flat metric for translational moduli) and six-by-six block in the lower right corner (corresponding to a Calabi–Yau metric with the vanishing Ricci tensor for orientational moduli). This metric can be read off from Eq. (5). The world-sheet theory is conformally invariant.

2. General considerations

It is known that the hadron spectrum is well described by linear Regge trajectories. In the early days of string theory this fact motivated people to consider dual resonance models as a theory of hadrons. It is believed that confinement in QCD is due formation of confining strings. In all known examples in which the confining strings are formed in a controllable way, say, the Abrikosov–Nielsen–Olesen (ANO) string [3] in the weakly coupled Abelian–Higgs model or the Seiberg–Witten strings in slightly deformed $\mathcal{N}=2$ super-Yang–Mills theory [4], the Regge trajectories will show linear behavior only at asymptotically large spins [5,6].

Indeed, consider an open string rotating with the spin J. In the (semi)classical approximation its length is determined by the relation $L^2 \sim J/T$ where T is the string tension. Its transverse size is given by the inverse mass m of the bulk fields forming the string, say, for the ANO string the masses of the gauge and Higgs fields.[1]

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[1] For BPS-saturated strings these masses are equal.

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At weak coupling these masses typically scale as $m \sim g\sqrt{T}$ where g is a small gauge coupling constant.

Clearly the string excitation spectrum can form linear Regge trajectories only if the string length is much larger than its transverse size. This gives the condition

$$mL \gg 1\,, \text{ or } J \gg 1/g^2\,. \tag{1}$$

At weak coupling g is small so spin J should be large.

At $J \sim 1$ the condition (1) is not met at weak coupling: the string is not developed. Rather, we deal with a sausage-like field configuration. The quark–antiquark mesons formed are closer to spherical symmetry. No linear Regge trajectory apply at $J \sim 1$.

Empirically in the real world QCD we have practically linear Regge trajectories at $J \sim 1$. Can we find *any* example of a four-dimensional bulk theory where confining string remains thin at $J \sim 1$? If so, the string must satisfy the condition

$$T \ll m^2\,, \tag{2}$$

to be referred to as the thin string condition. This condition cannot be met at weak coupling.

3. Strong coupling

We have to find an appropriate strongly coupled four-dimensional bulk theory. We will find super-Yang–Mills theory which supports vortices similar to critical strings (e.g. with conformal world-sheet theory) and then formulate a necessary conditions for the existence of the thin string regime.

4. Thin string regime

In the effective two-dimensional theory on the string world sheet the problem can be understood as follows. For the ANO string the effective theory on the string world sheet is given by the Nambu–Goto action *plus* higher derivative corrections. Higher derivative terms are needed to make the world-sheet theory UV complete [2]. This requirement can be used to constrain higher derivative terms order by order in the derivative expansion, see e.g. [7] and references therein.

Higher derivative corrections run in powers of the ratio ∂^2/m^2 where the mass of the bulk fields m is given by $m \sim g\sqrt{T}$ and typical energy in the numerator at $J \sim 1$ is determined by the string tension. Thus, higher derivative corrections materialize as powers of T/m^2. Obviously they all blow up at weak coupling – the string surface become "crumpled" [8]. This is the world-sheet implementation of the bulk picture of a short and thick "string."

We want to find a regime in which the string remains thin, see (2). This means that the higher derivative corrections should be parametrically small. In other words, the low-energy world-sheet theory[2] should be UV complete. This leads us to the following necessary conditions to have such a regime:

(i) The low-energy world-sheet theory on the string must be conformally invariant;

(ii) It must have the critical value of the Virasoro central charge.

These are the famous conditions satisfied by the fundamental string. In particular, the bosonic fundamental string becomes critical in $D = 26$, while the superstring becomes critical in $D = 10$. The low energy world-sheet theory for the ANO string is not critical in four dimensions. We will show below that the above conditions are met in a class of the non-Abelian vortices [10–13]. In particular, the solitonic vortex in question must have six orientational moduli, which, together with four translational moduli, will form a ten-dimensional space.

[2] By low energy-theory we mean a theory with no more than two derivatives in the Polyakov formulation [9], see Eq. (5).

5. Non-Abelian vortices

Non-Abelian vortices are supported in a large class of supersymmetric and non-supersymmetric gauge theories. We will focus on the bulk four-dimensional theories in which the non-Abelian vortices were first found: $\mathcal{N} = 2$ supersymmetric QCD with the $U(N)$ gauge group, N_f quark flavor multiplets ($N_f \geq N$) and the Fayet–Iliopoulos (FI) parameter ξ of the $U(1)$ factor of the gauge group. In this theory the vortices under consideration are BPS-saturated and preserve half of the bulk supersymmetry. Thus, they possess $\mathcal{N} = (2,2)$ supersymmetry on the world sheet. The string tension is determined exactly by

$$T_P = 2\pi\xi. \tag{3}$$

These strings are formed due to the (s)quark condensation; therefore, they confine monopoles. More precisely, in the $U(N)$ gauge theories the confined monopoles are implemented as junctions of two vortices of different kinds [14,12,13].

Dynamics of the translational modes in the Polyakov formulation [9] can be described by the action

$$S_{\text{tr.}} = \frac{T}{2}\int d^2\sigma\sqrt{h}\,h^{\alpha\beta}\,\partial_\alpha x^\mu\,\partial_\beta x_\mu, \tag{4}$$

where σ^α ($\alpha = 1,2$) are the world-sheet coordinates, x^μ ($\mu = 1,\ldots,4$) describe the string world sheet and $h = det(h_{\alpha\beta})$ where $h_{\alpha\beta}$ is the world-sheet metric which is understood as a independent variable.[3]

If $N_f = N$ the dynamics of the orientational zero modes of the vortex, which become orientational moduli fields on the world sheet, is described by two-dimensional $\mathcal{N} = (2,2)$-supersymmetric $CP(N-1)$ model. If one adds extra quark flavors, non-Abelian vortices become semilocal. They acquire size moduli [15].

Non-Abelian semilocal vortices in $\mathcal{N} = 2$ SQCD with $N_f > N$ were studied in [10,13,16–18]. The world-sheet theory for the orientational moduli of the semilocal vortex is given[4] by the weighted $CP(N,\tilde{N})$ sigma model where $\tilde{N} = (N_f - N)$. Its gauged formulation is as follows [19]. One introduces two types of complex fields, with the $U(1)$ charges ± 1: n^P ($P = 1,\ldots,N$) and ρ^K ($K = N+1,\ldots,N_f$). The orientational moduli are described by the N-plets n^P while the size moduli are parametrized by the $\tilde{N}$-plet ρ^K.

The effective two-dimensional theory on the world sheet has the action

$$S_{\text{or.}} = \int d^2\sigma\sqrt{h}\left\{h^{\alpha\beta}\left(\tilde{\nabla}_\alpha\bar{n}_P\,\nabla_\beta n^P + \nabla_\alpha\bar{\rho}_K\,\tilde{\nabla}_\beta\rho^K\right)\right.$$
$$\left. + \frac{e^2}{2}\left(|n^P|^2 - |\rho^K|^2 - 2\beta\right)^2\right\} + \text{fermions}, \tag{5}$$

[3] Effective world-sheet theories for both translational and orientational moduli are derived in the quasiclassical approximation. In this approximation two alternative string theory formulations – in terms of the induced metric and in terms of the metric $h_{\alpha\beta}$ as an independent variable – are equivalent.

[4] Both the orientational and the size moduli have logarithmically divergent norms, see e.g. [16]. After an appropriate infrared regularization, logarithmically divergent norms can be absorbed into the definition of relevant two-dimensional fields [16]. In fact, the world-sheet theory on the semilocal non-Abelian string is not exactly the weighted $CP(N,\tilde{N})$ model [18], there are minor differences unimportant for our purposes. The actual theory is called the *zn* model. We can ignore the above differences.