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Abolishing the maximum tension principle

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ABSTRACT

We find the series of example theories for which the relativistic limit of maximum tension $F_{max} = c^4/4G$ represented by the entropic force can be abolished. Among them the varying constants theories, some generalized entropy models applied both for cosmological and black hole horizons as well as some generalized uncertainty principle models.

1. Introduction

According to an early remark by Gibbons [1] and Schiller [2], due to the phenomenon of gravitational collapse and black hole formation, there exists a maximum force or maximum tension limit $F_{max} = c^4/4G$ in general relativity (c – the velocity of light, G – Newton gravitational constant). The fact is known as "The Principle of Maximum Tension". This is unlike in Newton's gravity, where the two point masses may approach each other arbitrarily close and so the force between them may reach infinity. The limit can nicely be derived by the application of the cosmic string deficit angle $\phi = (8\pi G/c^4)F$ not to exceed 2π [1]. It is interesting that the maximum tension limit holds also in string theory, where the tension T is given by the Regge slope parameter α', i.e. $F_{max} \propto T = 1/2\pi\alpha'$. The limit is slightly modified in the presence of the positive cosmological constant [3].

In this paper we would like to show that there exist some theories in which the Principle of Maximum Tension does not hold and we are able to recover the infinite Newtonian tension limit again. One group of them will be the varying constants theories and another the entropic force theories and black hole thermodynamics within the framework of generalized entropy models. The generalized uncertainty principle (GUP) framework will also be exemplified. Special comment about the Carroll limit of the Principle of Maximum Tension will be presented, too.

2. Maximum tension and the entropic force

The maximum tension or force between the two bodies in general relativity has been claimed to be [1]

$$F_{max} = \frac{c^4}{4G}. \tag{2.1}$$

It is advisable to note that the factor c^4/G appears in the Einstein field equations and is of the order of 10^{44} Newtons. If the field equations are presented in the form

$$T_{\mu\nu} = \frac{1}{8\pi}\frac{c^4}{G}G_{\mu\nu}, \tag{2.2}$$

where $T_{\mu\nu}$ is the stress tensor and $G_{\mu\nu}$ is the (geometrical) Einstein tensor, then we can consider their analogy with the elastic force equation:

$$F = kx, \tag{2.3}$$

where k is an elastic constant, and x is the displacement. In this analogy, we can think of gravitational waves being some perturbations of spacetime and the ratio c^4/G which appears in (2.1) plays the role of an elastic constant. Its large value means that the spacetime is extremely rigid or, in other words, it is extremely difficult to make it vibrate [4].

We make an observation that similar ratio c^4/G appears in the expression for the entropic force within the framework of entropic cosmology [5]. In order to calculate this force one has to apply the Hawking temperature [6]

$$T = \frac{\gamma\hbar c}{2\pi k_B r_h}, \tag{2.4}$$

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and the Bekenstein entropy [7]

$$S = \frac{k_B c^3 A}{4\hbar G} = \frac{\pi k_B c^3}{G\hbar} r_h^2, \tag{2.5}$$

on the horizon at $r = r_h$, where $A = 4\pi r_h^2$ is the horizon area, $\hbar$ and k_B are Planck constant and Boltzmann constant, respectively, γ is an arbitrary and non-negative parameter of the order of unity $O(1)$.

The entropic force is defined as [5]

$$F_r = -T\frac{dS}{dr_h} \tag{2.6}$$

and by the application of (2.4) and (2.5), one gets

$$F_r = -\gamma\frac{c^4}{G}, \tag{2.7}$$

where the minus sign means that the force points in the direction of increasing entropy. It emerges that up to a numerical factor $\gamma/4$ and the sign, this is the maximum force limit in general relativity. The entropic force is supposed to be responsible for the current acceleration as well as for an early exponential expansion of the universe. There is an extra entropic force term into the Friedmann equation and the acceleration equation which are obtained from Einstein field equations (2.2).

3. Abolishing maximum tension

3.1. Varying constants

One way to release the maximum tension limit is when we admit the speed of light c and the gravitational constant G in the formula (2.1) to vary. There is a Newtonian mechanics limit of (2.1) $c \to \infty$, $G \to 0$ which is one of the corners of the Bronshtein–Zelmanov–Okun cube [8] and it of course recovers infinite tension $F_{max} \to \infty$. Another interesting limit is the Carroll limit $c \to 0$ or $G \to \infty$ which gives $F_{max} \to 0$ [9,10]. The $c \to 0$ limit is also predicted within the framework of loop quantum cosmology (LQC) and one can easily understand why it is called the anti-Newtonian limit [11], while $G \to \infty$ is just the strong coupling limit of gravity [12]. Bearing in mind gravitational wave analogy (2.3), the elastic constant is zero and no gravitational wave can propagate or in other words, the spacetime is infinitely rigid.

Now consider the theory which allows that c and G vary in time in (2.2), (2.4), and (2.5). In cosmology, the horizon also depends on time $r = r(t)$ so that we can write

$$dS/dr_h = \dot{S}/\dot{r}. \tag{3.1}$$

In the varying constants theories [13] the entropic force is given by [14]

$$F = -T\frac{dS}{dr_h} = -\frac{\gamma c^4(t)}{2G(t)}\left[\frac{3\frac{\dot{c}(t)}{c(t)} - \frac{\dot{G}(t)}{G(t)} + 2\frac{\dot{r}_h}{r_h}}{\frac{\dot{r}_h}{r_h}}\right], \tag{3.2}$$

where we have applied the Hubble horizon

$$r_h = \frac{c}{H} \tag{3.3}$$

and get

$$F = -\frac{\gamma c^4(t)}{2G(t)}\left[\frac{5\frac{\dot{c}(t)}{c(t)} - \frac{\dot{G}(t)}{G(t)} - 2\frac{\dot{H}}{H}}{\frac{\dot{c}(t)}{c(t)} - \frac{\dot{H}}{H}}\right]. \tag{3.4}$$

This reduces to (2.7) for $\dot{c} = \dot{G} = 0$.

The following conclusions are in order. Namely, if the fundamental constants c and G are really constant, then the maximum force (3.4) reduces to a constant value given by (2.7). However, the variability of c and G modifies this claim in a way that the maximum force also varies in time. In particular, it seems to be infinite for a constant horizon value $\dot{r}_h = 0$ which corresponds to a model with $c(t) \propto H(t)$. The entropic force can also become infinite, if the derivatives of c and G are infinite.

3.2. Modified entropy models

Another way to abolish the Principle of Maximum Tension even without varying c and G is when one changes the definition of entropy (see the appendix in Ref. [15]). The point is that the Bekenstein entropy defined by (2.5) is the area entropy which means that it is proportional to r_h^2 so that taking the derivative of (2.6), and multiplying it by Hawking temperature (2.4) gives a constant (and a finite) value of the entropic force (2.7).

However, if one applies the volume entropy [15], then one has the entropy which is proportional to r_h^3. An example is a non-additive entropy [16] or nonextensive Tsallis entropy [17] which generalizes (2.5) to

$$S_3 \propto r_h^3. \tag{3.5}$$

Following Ref. [15], the correct expression for this volume entropy is

$$S_3 = \zeta\frac{\pi k_B c^3}{\hbar G} r_h^3, \tag{3.6}$$

which as applied to the entropic force definition (2.6) together with the Hawking temperature (2.4) gives

$$F_{r3} = -T\frac{dS_3}{dr_h} = -\frac{3}{2}\gamma\zeta\frac{c^4}{G} r_h, \tag{3.7}$$

where ζ is a dimensional constant. Since $r_h = r_h(t)$ according to (3.3), then the maximum force may reach infinity again, when the horizon size becomes infinitely large. Of course the same happens, if one of the conditions $c \to \infty$ or $G \to 0$ holds.

Another example is the quartic entropy defined as [15]

$$S_4 = \xi\frac{\pi k_B c^3}{\hbar G} r_h^4, \tag{3.8}$$

where ξ is a dimensional constant. It gives an entropic force in the form

$$F_{r4} = -T\frac{dS_4}{dr_h} = -2\gamma\xi\frac{c^4}{G} r_h^2. \tag{3.9}$$

Here again $r_h = r_h(t)$ according to (3.3), and so the maximum force may reach infinity when the horizon size becomes infinitely large and this happens much faster than for the volume entropy entropic force (3.7).

For any generalized entropy which is proportional to r_h^D, with D being an appropriate volume dimension, we have

$$S_D \propto r_h^D, \qquad F_{rD} \propto r_h^{D-2}. \tag{3.10}$$

For varying constants theories, a common generalization of the formula (3.4) and the formulas (3.7) and (3.9) reads as

$$F_{rD} = -\frac{\gamma c^4(t)}{2G(t)}\left[\frac{(3+D)\frac{\dot{c}(t)}{c(t)} - \frac{\dot{G}(t)}{G(t)} - D\frac{\dot{H}}{H}}{\frac{\dot{c}(t)}{c(t)} - \frac{\dot{H}}{H}}\right]\left(\frac{c}{H}\right)^{D-2}. \tag{3.11}$$