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Bimetric gravity is cosmologically viable

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A B S T R A C T

Bimetric theory describes gravitational interactions in the presence of an extra spin-2 field. Previous work has suggested that its cosmological solutions are generically plagued by instabilities. We show that by taking the Planck mass for the second metric, M_f, to be small, these instabilities can be pushed back to unobservably early times. In this limit, the theory approaches general relativity with an effective cosmological constant which is, remarkably, determined by the spin-2 interaction scale. This provides a late-time expansion history which is extremely close to ΛCDM, but with a technically-natural value for the cosmological constant. We find M_f should be no larger than the electroweak scale in order for cosmological perturbations to be stable by big-bang nucleosynthesis. We further show that in this limit the helicity-0 mode is no longer strongly-coupled at low energy scales.

"The reports of my death have been greatly exaggerated."
—*Metrics Twain*

1. Introduction

The Standard Model of particle physics contains fields with spins 0, 1/2, and 1, describing matter as well as the strong and electroweak forces. General relativity (GR) extends this to the gravitational interactions by introducing a massless spin-2 field. There is theoretical and observational motivation to seek physics beyond the Standard Model and GR. In particular, GR is nonrenormalizable and is associated with the cosmological constant, dark energy, and dark matter problems. To compound the puzzle, the GR-based Λ-cold dark matter (ΛCDM) model provides a very good fit to observational data, despite its theoretical problems. In order to be observationally viable, any modified theory of gravity must be able to mimic GR over a wide range of distances.

A natural possibility for extending the set of known classical field theories is to include additional spin-2 fields and interactions. While "massive" and "bimetric" theories of gravity have a long history [1,2], nonlinear theories of interacting spin-2 fields were found, in general, to suffer from the Boulware–Deser (BD) ghost instability [3]. Recently a particular bimetric theory (or bigravity) has been shown to avoid this ghost instability [4,5]. This theory describes nonlinear interactions of the gravitational metric with an additional spin-2 field. It is an extension of an earlier ghost-free theory of massive gravity (a massive spin-2 field on a nondynamical flat background) [6–8] for which the absence of the BD ghost at the nonlinear level was established in Refs. [5,9–11].

Including spin-2 interactions modifies GR, *inter alia*, at large distances. Bimetric theory is therefore a candidate to explain the accelerated expansion of the Universe [12,13]. Indeed, bigravity has been shown to possess Friedmann–Lemaître–Robertson–Walker (FLRW) solutions which can match observations of the cosmic

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expansion history, even in the absence of vacuum energy [14–20].[1] Linear perturbations around these cosmological backgrounds have also been studied extensively [28–41]. The epoch of acceleration is set by the mass scale m of the spin-2 interactions. Unlike a small vacuum energy, m is protected from large quantum corrections due to an extra diffeomorphism symmetry that is recovered in the limit $m \to 0$, just as fermion masses are protected by chiral symmetry in the Standard Model (see Ref. [42] for an explicit analysis in the massive gravity setup). This makes interacting spin-2 fields especially attractive from a theoretical point of view.

Cosmological solutions lie on one of two branches, called the finite and infinite branches.[2] The infinite-branch models can have sensible backgrounds [19,32], but the perturbations have been found to contain ghosts in both the scalar and tensor sectors [33,34,41]. Most viable background solutions lie on the finite branch [16–19]. While these avoid the aforementioned ghosts, they contain a scalar instability at early times [29,32,33] that invalidates the use of linear perturbation theory and could potentially rule these models out. For parameter values thought to be favored by data, this instability was found to be present until recent times (i.e., a similar time to the onset of cosmic acceleration) and thus seemed to spoil the predictivity of bimetric cosmology.

In this Letter we study a physically well-motivated region in the parameter space of bimetric theory that has been missed in earlier work due to a ubiquitous choice of parameter rescaling. We demonstrate how in this region the instability problem in the finite branch can be resolved while the model still provides late-time acceleration in agreement with observations.

Our search for viable bimetric cosmologies will be guided by the precise agreement of GR with data on all scales, which motivates us to study models of modified gravity which are close to their GR limit. Often this limit is dismayingly trivial; if a theory of modified gravity is meant to produce late-time self-acceleration in the absence of a cosmological constant degenerate with vacuum energy, then we would expect that self-acceleration to disappear as the theory approaches GR. We will see, however, that there exists a GR limit of bigravity which retains its self-acceleration, leading to a GR-like universe with an effective cosmological constant produced purely by the spin-2 interactions.

2. Bimetric gravity

The ghost-free action for bigravity containing metrics $g_{\mu\nu}$ and $f_{\mu\nu}$ is given by [4,43]

$$S = \int \mathrm{d}^4x \Big[-\frac{M_{\mathrm{Pl}}^2}{2}\sqrt{g}R(g) - \frac{M_f^2}{2}\sqrt{f}R(f) + m^2 M_{\mathrm{Pl}}^2 \sqrt{g}\, V(\mathbb{X}) + \sqrt{g}\mathcal{L}_m(g,\Phi_i)\Big]. \tag{1}$$

Here M_{Pl} and M_f are the Planck masses for $g_{\mu\nu}$ and $f_{\mu\nu}$, respectively, and we will frequently refer to their ratio,

$$\alpha \equiv \frac{M_f}{M_{\mathrm{Pl}}}. \tag{2}$$

The potential $V(\mathbb{X})$ is constructed from the elementary symmetric polynomials $e_n(\mathbb{X})$ of the eigenvalues of the matrix $\mathbb{X} \equiv \sqrt{g^{-1}f}$, defined by

$$\mathbb{X}^\mu{}_\alpha \mathbb{X}^\alpha{}_\nu \equiv g^{\mu\alpha} f_{\alpha\nu}, \tag{3}$$

and has the form [8,43],[3]

$$\sqrt{g}\, V(\mathbb{X}) = \sqrt{g}\beta_0 + \sqrt{g}\sum_{n=1}^{3}\beta_n e_n(\mathbb{X}) + \sqrt{f}\beta_4. \tag{4}$$

In the above, m is a mass scale and β_n are dimensionless interaction parameters. β_0 and β_4 parameterize the vacuum energies in the two sectors. Guided by the absence of ghosts and the weak equivalence principle, we take the matter sector to be coupled to $g_{\mu\nu}$.[4] Then the vacuum-energy contributions from the matter sector $\mathcal{L}_{\mathrm{m}}$ are captured in β_0. We can interpret $g_{\mu\nu}$ as the spacetime metric used for measuring distance and time, while $f_{\mu\nu}$ is an additional symmetric tensor that mixes nontrivially with gravity. As we discuss further below, the two metrics do not correspond to the spin-2 mass eigenstates but each contain both massive and massless components. Even before fitting to observational data, the parameters in the bimetric action are subject to several theoretical constraints. For instance, the squared mass of the massive spin-2 field needs to be positive, it must not violate the Higuchi bound [59,60], and ghost modes should be absent.

In terms of the Einstein tensor, $G_{\mu\nu}$, the equations of motion for the two metrics take the form

$$G_{\mu\nu}(g) + m^2 V^g_{\mu\nu} = \frac{1}{M_{\mathrm{Pl}}^2} T_{\mu\nu}, \tag{5}$$

$$\alpha^2 G_{\mu\nu}(f) + m^2 V^f_{\mu\nu} = 0, \tag{6}$$

where $V^{(g,f)}_{\mu\nu}$ are determined by varying the interaction potential, V. Taking the divergence of eq. (5) and using the Bianchi identity leads to the *Bianchi constraint*,

$$\nabla^\mu_{(g)} V^g_{\mu\nu} = 0. \tag{7}$$

The analogous equation for $f_{\mu\nu}$ carries no additional information due to the general covariance of the action.

Finally, note that the action (1) has a status similar to Proca theory on curved backgrounds. It is therefore expected to require an analogue of the Higgs mechanism, with new degrees of freedom, in order to have improved quantum behavior. The search for a ghost-free Higgs mechanism for gravity is still in progress [61].

3. The GR limit

When bigravity is linearized around proportional backgrounds $\bar{f}_{\mu\nu} = c^2 \bar{g}_{\mu\nu}$ with constant c,[5]

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{1}{M_{\mathrm{Pl}}}\delta g_{\mu\nu}, \tag{8}$$

$$f_{\mu\nu} = c^2 \bar{g}_{\mu\nu} + \frac{c}{M_f}\delta f_{\mu\nu}, \tag{9}$$

the canonically-normalized perturbations can be diagonalized into massless modes $\delta G_{\mu\nu}$ and massive modes $\delta M_{\mu\nu}$ as [4,62]

$$\delta G_{\mu\nu} \propto \left(\delta g_{\mu\nu} + c\alpha\, \delta f_{\mu\nu}\right), \tag{10}$$

$$\delta M_{\mu\nu} \propto \left(\delta f_{\mu\nu} - c\alpha\, \delta g_{\mu\nu}\right). \tag{11}$$

[1] Stable FLRW solutions do not exist in massive gravity [21–27].

[2] There is a third branch containing bouncing solutions, but these tend to have pathologies [41].

[3] This is a generalization of the massive-gravity potential [8] (to which it reduces for $f_{\mu\nu} = \eta_{\mu\nu}$ and a restricted set of β_n) given in Ref. [43].

[4] More general matter couplings not constrained by these requirements have been studied in Refs. [20,44–58].

[5] These correspond to Einstein spaces and, for nonvanishing α, solve the field equations only in vacuum. A quartic equation determines $c = c(\beta_n, \alpha)$.