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Ab initio approach to the non-perturbative scalar Yukawa model

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ABSTRACT

We report on the first non-perturbative calculation of the scalar Yukawa model in the single-nucleon sector up to four-body Fock sector truncation (one "scalar nucleon" and three "scalar pions"). The light-front Hamiltonian approach with a systematic non-perturbative renormalization is applied. We study the n-body norms and the electromagnetic form factor. We find that the one- and two-body contributions dominate up to coupling $\alpha \approx 1.7$. As we approach the coupling $\alpha \approx 2.2$, we discover that the four-body contribution rises rapidly and overtakes the two- and three-body contributions. By comparing with lower sector truncations, we show that the form factor converges with respect to the Fock sector expansion.

1. Introduction

Solving quantum field theories in the non-perturbative regime is not only a theoretical challenge but also essential to understand the structure of hadrons from first principles. The light-front (LF) Hamiltonian quantum field theory approach provides a natural framework to tackle this issue [1,2]. A great advantage of this approach is that it provides direct access to the hadronic observables. In the LF dynamics, the system is defined at a fixed LF time $x^+ \equiv t + z$. The physical states are obtained by diagonalizing the LF Hamiltonian operator. The vacuum in LF quantization is trivial. As a result, it is particularly convenient to expand the physical states in the Fock space. For example, a physical pion state can be written in terms of quarks (q), antiquarks ($\bar{q}$) and gluons (g) as $|\pi\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}gg\rangle + \cdots$.

In order to do practical calculations, the Fock space has to be truncated. A natural choice, taking advantage of the LF dynamics, is the Fock sector truncation, also known as the light-front Tamm–Dancoff (LFTD) method [2]. A number of non-perturbative renormalization schemes have been developed based on the LFTD [3–6]. Thus we arrive at a few-body problem and predictions can be systematically improved by including more Fock sectors. The LFTD method is a non-perturbative approach in Minkowski space, which can be compared with other non-perturbative methods, e.g., Lattice quantum field theory in Euclidean space. Of course, this approach only works if the Fock sector expansion converges in the non-perturbative region. In practice, one can compare successive Fock sector truncations and check numerically whether the relevant physical observables converge. We will see that good convergence is achieved for the scalar Yukawa model in a non-perturbative regime with a four-body Fock sector truncation. Similar results, though by a different method, were found in Refs. [7,8] for the Wick–Cutkosky model [9].

We apply this approach to a scalar version of the Yukawa model that describes the pion-mediated nucleon–nucleon interaction. The Lagrangian density of the model reads

$$\mathscr{L} = \partial_\mu N^\dagger \partial^\mu N - m^2|N|^2 + \tfrac{1}{2}\partial_\mu \pi \partial^\mu \pi - \tfrac{1}{2}\mu_0^2 \pi^2 + g_0|N|^2\pi + \delta m^2 |N|^2, \tag{1}$$

where g_0 is the bare coupling, δm^2 is the mass counterterm of the field $N(x)$. It is convenient to introduce a dimensionless coupling constant

$$\alpha = \frac{g^2}{16\pi m^2}.$$

For the sake of brevity, we refer to the fundamental degrees-of-freedoms (d.o.f.'s) $N(x)$ and $\pi(x)$ as "scalar nucleon" and "scalar pion" field respectively. We also introduce a Pauli–Villars (PV) scalar pion (with mass μ_1) to regularize the ultraviolet (UV) divergence [10]. Then, a sector dependent method known as the Fock sector dependent renormalization (FSDR) developed in Ref. [6] is used to renormalize the theory. FSDR is a systematic non-perturbative renormalization scheme based on the covariant light-front dynamics (CLFD, see Ref. [11] for a review) and Fock sector

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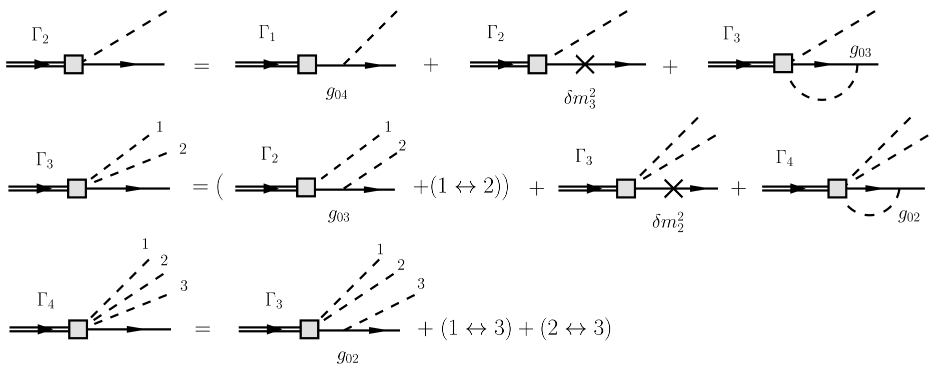

Fig. 1. The diagrammatic representation of the system of equations in the four-body truncation.

expansion. It has shown great promise in the application to the Yukawa model and QED [12,13].

The scalar Yukawa model is known to exhibit a vacuum instability [14]. It can be stabilized by either adding the quartic terms $\frac{1}{4!}\pi^4$, $\frac{1}{2}|N|^4$ and $\frac{1}{2}|N|^2\pi^2$ to the Lagrangian, or restricting the nucleon–antinucleon d.o.f. [15]. The latter leads to the exclusion of the pion self-energy correction, sometimes referred to as the "quenched approximation". For the sake of simplicity, here we study this restricted version of the theory. Then the bare mass of the scalar pion becomes the physical mass, $\mu_0 = \mu$. It should be emphasized, though, that our formalism is capable of dealing with the (scalar) antinucleon d.o.f. The scalar nucleon and scalar pion d.o.f.'s generate non-perturbative dynamics at large coupling sufficient for our purposes.

Previously, this model has been solved in the same approach up to three-body truncation (one scalar nucleon, two scalar pions) [6]. The results from the two- and three-body truncations agree at small couplings; yet they deviate in the large coupling region. Therefore, it is crucial to extend the non-perturbative calculation to higher Fock sectors. In this paper, we present the calculation of the four-body truncation (one scalar nucleon, three scalar pions). By comparing successive truncations, we can examine the convergence of the Fock sector expansion. We presented a preliminary version of this work in Ref. [16].

We first introduce our formalism in the next section. The LF Hamiltonian field theory will be briefly mentioned and the non-perturbative renormalization procedure will be explained. Then a set of coupled integral equations will be derived for the four-body truncation. In Section 3, we present the numerical results, including the calculation of the electromagnetic form factor. We conclude in Section 4.

2. Light-front Hamiltonian field theory

The LF Hamiltonian for the scalar Yukawa model is

$$\hat{P}^- = \int \mathrm{d}^3x \Big[\boldsymbol{\partial}_\perp N^\dagger \cdot \boldsymbol{\partial}_\perp N + m^2|N|^2 + \tfrac{1}{2}\boldsymbol{\partial}_\perp \pi \cdot \boldsymbol{\partial}_\perp \pi + \tfrac{1}{2}\mu_0^2\pi^2 - g_0|N|^2\pi - \delta m^2|N|^2\Big]_{x^+=0}. \quad (2)$$

The physical states can be obtained by solving the time-independent Schrödinger equation

$$\hat{P}^-|p\rangle = \frac{\boldsymbol{p}_\perp^2 + M^2}{p^+}|p\rangle, \quad (3)$$

where $\boldsymbol{p}_\perp$ and p^+ are the transverse and longitudinal momentum, respectively. Thanks to boost invariance in the LF dynamics, we can take $\boldsymbol{p}_\perp = 0$ without loss of generality.

The system is solved in the single-nucleon sector. The state vector admits a Fock space expansion,

$$|p\rangle = \sum_n \int D_n\, \psi_n(\boldsymbol{k}_{1\perp}, x_1, \ldots, \boldsymbol{k}_{n\perp}, x_n; p^2) \times |\boldsymbol{k}_{1\perp}, x_1, \ldots, \boldsymbol{k}_{n\perp}, x_n\rangle, \quad (4)$$

where $x_i \equiv \frac{k_i^+}{p^+}$, and

$$D_n = 2(2\pi)^3\delta^{(2)}(\boldsymbol{k}_{1\perp} + \cdots + \boldsymbol{k}_{n\perp})\delta(x_1 + \cdots + x_n - 1) \times \prod_{i=1}^{n} \frac{\mathrm{d}^2k_{i\perp}\mathrm{d}x_i}{(2\pi)^3 2x_i}.$$

The n-body Fock state $|\boldsymbol{k}_{1\perp}, x_1, \ldots, \boldsymbol{k}_{n\perp}, x_n\rangle$ consists $(n-1)$ scalar pions and 1 scalar nucleon. We use the last pair $(\boldsymbol{k}_{n\perp}, x_n)$ to denote the momentum of the scalar nucleon. ψ_n, known as the LF wave function (LFWF), is a boost invariant. The LFWFs are normalized to unity, $\sum_n I_n = 1$, where

$$I_n = \frac{1}{(n-1)!}\int D_n \Big|\psi_n(\boldsymbol{k}_{1\perp}, x_1, \ldots, \boldsymbol{k}_{n\perp}, x_n; p^2)\Big|^2 \quad (5)$$

is the probability that the system appears in the n-body Fock sector. In the scalar Yukawa model, these quantities are regulator independent, in contrast to more realistic theories such as Yukawa and QED. Note that $\psi_1 = \sqrt{I_1}$ is a constant.

It is convenient to introduce the n-body vertex functions,

$$\Gamma_n(\boldsymbol{k}_{1\perp}, x_1, \ldots, \boldsymbol{k}_{n-1\perp}, x_{n-1}; p^2) = (s_{1,\ldots,n-1} - p^2)\psi_n(\boldsymbol{k}_{1\perp}, x_1, \ldots, \boldsymbol{k}_{n\perp}, x_n; p^2) \quad (6)$$

for $n > 1$ and $\Gamma_1 = (m^2 - p^2)\psi_1$, where

$$s_{i_1,\ldots,i_{n-1}} \equiv (k_{i_1} + \cdots + k_{i_{n-1}} + k_n)^2 = \sum_{i=i_1}^{i_{n-1}} \frac{\boldsymbol{k}_{i\perp}^2 + \mu_{j_i}^2}{x_i} + \frac{\boldsymbol{k}_{n\perp}^2 + m^2}{x_n}$$

is the invariant mass squared of the Fock state, and μ_{j_i} $(j_i = 0, 1)$ is the mass of the i-th scalar pion. We have suppressed $\boldsymbol{k}_{n\perp}$ and x_n in Γ_n, by virtue of the momentum conservations $\boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp} + \cdots + \boldsymbol{k}_{n\perp} = 0$, $x_1 + x_2 + \cdots + x_n = 1$. For simplicity we will also omit the dependence on p^2 in Γ_n for the ground state $p^2 = m^2$.

Written in terms of the vertex functions Γ, Eq. (3) can be represented diagrammatically using the LF graphical rules [17,18] (see Ref. [11] for a review). Fig. 1 shows the diagrams for the four-body truncation.

The two-body vertex function Γ_2 plays a particular role in renormalization. It comprises all radiative corrections allowed