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Quantum fields and entanglement on a curved lightfront

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ABSTRACT

We consider field quantization on an arbitrary null hypersurface in curved spacetime. We discuss the de Sitter horizon as the simplest example, relating the horizon quantization to the standard Fock space in the cosmological patch. We stress the universality of null-hypersurface kinematics, using it to generalize the Unruh effect to vacuum or thermal states with respect to null "time translations" on arbitrary (e.g. non-stationary) horizons. Finally, we consider a general pure state on a null hypersurface, which is divided into past and future halves, as when a bifurcation surface divides an event horizon. We present a closed-form recipe for reducing such a pure state into a mixed state on each half-hypersurface. This provides a framework for describing entanglement between spacetime regions directly in terms of their causal horizons. To illustrate our state-reduction recipe, we use it to derive the Unruh effect.

1. Introduction and summary

Lightfront quantization [1–4] is an approach to quantum field theory that replaces constant-time hypersurfaces with null hyperplanes. In this paper, we consider the analogous quantization on arbitrary null hypersurfaces (hereafter, "horizons") in curved spacetime. A key advantage of lightfront quantization is that the physical vacuum can be constructed kinematically, regardless of interactions [1]. This is accomplished by defining the vacuum in terms of the generator of null "time translations" along the lightfront. We will perform a similar construction for an arbitrary choice of null "time" on a general horizon. As the simplest example, we will discuss de Sitter space, where the Bunch–Davies vacuum [5] can be viewed [6] as the vacuum with respect to an affine null time along the cosmological horizon. We will rephrase the latter argument within the lightfront approach, stressing that it extends to interacting theories. We will then show how the natural Fock space on the de Sitter horizon captures the standard spatial momentum modes in the cosmological patch.

Physically, null hypersurfaces act as causal boundaries between spacetime regions. In particular, a pair of intersecting horizons divides spacetime into quadrants, of which the two spacelike-separated ones contain the evolution of two "halves of space". The entanglement between such regions is an important subject in quantum field theory, with implications for black hole thermodynamics [7,8] and renormalization flows [9,10]. It is most often described in terms of Hilbert spaces on *spacelike* hypersurfaces that lie in the appropriate spacetime regions. However, a more natural description would be in terms of the horizons themselves. This is the central goal of this paper. Specifically, we consider a horizon divided into halves along a spatial surface (or, equivalently, an intersection with a second horizon). We then present a recipe for reducing a pure state on the horizon into mixed states on its two halves. In some cases, e.g. de Sitter horizons and null hyperplanes in flat spacetime, these half-horizon states are causally equivalent to states in the two "halves of space" (in the flat case, up to data on a single lightray at null infinity). In other cases, e.g. a Schwarzschild horizon, reconstructing the spatial state requires additional boundary data. However, even then, the state on the horizon may capture the relevant entanglement, as in the Hawking–Unruh effect [11].

To illustrate our recipe for restricting states to half-horizons, we will use it to derive the Unruh effect: the vacuum state with respect to a null "time" u on a horizon is thermal with temperature $1/2\pi$ with respect to the "time" $\tau = \ln u$ on the half-horizon $u > 0$. Irrespective of our particular derivation, we stress that the universal form of null horizon kinematics allows us to immediately generalize the Unruh effect to arbitrary null "time" parameters on arbitrary curved horizons. We will use this fact to obtain the restriction of a global *thermal* state to a half-horizon, with an application to the causal diamonds of a de Sitter observer.

We assume that the null horizons under consideration are free of caustics. On the other hand, we do not require the horizons to be geodesically complete, so they may be truncated before a

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caustic is reached. We leave the issue of zero modes in lightfront quantization [12,13] outside the scope of the paper. For simplicity, we pretend that our quantum field theory contains only scalar fields, with a standard kinetic term and arbitrary potential. For interacting theories, the assumption of a standard kinetic term should be taken with caution, even though it is commonplace in the QCD lightfront quantization literature. The associated subtleties will be briefly discussed in Appendix A.

2. Operator algebra

The phase space of a field φ on a spacelike or null hypersurface Σ consists of the field's value and normal derivative, with symplectic form:

$$\Omega(\delta\varphi_1, \delta\varphi_2) = \int_\Sigma d^3x \left(\delta\varphi_1 S^\mu \partial_\mu \delta\varphi_2 - (1 \leftrightarrow 2)\right), \quad (1)$$

where S^μ is the densitized normal to Σ. On a null horizon, S^μ is the area current, pointing along the horizon's constituent lightrays. A key feature of the null case is that the normal S^μ is also *tangent* to the horizon; therefore, the values of φ on Σ determine also $S^\mu\partial_\mu\varphi$, and thus span the entire phase space. In this case, the symplectic form (1) becomes:

$$\Omega(\delta\varphi_1, \delta\varphi_2) = \int_\Sigma d^2x\, du \left(\delta\varphi_1 \sqrt{\gamma(u,x)}\partial_u \delta\varphi_2 - (1 \leftrightarrow 2)\right). \quad (2)$$

Here, x are 2d coordinates labeling the lightrays, u is a null coordinate along each ray, and $\sqrt{\gamma}$ is the area density of the 2d metric in the x directions. If we now define a rescaled field $\hat\phi$ by:

$$\hat\phi(u,x) \equiv \sqrt[4]{\gamma(u,x)}\,\hat\varphi(u,x), \quad (3)$$

the symplectic form (2) becomes:

$$\Omega(\delta\phi_1, \delta\phi_2) = \int_\Sigma d^2x\, du \left(\delta\phi_1 \partial_u \delta\phi_2 - (1 \leftrightarrow 2)\right). \quad (4)$$

On horizons where the metric is constant in u, the rescaling (3) becomes trivial; this case was studied in [14]. In general, the rescaling is important as it absorbs the dependence on the metric into the definition of the field ϕ, thus rendering the symplectic form (4) independent of γ. Since the symplectic form, as a functional of the field variations, is independent of the metric, we can import some well-known flat results. In particular, the commutators, obtained by quantizing the Poisson brackets found by inverting the symplectic form (4), can be written as:

$$\left[\hat\phi(u,x), \hat\phi(u',x')\right] = \frac{i}{4}\,\delta^{(2)}(x,x')\,\mathrm{sign}(u'-u). \quad (5)$$

Note that, since they are causally separated, fields on the same lightray do not commute. The expressions (3)–(5) (with additional factors) have appeared in the Poisson brackets [15] for null initial data in General Relativity.

We define creation and annihilation operators by Fourier-transforming $\phi(u,x)$ with respect to the null "time" u:

$$\hat a(\omega,x) = \sqrt{2\omega}\int_{-\infty}^{\infty} du\, e^{i\omega u}\hat\phi(u,x); \quad (6)$$

$$\hat a^\dagger(\omega,x) = \sqrt{2\omega}\int_{-\infty}^{\infty} du\, e^{-i\omega u}\hat\phi(u,x). \quad (7)$$

Using (5), we see that these satisfy the appropriate commutation relations:

$$\left[\hat a(\omega,x), \hat a^\dagger(\omega',x')\right] = 2\pi\,\delta(\omega-\omega')\,\delta^{(2)}(x,x');$$
$$\left[\hat a(\omega,x), \hat a(\omega',x')\right] = \left[\hat a^\dagger(\omega,x), \hat a^\dagger(\omega',x')\right] = 0. \quad (8)$$

Equations (5) and (8) giving the commutators of the field ϕ and its Fourier modes are the same as one would obtain for φ if the horizon was flat. So, while ϕ has simple commutation relations, the corresponding relations for φ will in general be more complicated.

The operators (6)–(7) can be used in the standard way to construct e.g. vacuum or thermal states with respect to the "time translation" generator $i\partial_u$. All of the above is independent of the field's mass and dynamics as encoded in its potential, up to issues with loop corrections that will be discussed in Appendix A.

3. De Sitter horizon

As an example, consider a cosmological horizon in de Sitter space. We define de Sitter space as the hyperboloid $v_\mu v^\mu = 1$ within $\mathbb{R}^{1,4}$, invariant under the isometry group $O(4,1)$. The horizon is a 2-sphere of lightrays defined by $\ell_\mu v^\mu = 0$, where $\ell^\mu = (1,1,\vec 0)$ is a null vector in $\mathbb{R}^{1,4}$. The horizon's points can be coordinatized in $\mathbb{R}^{1,4}$ as:

$$v^\mu = (u, u, \vec n). \quad (9)$$

Here, the unit 3d vector $\vec n$ plays the role of the lightray label x, while u is an affine null time along the rays.

The horizon creation and annihilation operators (6)–(7) have a simple meaning in terms of the Poincaré coordinates $(\eta, \vec y)$, which span the cosmological patch to the horizon's future. These are related to the $4+1$d radius-vector v^μ through:

$$v^\mu = -\frac{1}{\eta}\left(\frac{y^2-\eta^2+1}{2}, \frac{y^2-\eta^2-1}{2}, \vec y\right); \quad \eta < 0. \quad (10)$$

The metric is given by:

$$ds^2 = dv_\mu dv^\mu = \frac{1}{\eta^2}(-d\eta^2 + dy^2). \quad (11)$$

For momentum modes with respect to $\vec y$, the "infinite past" $\eta \to -\infty$ is a UV limit, due to the warp factor in (11). Suppose now that our field theory is well-defined in the UV, by means of a conformal fixed point. Then, although the metric is only *conformally* flat, one can define a Minkowski vacuum at $\eta \to -\infty$ (noting that any curvature corrections from the conformal transformation are irrelevant in the UV limit). This will be the Bunch–Davies vacuum of the full theory in de Sitter space.

Now, the horizon (9) can be expressed in the Poincaré coordinates (10) as a particular form of the $\eta \to -\infty$ limit:

$$\vec y = (-\eta + u)\vec n; \quad \eta \to -\infty. \quad (12)$$

In this limit, the time translation ∂_η becomes the null translation ∂_u. We conclude that the vacuum annihilated by the horizon operators (6) is the Minkowski vacuum at $\eta \to -\infty$, i.e. the Bunch–Davies vacuum.

Note further that spatial translations $\vec y \to \vec y + \delta\vec y$ of the Poincaré coordinates act on the horizon as an $\vec n$-dependent shift $u \to u + \vec n\cdot\delta\vec y$ along the lightrays. From here, it's easy to see that the creation operators $\hat a^\dagger(\omega,\vec n)$ from (7) *create particles with spatial momentum $\vec p = \omega\vec n$ in Poincaré coordinates*. This relates the horizon Fock space to the standard cosmological basis of comoving momenta.