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Small-x evolution of jet quenching parameter

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ABSTRACT

Concept of transverse deflection probability of a parton that travels through strongly interacting medium, recently introduced by D'Eramo, Liu and Rajagopal, has been used to derive high energy evolution equation for the jet quenching parameter in stochastic multiple scatterings regime. Jet quenching parameter, $\hat{q}(x)$, appears to evolve with x, with an exponent $0.9\bar{\alpha}_s$, which is slightly less than that of $x\mathcal{G}(x)$ where $\mathcal{G}(x)$ is the gluon distribution function.

1. Introduction

The heavy-ion collision programs at CERN's Large Hadron Collider have opened new access for exploration of extreme hot and dense nuclear matter. Precision tomography of the nuclear matter is now becoming feasible as an accurate test of the underlying quantum chromodynamic (QCD) theory. This is instrumental in discovering yet unexplored characteristics of various nuclear effects and collective phenomena that the nuclear matter may possess. One possibility is to explore the QCD scale/energy evolution of various observables in this extreme ambiance. Advancement of study for hard sector observables at the LHC elevated the medium modification of high energy jets as prevailing topic of investigation. In this context study on scale/energy evolution of the jet quenching parameter, attributed as the stopping power of the medium for a certain probe of the medium, is now viable. High energy quarks and gluons passing through the interacting nuclear matter have their transverse momentum distribution broadened due to multiple scatterings with the constituents of the medium. While travelling through the strongly coupled medium the hard parton loses energy as well as its direction of momentum changes. Change in the direction of momentum is referred to as 'transverse momentum broadening' for the travelling parton. In the context of jet-medium interaction the broadening refers to the effect on the jet when the direction of the momenta of an ensemble of partons changes due to the random kicks. Even though there is no apparent change in mean momenta, the spread of the momentum distribution of individual parton within that ensemble broadens.

Evolution of the momentum broadening was first studied by Liou, Mueller and Wu [3] by introducing radiative modification over the leading order momentum broadening effect. The authors showed that average momentum broadening $\langle p_\perp^2 \rangle$ has both double and single logarithmic terms. Both the double logarithmic terms, $\ln^2(L/l_0)$, and single logarithmic terms, $\ln(L/l_0)$ are coming from gluon radiation induced by the medium interactions. Here the length of the nuclear matter is L and l_0 is the size of constituents of the matter. Their estimation showed that the radiative contribution is to be a sizeable correction to the nonradiative leading value of $\langle p_\perp^2 \rangle$. Later an evolution equation has been obtained for the inclusive one-gluon distribution, through the concepts of classical branching process and cascade of partons [4]. This explicitly takes into account the dependence of the observed gluon spectrum upon the energy and the transverse momentum. The explicit transverse momentum dependence of the splitting kernel then enables one to identify large corrections to the jet quenching parameter. Subsequent studies on non-linear evolution lead to prescribe the renormalization of the jet-quenching parameter [5,6].

In this paper, in order to study the energy evolution of jet quenching parameter, we have adopted the idea of transverse deflection probability of a parton, that travels through the nuclear medium. Following a recent work by D'Eramo, Liu and Rajagopal [1] we then relate the momentum broadening to the S-matrix of the nuclear interaction for a dipole. The Balitsky–Kovchegov equation as the evolution equation of the S-matrix is then used to derive high energy evolution equation for the jet quenching parameter in stochastic multiple scatterings regime. The known result of double log enhancement emerges as a special case in the limit when the single scattering is only contributing. Power-counting techniques borrowed from Soft-Collinear-Effective-Theory (SCET) [17–20] have been used to identify the leading contributions in the stochastic multiple scatterings region. For an almost constant $\hat{q}(\omega)$ we recovered the double log result (in the limit $Q_s^2 \to \hat{q}L$) first derived in [3] and subsequent other studies [4–6].

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We have also shown that the double log enhancement gets diluted when we go beyond single scattering limit as previously argued in [3]. Jet quenching parameter $\hat{q}(x)$ is found to evolve with x, slightly weaker than $x\mathcal{G}(x)$ where $\mathcal{G}(x)$ is the gluon distribution function.

2. Probability density for transverse deflection $P(k_\perp)$

The transverse momentum broadening of a parton can be studied by introducing concept of a probability density, denoted in this article as $P(k_\perp)$. It signifies the probabilistic weight for the event where, after travelling a medium of length L, amount of transverse momentum that the parton acquired is $k_\perp$ [1]. The probability density $P(k_\perp)$ is chosen to be normalized in the following way,

$$\int \frac{d^2k_\perp}{(2\pi)^2} P(k_\perp) = \frac{1}{4\pi} \int dk_\perp^2 \; P(k_\perp) = 1. \tag{1}$$

Using this probability density one can quickly estimate mean transverse momentum picked up by the hard parton per unit distance travelled,

$$\hat{q} \equiv \frac{\langle k_\perp^2 \rangle}{L} = \frac{1}{4\pi L} \int dk_\perp^2 \; k_\perp^2 \; P(k_\perp). \tag{2}$$

This defines the jet broadening (or quenching) parameter $\hat{q}$ that may have intrinsic dependence on x and Q of the probe through $P(k_\perp)$.

Its important to make a certain caveat when defining the jet quenching parameter, $\hat{q}$, as done in Eq. (2). In absence of any finite upper bound to the integral, the right-hand side of Eq. (2) could be diverging, e.g., when $P(k_\perp)$ behaves as power law, $P(k_\perp) \sim 1/k_\perp^4$ in the event of lowest order perturbative gluon production at large-$k_\perp$. This gives a logarithmic UV divergence, resulting both average transverse momentum $\langle k_\perp^2 \rangle$ as well as jet quenching parameter, $\hat{q}$, infinite. Although, in the case of multiple scattering with Sudakov like form factor or in the event of multiple stochastic scatterings where $P(k_\perp)$ takes the form of an exponentially damping factor, right hand side of Eq. (2) should be finite even without any apparent upper bound in the integral.

In this paper we will discuss the event of multiple stochastic scatterings, where the probe receives random transverse kicks and $P(k_\perp)$ takes the form of a Gaussian with a variance of $\hat{q}L/2$,

$$P(k_\perp) = \frac{4\pi}{\hat{q}L} \; \exp\left(-\frac{k_\perp^2}{\hat{q}L} \right). \tag{3}$$

In this study we are looking for transverse momentum broadening of the hard parton that has initial light cone momentum $p(p^+, p^-, p_\perp) \sim Q\,(0, 1, 0)$ and enters in a brick of strongly interacting medium of length L. In order to do the power counting, we have introduced the dimensionless small parameter λ. Power corrections in λ to some hard process are generally suppressed in the presence of a hard scale, $Q^2 \gg \Lambda_{QCD}^2$, which act as base for the power corrections. This is a concept borrowed from the soft collinear effective theory (SCET) studies [17–20]. In SCET studies the power counting protocol is as follows, terms that are subleading in two orders of magnitude can be dropped but the terms that are suppressed by one order of magnitude should be kept. While travelling through nuclear medium the hard partons interact repeatedly with the Glauber gluons which scale as $Q\,(\lambda^2, \lambda^2, \lambda)$. After a first few scatterings the hard parton's momentum becomes of order $Q\,(\lambda^2, 1, \lambda)$, though still quite collinear. In this scenario of high energy regime where Glauber gluons are mostly effective, it has been shown in Ref. [1] that $P(k_\perp)$ can be expressed as the Fourier transform in $r_\perp$ of the expectation value of two light-like path-ordered Wilson lines transversely separated by $r_\perp$,

$$P(k_\perp) = \int d^2r_\perp \; e^{-ik_\perp r_\perp} \; \mathcal{S}(r_\perp), \tag{4}$$

where,

$$\mathcal{S}(r_\perp) \equiv \frac{1}{N_c} \langle \, \mathrm{Tr}\,[\mathcal{W}(0, r_\perp)\,\mathcal{W}(0, 0)] \, \rangle, \tag{5}$$

with,

$$\mathcal{W}(y^+, y_\perp) \equiv \mathcal{P}\left\{ \exp\left[ig \int_0^{L^-} dy^- A^+\left(y^+, y^-, y_\perp\right) \right] \right\}. \tag{6}$$

Within a dipole picture transverse separation $y_\perp - y'_\perp$ can be taken as the transverse size of the dipole $r_\perp$. The length of the medium is $L = L^-/\sqrt{2}$. In this work we have assumed that $P(k_\perp)$ as the Fourier transform of $\mathcal{S}(r_\perp)$ is well valid while in high energy regime i.e. at small-x and all the evolution characteristics for $P(k_\perp)$ exclusively contained in $\mathcal{S}(r_\perp)$, so that we may write,

$$P(k_\perp, Y) = \int d^2r_\perp \; e^{-ik_\perp r_\perp} \; \mathcal{S}(r_\perp, Y), \tag{7}$$

and,

$$\frac{\partial P(k_\perp, Y)}{\partial Y} = \int d^2r_\perp \; e^{-ik_\perp r_\perp} \; \frac{\partial \mathcal{S}(k_\perp, Y)}{\partial Y}, \tag{8}$$

and therefore from Eq. (2),

$$\frac{\partial \hat{q}(Y)}{\partial Y} = \frac{1}{4\pi L} \int dk_\perp^2 \; k_\perp^2 \int d^2r_\perp \; e^{-ik_\perp r_\perp} \; \frac{\partial \mathcal{S}(r_\perp, Y)}{\partial Y} = \hat{\mathcal{F}}_{[k_\perp, r_\perp]} \left[\frac{\partial \mathcal{S}(r_\perp, Y)}{\partial Y} \right]. \tag{9}$$

Where for brevity in notations we have introduced $\hat{\mathcal{F}}$,

$$\hat{\mathcal{F}}_{[k_\perp, r_\perp]}[\mathcal{O}] \equiv \frac{1}{4\pi L} \int dk_\perp^2 \; k_\perp^2 \int d^2r_\perp \; e^{-ik_\perp r_\perp} \; \mathcal{O}. \tag{10}$$

It can also be shown that,

$$\hat{\mathcal{F}}_{[k_\perp, r_\perp]} \left[r_\perp^2 \exp\left(-\frac{\hat{q}}{4} L r_\perp^2 \right) \right] = -\frac{4}{L}, \tag{11}$$

$$\hat{\mathcal{F}}_{[k_\perp, r_\perp]} \left[r_\perp^4 \exp\left(-\frac{\hat{q}}{4} L r_\perp^2 \right) \right] = 0. \tag{12}$$

Now, the non-linear evolution of the S-matrix, $\mathcal{S}(r_\perp = y_\perp - y'_\perp)$, is governed by the Balitsky–Kovchegov equation (BK) [2,7–9], in large-N_c limit as,

$$\frac{\partial \mathcal{S}(y_\perp, y'_\perp; Y)}{\partial Y} = -\frac{\alpha_s N_c}{2\pi^2} \int d^2z_\perp \frac{(y_\perp - y'_\perp)^2}{(y_\perp - z_\perp)^2 (z_\perp - y'_\perp)^2} \left[\mathcal{S}(y_\perp, y'_\perp; Y) - \mathcal{S}(y_\perp, z_\perp; Y) \mathcal{S}(z_\perp, y'_\perp, Y) \right]. \tag{13}$$

Using Eq. (9) and Eq. (13) with $r_\perp = y_\perp - y'_\perp$, together with fact that the medium is transnationally invariant, evolution equation for $\hat{q}(Y)$, can now be written as,

$$\frac{\partial \hat{q}(Y)}{\partial Y} = -\frac{\alpha_s N_c}{2\pi^2} \hat{\mathcal{F}}_{[k_\perp, r_\perp]}[\mathcal{M}(r_\perp)], \tag{14}$$

where $\mathcal{M}(r_\perp)$ is an integral over the daughter dipoles' transverse coordinates,

$$\mathcal{M}(r_\perp) = \int d^2B_\perp \frac{r_\perp^2}{(r_\perp - B_\perp)^2 B_\perp^2} \left[\mathcal{S}(r_\perp, Y) - \mathcal{S}(r_\perp - B_\perp, Y) \mathcal{S}(B_\perp, Y) \right] \tag{15}$$