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Wormhole inspired by non-commutative geometry

Farook Rahaman [a], Sreya Karmakar [b], Indrani Karar [c], Saibal Ray [d,*]

[a] Department of Mathematics, Jadavpur University, Kolkata 700032, West Bengal, India
[b] Department of Physics, Calcutta Institute of Engineering and Management, Kolkata 700040, West Bengal, India
[c] Department of Mathematics, Saroj Mohan Institute of Technology, Guptipara, West Bengal, India
[d] Department of Physics, Government College of Engineering & Ceramic Technology, Kolkata 700010, West Bengal, India

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ABSTRACT

In the present Letter we search for a new wormhole solution inspired by noncommutative geometry with the additional condition of allowing conformal Killing vectors (CKV). A special aspect of noncommutative geometry is that it replaces point-like structures of gravitational sources with smeared objects under Gaussian distribution. However, the purpose of this letter is to obtain wormhole solutions with noncommutative geometry as a background where we consider a point-like structure of gravitational object without smearing effect. It is found through this investigation that wormhole solutions exist in this Lorentzian distribution with viable physical properties.

1. Introduction

A *wormhole*, which is similar to a tunnel with two ends each in separate points in spacetime or two connecting black holes, was conjectured first by Weyl [1] and later on by Wheeler [2]. In a more concrete physical definition it is essentially some kind of hypothetical topological feature of spacetime which may acts as *shortcut* through spacetime topology.

It is argued by Morris et al. [3,4] and others [5–7] that in principle a wormhole would allow travel in time as well as in space and can be shown explicitly how to convert a wormhole traversing space into one traversing time. However, there are other types of wormholes available in the literature where the traversing path does not pass through a region of exotic matter [8,9]. Following the work of Visser [8], a new type of thin-shell wormhole, which was constructed by applying the cut-and-paste technique to two copies of a charged black hole [10], is of special mention in this regard.

Thus a traversable wormhole, tunnel-like structure connecting different regions of our Universe or of different universes altogether, has been an issue of special investigation under Einstein's general theory of relativity [11]. It is argued by Rahaman et al. [12] that although just as good a prediction of Einstein's theory as black holes, wormholes have so far eluded detection. As one of the peculiar features a wormhole requires the violation of the null energy condition (NEC) [4]. One can note that phantom dark energy also violates the NEC and hence could have deep connection to in formation of wormholes [13,14].

It is believed that some perspective of quantum gravity can be explored mathematically in a better way with the help of noncommutative geometry. This is based on the non-commutativity of the coordinates encoded in the commutator, $[x_\mu, x_\nu] = \theta_{\mu\nu}$, where $\theta_{\mu\nu}$ is an anti-symmetric and real second-ordered matrix which determines the fundamental cell discretization of spacetime [15–19]. We also invoke the inheritance symmetry of the spacetime under conformal Killing vectors (CKV). Basically CKVs are motions along which the metric tensor of a spacetime remains invariant up to a certain scale factor. In a given spacetime manifold M, one can define a globally smooth conformal vector field ξ, such that for the metric g_{ab} it can be written as

$$\xi_{a;b} = \psi g_{ab} + F_{ab}, \tag{1}$$

where $\psi : M \to R$ is the smooth conformal function of ξ and F_{ab} is the conformal bivector of ξ. This is equivalent to the following form:

$$L_\xi g_{ik} = \xi_{i;k} + \xi_{k;i} = \psi g_{ik}, \tag{2}$$

where L signifies the Lie derivatives along the CKV ξ^α.

* Corresponding author.
E-mail addresses: rahaman@iucaa.ernet.in (F. Rahaman), sreya.karmakar@gmail.com (S. Karmakar), indrani.karar08@gmail.com (I. Karar), saibal@iucaa.ernet.in (S. Ray).

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In favor of the prescription of this mathematical technique CKV we find out the following features: (1) it provides a deeper insight into the spacetime geometry and facilitates the generation of exact solutions to the Einstein field equations in a more comprehensive forms, (2) the study of this particular symmetry in spacetime is physically very important as it plays a crucial role of discovering conservation laws and to devise spacetime classification schemes, and (3) because of the highly non-linearity of the Einstein field equations one can reduce easily the partial differential equations to ordinary differential equations by using CKV. Interested readers may look at the recent works on CKV technique available in the literature [20–22].

In this Letter therefore we search for some new solutions of wormhole admitting conformal motion of Killing vectors. It is a formal practice to consider the inheritance symmetry to establish a natural relationship between spacetime geometry and matter–energy distribution for a astrophysical system. Thus our main goal here is to examine the solutions of Einstein field equations by admitting CKV under non-commutative geometry. The scheme of the investigation is as follows: in Section 2 we provide the mathematical formalism and Einstein's field equations under the framework of this technique. A specific matter–energy density profile has been employed in Section 3 to obtain various physical features of the wormhole under consideration by addressing the issues like the conservation equation, stability of the system, active gravitational mass and gravitational energy. Section 4 is devoted for some concluding remarks.

2. Conformal Killing vector and basic equations

We take the static spherically symmetric metric in the following form

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2), \tag{3}$$

where r is the radial coordinate. Here ν and λ are the metric potentials which have functional dependence on r only.

Thus, the only survived Einstein's field equations in their explicit forms (rendering $G = c = 1$) are

$$e^{-\lambda}\left[\frac{\lambda'}{r} - \frac{1}{r^2}\right] + \frac{1}{r^2} = 8\pi\rho, \tag{4}$$

$$e^{-\lambda}\left[\frac{1}{r^2} + \frac{\nu'}{r}\right] - \frac{1}{r^2} = 8\pi p_r, \tag{5}$$

$$\frac{1}{2}e^{-\lambda}\left[\frac{1}{2}(\nu')^2 + \nu'' - \frac{1}{2}\lambda'\nu' + \frac{1}{r}(\nu' - \lambda')\right] = 8\pi p_t, \tag{6}$$

where ρ, p_r and p_t are matter–energy density, radial pressure and transverse pressure respectively for the fluid distribution. Here $'$ over ν and λ denotes partial derivative w.r.t. radial coordinate r.

The conformal Killing equations, as mentioned in Eq. (2), then yield as follows:

$$\xi^1\nu' = \psi,$$

$$\xi^4 = C_1 = constant,$$

$$\xi^1 = \frac{\psi r}{2},$$

$$\xi^1\lambda' + 2\xi^1_{,1} = \psi,$$

where ξ^α are the conformal 4-vectors and ψ is the conformal function as mentioned earlier.

This set of equations, in a straight forward way, imply the following simple forms:

$$e^\nu = C_2^2 r^2, \tag{7}$$

$$e^\lambda = \frac{{C_3}^2}{\psi^2}, \tag{8}$$

$$\xi^i = C_1\delta^i_4 + \left(\frac{\psi r}{2}\right)\delta^i_1, \tag{9}$$

where C_2 and C_3 are integration constants. Here the non-zero components of the conformal Killing vector ξ^a are ξ^0 and ξ^1.

Now using solutions (7) and (8), Eqs. (3)–(5) take the following form as

$$\frac{1}{r^2}\left[1 - \frac{\psi^2}{C_3^2}\right] - \frac{2\psi\psi'}{C_3^2 r} = 8\pi\rho, \tag{10}$$

$$\frac{1}{r^2}\left[1 - \frac{3\psi^2}{C_3^2}\right] = -8\pi p_r, \tag{11}$$

$$\left[\frac{\psi^2}{C_3^2 r^2}\right] + \frac{2\psi\psi'}{C_3^2 r} = 8\pi p_t. \tag{12}$$

These are the equations forming master set which has all the information of the fluid distribution under the framework of Einstein's general theory of relativity with the associated non-commutative geometry and conformal Killing vectors.

3. The matter–energy density profile and physical features of the wormhole

As stated by Rahaman et al. [11], the necessary ingredients that supply fuel to construct wormholes remain an elusive goal for theoretical physicists and there are several proposals that have been put forward by different authors [23–28]. However, in our present work we consider cosmic fluid as source and thus have provided a new class of wormhole solutions. Keeping the essential aspects of the noncommutativity approach which are specifically sensitive to the Gaussian nature of the smearing as employed by Nicolini et al. [18], we rather get inspired by the work of Mehdipour [29] to search for a new fluid model admitting conformal motion. Therefore, we assume a Lorentzian distribution of particle-like gravitational source and hence the energy density profile as given in Ref. [29] as follows:

$$\rho(r) = \frac{M\sqrt{\phi}}{\pi^2(r^2 + \phi)^2}, \tag{13}$$

where ϕ is the noncommutativity parameter and M is the smeared mass distribution.

Now, solving Eq. (10) we get

$$\psi^2 = C_3^2 - \left(\frac{4C_3^2 M}{\pi r}\right)\left[tan^{-1}\left(\frac{r}{\sqrt{\phi}}\right) - \frac{r\sqrt{\phi}}{r^2 + \phi}\right] + \frac{D_1}{r}, \tag{14}$$

where D_1 is an integration constant and can be taken as zero.

The parameters, like the radial pressure, tangential pressure and metric potentials, are found as

$$p_r = \frac{1}{8\pi}\left[\frac{2}{r^2} - \frac{12M}{\pi r^3}\left\{tan^{-1}\left(\frac{r}{\sqrt{\phi}}\right) - \frac{r\sqrt{\phi}}{r^2 + \phi}\right\}\right], \tag{15}$$

$$p_t = \frac{1}{8\pi}\left[\frac{1}{r^2} - 8\pi\left(\frac{M\sqrt{\phi}}{\pi^2(r^2 + \phi)^2}\right)\right], \tag{16}$$

$$e^\nu = C_2^2 r^2, \tag{17}$$

$$e^\lambda = \frac{1}{\left[1 - \left(\frac{4M}{\pi r}\right)\left(tan^{-1}\left(\frac{r}{\sqrt{\phi}}\right) - \frac{r\sqrt{\phi}}{r^2+\phi}\right)\right]}. \tag{18}$$