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Higher-order Lorentz-invariance violation, quantum gravity and fine-tuning

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ABSTRACT

The issue of Lorentz fine-tuning in effective theories containing higher-order operators is studied. To this end, we focus on the Myers–Pospelov extension of QED with dimension-five operators in the photon sector and standard fermions. We compute the fermion self-energy at one-loop order considering its even and odd *CPT* contributions. In the even sector we find small radiative corrections to the usual parameters of QED which also turn to be finite. In the odd sector the axial operator is shown to contain unsuppressed effects of Lorentz violation leading to a possible fine-tuning. We use dimensional regularization to deal with the divergencies and a generic preferred four-vector. Taking the first steps in the renormalization procedure for Lorentz violating theories we arrive to acceptable small corrections allowing to set the bound $\xi < 6 \times 10^{-3}$.

1. Introduction

New physics from the Planck scale has been hypothesized to show up at low energies as small violations of Lorentz symmetry [1]. This possibility has been supported by the idea that spacetime may change drastically at high energies giving place to some level or discreteness or spacetime foam. In the language of effective theory the Lorentz symmetry departures are implemented with Planck mass suppressed operators in the Lagrangians. The effective approach has been shown to be extremely successful in order to contrast the possible Lorentz and CPT symmetry violations with experiments. A great part of these searches have been given within the framework of the standard model extension with several bounds on Lorentz symmetry violation provided [2–4]. In general most of the studies on Lorentz symmetry violation have been performed with operators of mass dimension $d \leq 4$ [5]. In part because the higher-order theories present some problems in their quantization [6]. However, in the last years these operators have received more attention and several bounds have been put forward [7–11]. Moreover, a generalization has been constructed to include non-minimal terms in the effective framework of the standard model extension [12].

Many years ago Lee and Wick [13] and Cutkosky [14] studied the unitarity of higher-order theories using the formalism of indefinite metrics in Hilbert space. They succeeded to prove that unitarity can be conserved in some higher-order models by restricting the space of asymptotic states. This has stimulated the construction of several higher-order models beyond the standard model [15]. One example is the Myers and Pospelov model based on dimension-five operators describing possible effects of quantum gravity [16,17]. In the model the Lorentz symmetry violation is characterized by a preferred four-vector n [18,19]. The preferred four-vector may be thought to come from a spontaneous symmetry breaking in an underlying fundamental theory. One of the original motivations to incorporate such terms was to produce cubic modifications in the dispersion relation, although an exact calculation yields a more complicated structure usually with the Gramian of the two vectors k and n involved. The Myers and Pospelov model has become an important arena to study higher-order effects of Lorentz-invariance violation [8,20–22].

This work aims to contribute to the discussion on the fine-tuning problem due to Lorentz symmetry violation [23], in particular when higher-order operators are present. There are different approaches to the subject, for example using the ingredient of discreteness [24] or supersymmetry [25]. For renormalizable operators, including higher space derivatives, large Lorentz violations can or not appear depending on the model and regularization

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scheme [26]. However, higher-order operators are good candidates to produce strong Lorentz violations via induced lower dimensional operators [27]. Some attempts to deal with the fine tuning problem considers modifications in the tensor contraction with a given Feynman diagram [16] or just restrict attention to higher-order corrections [28]. However in both cases the problem comes back at higher-order loops [29]. Here we analyze higher-order Lorentz violation by explicitly computing the radiative corrections in the Myers and Pospelov extension of QED. We use dimensional regularization which eventually preserves unitarity, thus extending some early treatments [18,20].

2. The QED extension with dimension-5 operators

The Myers–Pospelov Lagrangian extension of QED with modifications in the photon sector can be written as [16]

$$\mathcal{L} = \bar{\psi}(\gamma^\mu \partial_\mu - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\xi}{2m_{\rm Pl}} n_\mu \epsilon^{\mu\nu\lambda\sigma} A_\nu (n\cdot\partial)^2 F_{\lambda\sigma}\,, \tag{1}$$

where $m_{\rm Pl}$ is the Planck mass, ξ a dimensionless coupling parameter and n is a four-vector defining a preferred reference frame. In addition we introduce the gauge fixing Lagrangian term, $\mathcal{L}_{G.F} = -B(x)(n\cdot A)$, where $B(x)$ is an auxiliary field.

The field equations for A_μ and B derived from the Lagrangian $\mathcal{L} + \mathcal{L}_{G.F}$ read,

$$\partial_\mu F^{\mu\nu} + g\epsilon^{\nu\alpha\lambda\sigma} n_\alpha (n\cdot\partial)^2 F_{\lambda\sigma} = Bn^\nu\,, \tag{2}$$

$$n\cdot A = 0\,. \tag{3}$$

where $g = \frac{\xi}{m_{\rm Pl}}$. Contracting Eq. (2) with ∂_ν gives $(\partial\cdot n)B = 0$, which allows us to set $B = 0$. In the same way, the contraction of Eq. (2) with n_ν in momentum space leads to $k\cdot A = 0$.

We can choose the polarization vectors $e^{(a)}_\mu$ with $a = 1, 2$ to lie on the orthogonal hyperplane defined by k and n [30], satisfying $e^{(a)}\cdot e^{(b)} = -\delta^{ab}$ and

$$-\sum_a (e^{(a)}\otimes e^{(a)})_{\mu\nu} = -(e^{(1)}_\mu e^{(1)}_\nu + e^{(2)}_\mu e^{(2)}_\nu) \equiv e_{\mu\nu}\,, \tag{4}$$

$$\sum_a (e^{(a)}\wedge e^{(a)})_{\mu\nu} = e^{(1)}_\mu e^{(2)}_\nu - e^{(2)}_\mu e^{(1)}_\nu \equiv \epsilon_{\mu\nu}\,. \tag{5}$$

In particular, one can choose

$$e^{\mu\nu} = \eta^{\mu\nu} - \frac{(n\cdot k)}{D}(n^\mu k^\nu + n^\nu k^\mu) + \frac{k^2}{D}n^\mu n^\nu + \frac{n^2}{D}k^\mu k^\nu\,, \tag{6}$$

$$\epsilon^{\mu\nu} = \frac{1}{\sqrt{D}}\epsilon^{\mu\alpha\rho\nu} n_\alpha k_\rho\,, \tag{7}$$

with $D = (n\cdot k)^2 - n^2k^2$. With these elements the photon propagator can be written as

$$\Delta_{\mu\nu}(k) = -\sum_{\lambda=\pm1} \frac{P^{(\lambda)}_{\mu\nu}(k)}{k^2 + 2g\lambda(k\cdot n)^2\sqrt{D}}\,, \tag{8}$$

where $P^{(\lambda)}_{\mu\nu} = \frac{1}{2}(e_{\mu\nu} + i\lambda\epsilon_{\mu\nu})$ is an orthogonal projector.

3. The fermion self-energy

We compute the fermion self-energy with the modifications introduced only via the Lorentz violating photon propagator (8). The one loop-order approximation to the fermion self-energy is

$$\Sigma_2(p) = ie^2 \int \frac{d^4k}{(2\pi)^4}\gamma^\mu \left(\frac{\not p - \not k + m}{(p-k)^2 - m^2}\right)\gamma^\nu \Delta_{\mu\nu}(k)\,, \tag{9}$$

which can be decomposed into a *CPT* even part

$$\Sigma_2^{(+)}(p) = -\frac{ie^2}{2}\sum_\lambda \int \frac{d^4k}{(2\pi)^4}\gamma^\mu \left(\frac{\not p - \not k + m}{(p-k)^2 - m^2}\right) \times \frac{\gamma^\nu e_{\mu\nu}}{k^2 + 2g\lambda(k\cdot n)^2\sqrt{D}}\,, \tag{10}$$

and a *CPT* odd part

$$\Sigma_2^{(-)}(p) = -\frac{ie^2}{2}\sum_\lambda \int \frac{d^4k}{(2\pi)^4}\gamma^\mu \left(\frac{\not p - \not k + m}{(p-k)^2 - m^2}\right) \times \frac{\gamma^\nu i\lambda\epsilon_{\mu\nu}}{k^2 + 2g\lambda(k\cdot n)^2\sqrt{D}}\,. \tag{11}$$

Next we expand in powers of external momenta obtaining

$$\Sigma_2(p) = \Sigma_2(0) + p_\alpha \left(\frac{\partial\Sigma_2(p)}{\partial p_\alpha}\right)_{p=0} + \Sigma_g\,, \tag{12}$$

where Σ_g are convergent terms in the limit $g\to 0$ depending on quadratic and higher powers of p.

In order to compute the corrections our strategy will be i) perform a Wick rotation and extend analytically any four vector to the Euclidean $x_E = (ix_0, \vec{x})$, and ii) use dimensional regularization in spherical coordinates for the divergent integrals. To begin, we are interested on the first two even contributions in Eqs. (10) and (12), which are

$$\Sigma_2^{(+)}(0) = -\frac{ie^2}{2}m\sum_\lambda \int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m^2)}\frac{\gamma^\mu e_{\mu\nu}\gamma^\nu}{k^2 + 2g\lambda(k\cdot n)^2\sqrt{D}}\,, \tag{13}$$

$$\frac{\partial\Sigma_2^{(+)}(0)}{\partial p_\alpha} = -\frac{ie^2}{2}\sum_\lambda \int \frac{d^4k}{(2\pi)^4}\left[\frac{1}{(k^2-m^2)} - \frac{2k_\alpha^2}{(k^2-m^2)^2}\right] \times \frac{\gamma^\mu\gamma^\alpha\gamma^\nu e_{\mu\nu}}{k^2 + 2g\lambda(k\cdot n)^2\sqrt{D}}\,. \tag{14}$$

Applying our strategy leads to

$$\Sigma_2^{(+)}(0) = e^2 m\sum_\lambda \int \frac{d^4k_E}{(2\pi)^4}\frac{1}{(k_E^2+m^2)(k_E^2 - 2g\lambda(k_E\cdot n_E)^2\sqrt{D_E})}\,,$$

$$\frac{\partial\Sigma_2^{(+)}(0)}{\partial p_\alpha} = -\frac{e^2}{2}\left(n_\nu n^\alpha - \frac{n_E^2}{2}\eta^\alpha_\nu\right)\gamma^\nu \times \sum_\lambda \int \frac{d^4k_E}{(2\pi)^4}\left[\frac{1}{(k_E^2+m^2)} - \frac{k_E^2}{2(k_E^2+m^2)^2}\right] \times \frac{k_E^2}{D_E}\frac{1}{(k_E^2 - 2g\lambda(k_E\cdot n_E)^2\sqrt{D_E})}\,, \tag{15}$$

where we have used $\gamma^\mu e_{\mu\nu}\gamma^\nu = 2$ and $D_E = (n_E\cdot k_E)^2 - k_E^2 n_E^2$. The calculation produces

$$\Sigma_2^{(+)}(0) = \frac{e^2 m}{8\pi^2}\left(1 - \ln\left(\frac{g^2m^2(n_E^2)^3}{16}\right)\right)\,,$$

$$p_\alpha\frac{\partial\Sigma_2^{(+)}(0)}{\partial p_\alpha} = -\frac{e^2}{16\pi^2}\left(\frac{1}{2}\not p - \frac{\not n(n\cdot p)}{n_E^2}\right) \times \left(1 + \ln\left(\frac{g^2m^2(n_E^2)^3}{16}\right)\right)\,. \tag{16}$$

Let us emphasize that the renormalization in the even sector involves small corrections without any possible fine-tuning. Also, the