Physics Letters B 746 (2015) 237–241

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Relating the small parameters of neutrino oscillations

Soumita Pramanick, Amitava Raychaudhuri

Department of Physics, University of Calcutta, 92 Acharya Prafulla Chandra Road, Kolkata 700009, India

A R T I C L E   I N F O

Article history:
Received 1 March 2015
Received in revised form 3 May 2015
Accepted 3 May 2015
Available online 6 May 2015
Editor: J. Hisano

Keywords:
Neutrino mixing
θ_{13}
Leptonic CP-violation
Neutrino mass ordering
Perturbation

A B S T R A C T

Neutrino oscillations reveal several small parameters, namely, θ_{13}, the solar mass splitting *vis-à-vis* the atmospheric one, and the deviation of θ_{23} from maximal mixing. Can these small quantities all be traced to a single source and, if so, how could that be tested? Here a see-saw model for neutrino masses is presented wherein a dominant term generates the atmospheric mass splitting with maximal mixing in this sector, keeping $\theta_{13} = 0$ and zero solar splitting. A Type-I see-saw perturbative contribution results in non-zero values of θ_{13}, Δm^2_{solar}, θ_{12}, as well as allows θ_{23} to deviate from $\pi/4$ in consistency with the data while interrelating them all. CP-violation is a natural consequence and is large ($\delta \sim \pi/2, 3\pi/2$) for inverted mass ordering. The model will be tested as precision on the neutrino parameters is sharpened.

Information on neutrino mass and mixing have been steadily emerging from oscillation experiments. Among them the angle[1] θ_{13} is small ($\sin\theta_{13} \sim 0.1$) [1] while global fits to the solar, atmospheric, accelerator, and reactor neutrino oscillation data indicate that θ_{23} is near maximal ($\sim \pi/4$) [2,3]. On the other hand, the solar mass square difference is two orders smaller than the atmospheric one. These mixing parameters and the mass ordering are essential inputs for identifying viable models for neutrino masses.

A natural choice could be to take the mixing angles to be initially either $\pi/4$ (θ_{23}) or zero (θ_{13}, θ_{12}) and the solar splitting absent. In this spirit, here a proposal is put forward under which the atmospheric mass splitting and maximal mixing in this sector arise from a zero-order mass matrix while the smaller solar mass splitting and realistic θ_{13} and θ_{23} are generated by a Type-I see-saw [4] which acts as a perturbation. θ_{12} also arises out of the same perturbation and as a consequence of degeneracy is not constrained to be small. Attempts to generate *some* of the neutrino parameters by perturbation theory are not new [5,6], but to our knowledge there is no work in the literature that indicates that *all* the small parameters could have the same perturbative origin and agree with the current data.

The unperturbed neutrino mass matrix in the mass basis is $M^0 = \text{diag}\{m_1^{(0)}, m_1^{(0)}, m_3^{(0)}\}$ with the mixing matrix of the form

$$U^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \\ 0 & -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix} . \quad (1)$$

Here $\Delta m^2_{atm} = (m_3^{(0)})^2 - (m_1^{(0)})^2$. By suitably choosing the Majorana phases the masses $m_1^{(0)}$, $m_3^{(0)}$ are taken to be real and positive. The columns of U^0 are the unperturbed flavour eigenstates.[2] As stated, $\Delta m^2_{solar} = 0$ and $\theta_{13} = 0$. Since the first two states are degenerate in mass, one can also take $\theta_{12} = 0$. It is possible to generate this mass matrix from a Type-II see-saw.

In the flavour basis the mass matrix is $(M^0)^{flavour} = U^0 M^0 U^{0T}$ which in terms of $m^\pm = m_3^{(0)} \pm m_1^{(0)}$ is

$$(M^0)^{flavour} = \frac{1}{2}\begin{pmatrix} 2m_1^{(0)} & 0 & 0 \\ 0 & m^+ & m^- \\ 0 & m^- & m^+ \end{pmatrix} . \quad (2)$$

The perturbation is obtained by a Type-I see-saw. To reduce the number of independent parameters, in the flavour basis the Dirac mass term is taken to be proportional to the identity, i.e.,

$$M_D = m_D\, \mathbb{I} \, . \quad (3)$$

E-mail addresses: soumitaparamanick5@gmail.com (S. Pramanick), palitprof@gmail.com (A. Raychaudhuri).

[1] For the lepton mixing matrix the standard PMNS form is used.

[2] In the flavour basis the charged lepton mass matrix is diagonal.

http://dx.doi.org/10.1016/j.physletb.2015.05.007

In this basis, in the interest of minimality the right-handed neutrino Majorana mass matrix is taken with only two non-zero complex entries.

$$M_R^{flavour} = m_R \begin{pmatrix} 0 & xe^{-i\phi_1} & 0 \\ xe^{-i\phi_1} & 0 & 0 \\ 0 & 0 & ye^{-i\phi_2} \end{pmatrix} , \tag{4}$$

where x, y are dimensionless constants of $\mathcal{O}(1)$. No generality is lost by keeping the Dirac mass real.

As a warm-up consider first the real case, i.e., $\phi_1 = 0$ or π, $\phi_2 = 0$ or π. For notational convenience in the following the phase factors are not displayed; instead x (y) is taken as positive or negative depending on whether ϕ_1 (ϕ_2) is 0 or π. Negative x and y offer interesting variants which are stressed at the appropriate points.

The Type-I see-saw contribution in the mass basis is:

$$M'^{mass} = U^{0T} \left[M_D^T (M_R^{flavour})^{-1} M_D \right] U^0$$
$$= \frac{m_D^2}{\sqrt{2}\, xy m_R} \begin{pmatrix} 0 & y & y \\ y & \frac{x}{\sqrt{2}} & -\frac{x}{\sqrt{2}} \\ y & -\frac{x}{\sqrt{2}} & \frac{x}{\sqrt{2}} \end{pmatrix} . \tag{5}$$

The effect on the solar sector is governed by the submatrix of M'^{mass} in the subspace of the two degenerate states,

$$M'^{mass}_{2\times 2} = \frac{m_D^2}{\sqrt{2}\, xy m_R} \begin{pmatrix} 0 & y \\ y & x/\sqrt{2} \end{pmatrix} . \tag{6}$$

To first order in the perturbation:

$$\tan 2\theta_{12} = 2\sqrt{2} \left(\frac{y}{x} \right) . \tag{7}$$

For $y/x = 1$ one obtains the tribimaximal mixing value of θ_{12} which, though allowed by the data[3] at 3σ, is beyond the 1σ region. Since for the entire range of θ_{12} one has $\tan 2\theta_{12} > 0$, x and y must be chosen of the same sign. Therefore, either $\phi_1 = 0 = \phi_2$ or $\phi_1 = \pi = \phi_2$. From the global fits to the experimental results one finds:

$$0.682 < \frac{y}{x} < 1.075 \text{ at } 3\sigma . \tag{8}$$

Further, from Eq. (6),

$$\Delta m^2_{solar} = \frac{m_D^2}{xy m_R} m_1^{(0)} \sqrt{x^2 + 8y^2} . \tag{9}$$

To first order in the perturbation the corrected wave function $|\psi_3\rangle$ is:

$$|\psi_3\rangle = \begin{pmatrix} \kappa \\ \frac{1}{\sqrt{2}}(1 - \frac{\kappa}{\sqrt{2}} \frac{x}{y}) \\ \frac{1}{\sqrt{2}}(1 + \frac{\kappa}{\sqrt{2}} \frac{x}{y}) \end{pmatrix} , \tag{10}$$

where

$$\kappa \equiv \frac{m_D^2}{\sqrt{2}\, x m_R m^-} . \tag{11}$$

For positive x the sign of κ is fixed by that of m^-. Since by convention all the mixing angles θ_{ij} are in the first quadrant, from Eq. (10) one must identify:

$$\sin\theta_{13} \cos\delta = \kappa = \frac{m_D^2}{\sqrt{2}\, x m_R m^-} , \tag{12}$$

[3] We use the 3σ ranges $7.03 \le \Delta m^2_{21}/10^{-5}$ eV$^2 \le 8.03$ and $31.30° \le \theta_{12} \le 35.90°$ [2].

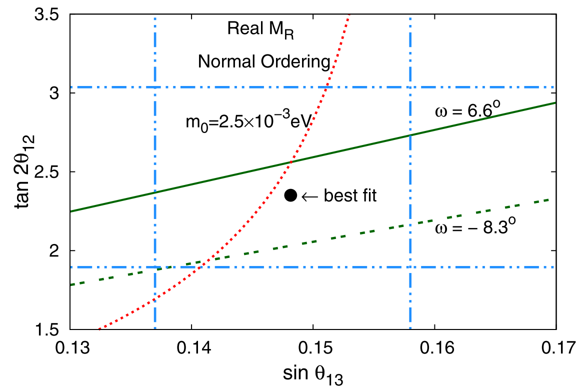

Fig. 1. The blue dot-dashed box is the global-fit 3σ allowed range of $\sin\theta_{13}$ and $\tan 2\theta_{12}$. The best-fit point is shown as a black dot. The red dotted curve is from Eq. (13) with $m_0 = 2.5$ meV when the best-fit values of the two mass-splittings are used. The portion below the green solid (dashed) straight line is excluded by θ_{23} at 3σ – Eq. (17) – for the first (second) octant. In case of inverted ordering no solution of Eq. (13) is allowed for real M_R. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

where for $x > 0$ the PMNS phase $\delta = 0$ for normal mass ordering (NO) and $\delta = \pi$ for inverted mass ordering (IO). Needless to say, both these cases are CP conserving. If x is negative then NO (IO) would correspond to $\delta = \pi$ (0).

An immediate consequence of Eqs. (12), (7), and (9) is

$$\Delta m^2_{solar} = \text{sgn}(x)\, m^- m_1^{(0)} \frac{4 \sin\theta_{13} \cos\delta}{\sin 2\theta_{12}} , \tag{13}$$

which exhibits how the solar sector and θ_{13} are intertwined. The positive sign of Δm^2_{solar}, preferred by the data, is trivially verified since $\text{sgn}(x)\, m^- \sin\theta_{13} \cos\delta > 0$ from Eq. (12). However, Eq. (13) excludes inverted ordering. Once the neutrino mass square splittings, θ_{12}, and θ_{13} are chosen, Eq. (13) determines the lightest neutrino mass, m_0. Defining $z = m^- m_1^{(0)}/\Delta m^2_{atm}$ and $m_0/\sqrt{|\Delta m^2_{atm}|} = \tan\xi$, one has

$$z = \sin\xi/(1 + \sin\xi) \text{ (normal ordering)},$$
$$z = 1/(1 + \sin\xi) \text{ (inverted ordering)} . \tag{14}$$

It is seen that $0 \le z \le 1/2$ for NO and $1/2 \le z \le 1$ for IO, with $z \to 1/2$ corresponding to quasidegeneracy, i.e., $m_0 \to$ large, in both cases. From Eq. (13)

$$z = \left(\frac{\Delta m^2_{solar}}{|\Delta m^2_{atm}|} \right) \left(\frac{\sin 2\theta_{12}}{4 \sin\theta_{13} |\cos\delta|} \right) , \tag{15}$$

with $|\cos\delta| = 1$ for real M_R. As shown below, the allowed ranges of the oscillation parameters imply $z \sim 10^{-2}$ and so inverted mass ordering is disallowed.

From Eq. (10) one further finds:

$$\tan\theta_{23} \equiv \tan(\pi/4 - \omega) = \frac{1 - \frac{\kappa}{\sqrt{2}} \frac{x}{y}}{1 + \frac{\kappa}{\sqrt{2}} \frac{x}{y}} , \tag{16}$$

where, using Eqs. (7) and (12),

$$\tan\omega = \frac{2 \sin\theta_{13} \cos\delta}{\tan 2\theta_{12}} . \tag{17}$$

θ_{23} will be in the first (second) octant, i.e., the sign of ω will be positive (negative) if $\delta = 0$ (π). Recall, this corresponds to $x > 0$ ($x < 0$).

In Fig. 1 the global-fit 3σ range of $\sin\theta_{13}$ and $\tan 2\theta_{12}$ is shown as the blue dot-dashed box with the best-fit value indicated by