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Graviton modes in multiply warped geometry

Mathew Thomas Arun [a], Debajyoti Choudhury [a], Ashmita Das [b], Soumitra SenGupta [b,*]

[a] Department of Physics and Astrophysics, University of Delhi, Delhi 110007, India
[b] Department of Theoretical Physics, Indian Association for the Cultivation of Sciences, 2A&B R.S.C. Mullick Road, Kolkata 700 032, India

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ABSTRACT

The negative results in the search for Kaluza–Klein graviton modes at the LHC, when confronted with the discovery of the Higgs, have been construed to have severely limited the efficacy of the Randall–Sundrum model as an explanation of the hierarchy problem. We show, though, that the presence of multiple warping offers a natural resolution of this conundrum through modifications in both the graviton spectrum and their couplings to the Standard Model fields.

1. Introduction

Despite the spectacular success of the Standard Model (SM) of elementary particles, the search for new physics beyond the SM continues. One of the primary motivations for this is to resolve the well-known gauge hierarchy/naturalness problem in connection with the fine tuning of the higgs mass against large radiative corrections. Among several proposals to address this problem, models with extra spatial dimensions draw special attention. In this context, the warped geometry model proposed by Randall and Sundrum (RS) [1] turned out to be particularly successful for (*i*) it resolves the gauge hierarchy problem without bringing in any other intermediate scale in the theory in contrast to the large extra dimensional models; (*ii*) the modulus of the extra dimensional model can be stabilized to a desired value by the Goldberger–Wise mechanism [2], and (*iii*) a similar warped solution can be obtained from a more fundamental theory like string theory where extra dimensions appear naturally [3]. As a result, several search strategies at the LHC were designed specifically [4–7] to detect the indirect/direct signatures of these warped extra dimensions e.g. through the dileptonic decays of Kaluza–Klein (KK) excitations of the graviton which appear in these models at the TeV scale.

The original RS model was defined as a slice of AdS_5 space with an S^1/Z_2 orbifolding and a pair of three-branes located at the orbifold fixed points, viz. $y = 0, \pi$ (with the SM fields being localized on the last mentioned). The parameters characterizing the theory are the 5-dimensional fundamental (gravitational) scale M_5 and the bulk cosmological constant Λ_5. The solution to Einstein's equations, on demanding a $(1+3)$-dimensional Lorentz symmetry, then leads to a warp-factor in the metric of the form $\exp(-k_5 r_c y)$ where r_c is the compactification radius and $k_5 = \sqrt{-\Lambda_5/24 M_5^3}$. Clearly, the applicability of the semiclassical treatment (as opposed to a full quantum gravity calculation) requires that the bulk curvature k_5 be substantially smaller than M_5. An analogous string theoretic argument [8] relating the D3 brane tension to the string scale (related, in turn, to M_5 through Yang–Mills gauge couplings) demands the same, leading to $k_5/M_5 \lesssim 0.1$. On the other hand, too small a value for this ratio would, typically, necessitate a considerable hierarchy between r_c^{-1} and M_5, thereby taking away from the merits of the scenario. Thus, it is normally accepted that one should consider only $0.01 \le k_5/M_5 \le 0.1$. Indeed, this constraint plays a crucial role in most of the phenomenological studies of this scenario, and certainly for the aforementioned results reported by the ATLAS and the CMS groups. Throughout our analysis we shall impose an analogous condition on the bulk curvature as an important restriction to ensure the applicability of our semiclassical calculations.

In the context of the original RS model, the large exponential warping is held responsible for the apparent lightness of the Higgs vacuum expectation value v (and its mass), as perceived on our brane, related as it is to some naturally high scale $\tilde{v} \sim \mathcal{O}(M_5)$, applicable at the other brane, through the relation

$$v = \tilde{v}\, e^{-\pi k_5 r_c} \, . \tag{1}$$

Here $\tilde{v}$ is determined by the natural scale of higher dimensional model ∼ five dimensional Planck scale M_5 and $k_5 r_c \approx 12$ would explain the hierarchy with r_c being stabilized to this value by

* Corresponding author.
E-mail addresses: thomas.mathewarun@gmail.com (M.T. Arun), debajyoti.choudhury@gmail.com (D. Choudhury), ashmita.phy@gmail.com (A. Das), soumitraiacs@gmail.com (S. SenGupta).

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some mechanism [2]. The compactification leads to a non-trivial KK tower of gravitons with the levels being given by

$$m_n = x_n k_5 e^{-\pi k_5 r_c} \tag{2}$$

where x_n's are the roots of the Bessel function of order one. With only the lowest (massless) graviton wavefunction being localized away from our brane, its coupling to the SM fields is small, viz. $\mathcal{O}(M_5^{-1})$. As the couplings of the others to the SM fields suffer no such suppression, they are, presumably, accessible to collider searches. The ATLAS Collaboration [5], though, has reported negative results ruling out a level-1 KK graviton in the mass range below 1.03 (2.23) TeV, with the exact lower bound depending on the value chosen for k_5/M_5.

This result immediately brings forth a potential problem for the model, for Eqs. (1) and (2) together demand that

$$\frac{m_1}{m_H} \sim \frac{m_1}{v} = x_1 \frac{k_5}{\widetilde{v}} = x_1 \frac{k_5}{M_5} \frac{M_5}{\widetilde{v}} \tag{3}$$

Since $k_5/M_5 \lesssim 0.1$, it is immediately apparent that, unless $\widetilde{v}$ is at least two orders of magnitude smaller than M_5, a 126 GeV Higgs [9,10] would cry out for a KK graviton below a TeV. Indeed, this argument has been inverted in the literature [11] to argue for a much lower cutoff (in other words $\widetilde{v}$) in the theory. In other words, some new physics would need to appear at least two orders of magnitude below the fundamental scale M_5, which, in the RS scenario is very close to the four-dimensional Planck scale itself.

Let us remind ourselves of the nature of cutoffs in the effective four-dimensional theory, considered as a theory of the SM fields augmented by the RS gravitons. While the SM is operative below the scale of the first KK graviton, the *new* four-dimensional theory is operative all the way up to the compactification scale $\sim r_c^{-1}$ when each of the KK graviton is expected to take part in the amplitude estimation as the beam energy is increased. Beyond the energy $\sim r_c^{-1}$, we indeed encounter new physics by probing into the extra dimension where the theory can no longer be defined as an effective theory in four dimensions defined by standard model and KK gravitons.

It is important to realize, at this stage, that part of the aforementioned problem lies in the very restrictive nature of the RS model as it is impossible to lower r_c^{-1} by two orders without disturbing the value of the warped factor significantly. This, in turn, would introduce a little hierarchy necessitating a fine tuning of 2–3 orders so that the Higgs mass may be kept $\sim$ 125 GeV. This feature would worsen further if a graviton KK mode continues to elude us in the forthcoming runs of the LHC, as well as in future collider experiments.

On the other hand, within the context of a generalization of the RS model with additional warped extra dimensions, a lower cutoff appears *naturally*, in the form of a larger compactification radius. In other words, the problem is circumvented without the need for any additional (small) fine tuning. Indeed, once we admit more than four dimensions, there is no particular reason to restrict the number to five, especially with constructs such as string theoretic models arguing in favour of many more. Such variants of the RS model have been proposed earlier [12–15,28] with these, typically, considering several independent S^1/Z_2 orbifolded dimensions along with $M^{(1,3)}$. For example, codimension-2 brane models [16] have been invoked to address aspects like Hubble expansion and inflation [17–19], Casimir densities [20,21], little RS hierarchy [22], gravity and matter field localizations [23,24], fermion mass generations [25,26], moduli stabilization [27], etc.

We begin our study, with a brief discussion of the basic features of warped geometry model in 6-dimension with two successive S_1/Z_2 orbifoldings.

2. Multiply warped brane world model in 6D

Consider a doubly warped compactified six-dimensional space–time with successive Z_2 orbifolding in each of the extra dimensions, viz. $M^{1,5} \to [M^{1,3} \times S^1/Z_2] \times S^1/Z_2$. Demanding four-dimensional (x^μ) Lorentz symmetry within the set up, requires the line element to be given by [28]

$$ds_6^2 = b^2(z)[a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + R_y^2 dy^2] + r_z^2 dz^2 \,, \tag{4}$$

where the compact directions are represented by the angular coordinates $y, z \in [0, \pi]$ with R_y and r_z being the corresponding moduli. Just as in the RS case, non-trivial warp factors $a(y)$ and $b(z)$, when accompanied by the orbifolding necessitates the presence of localized energy densities at the orbifold fixed points, and in the present case, these appear in the form of tensions associated with the four end-of-the-world 4-branes.

The total bulk-brane action for the six dimensional space time is, thus,

$$\begin{aligned}
\mathcal{S} &= \mathcal{S}_6 + \mathcal{S}_5 \\
\mathcal{S}_6 &= \int d^4x\, dy\, dz \sqrt{-g_6}\,(M_6^4 R_6 - \Lambda) \\
\mathcal{S}_5 &= \int d^4x\, dy\, dz \sqrt{-g_5}\,[V_1(z)\,\delta(y) + V_2(z)\,\delta(y-\pi)] \\
&\quad + \int d^4x\, dy, dz \sqrt{-\tilde{g}_5}\,[V_3(y)\,\delta(z) + V_4(y)\,\delta(z-\pi)] \,,
\end{aligned} \tag{5}$$

where Λ is the (six dimensional) bulk cosmological constant and M_6 is the natural scale (quantum gravity scale) in six dimensions. The five-dimensional metrics in $\mathcal{S}_5$ are those induced on the appropriate 4-branes, which accord a rectangular box shape to the space. Furthermore, the SM (and other) fields may be localized on additional 3-branes located at the four corners of the box, viz.

$$\mathcal{S}_4 = \sum_{y_i, z_i = 0, \pi} \int d^4x\, dy\, dz \sqrt{-g_4}\, \mathcal{L}_i\, \delta(y - y_i)\, \delta(z - z_i) \,.$$

These terms, however, are not germane to the discussions of this paper, and we shall not discuss $\mathcal{S}_4$ any further.

For a negative bulk cosmological constant Λ, the solutions for the 6-dimensional Einstein field equations are given by [28]

$$\begin{aligned}
a(y) &= e^{-c|y|} & c &= \frac{R_y k}{r_z \cosh k\pi} \\
b(z) &= \frac{\cosh(kz)}{\cosh(k\pi)} & k &= r_z \sqrt{\frac{-\Lambda}{10 M_6^4}} \equiv r_z k' \,.
\end{aligned} \tag{6}$$

The Israel junction conditions specify the brane tensions. The smoothness of the warp factor at $z = 0$ implies $V_3(y)$ be vanishing, while the fixed point at $z = \pi$ necessitates a negative tension, viz.

$$V_3(y) = 0, \qquad V_4(y) = \frac{-8M^4 k}{r_z} \tanh(k\pi) \,. \tag{7}$$

With the warping in the y-direction being similar to that in the 5D RS model, the two 4-branes sitting at $y = 0$ and $y = \pi$ have equal and opposite energy densities. However, the z-warping dictates that, rather than being constants, these energy densities must be z-dependent, viz.

$$V_1(z) = -V_2(z) = 8M^2 \sqrt{\frac{-\Lambda}{10}}\, \mathrm{sech}(kz) \,. \tag{8}$$

Such a z-dependence can arise from a scalar field distribution confined on the brane. For a detailed discussion on this we refer our