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Decoupling of heavy quarks at four loops and effective Higgs-fermion coupling

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ABSTRACT

We compute the decoupling constant ζ_m relating light quark masses of effective n_l-flavour QCD to (n_l+1)-flavour QCD to four-loop order. Immediate applications are the evaluation of the $\overline{\text{MS}}$ charm quark mass with five active flavours and the bottom quark mass at the scale of the top quark or even at GUT scales. With the help of a low-energy theorem ζ_m can be used to obtain the effective coupling of a Higgs boson to light quarks with five-loop accuracy. We briefly discuss the influence on $\Gamma(H \to b\bar{b})$.

1. Introduction and notation

Perturbative calculations in QCD are quite advanced and have reached, at least for some observables, the four and even five-loop level (see Refs. [1,2] for a recent review). This concerns in particular the renormalization group functions which have been computed at four loops in Refs. [3–7]. The first five-loop result has been obtained recently in Ref. [8] where the quark mass anomalous dimension has been computed to this order.

In order to consistently relate the quark masses and strong coupling constant evaluated at different energy scales, both the renormalization group functions and also the decoupling relations have to be available. The latter take care of integrating out heavy quark fields. In fact, N-loop running goes along with $(N-1)$-loop decoupling. Thus, besides the five-loop anomalous dimensions also the four-loop decoupling relations are needed. In Refs. [9,10] a first step has been undertaken in this direction and the four-loop decoupling constant for α_s has been computed (although the five-loop beta function is not yet available). In this paper we complement the result by computing the four-loop corrections to the decoupling constant for the light quark masses, which supplements the five-loop result for γ_m [8].

In Ref. [11] a formalism has been derived which allows for an effective calculation of the N-loop decoupling constants with the help of N-loop vacuum integrals. In the following we present the formulae which are relevant for the calculation of the quark mass decoupling constant.

The bare decoupling constant ζ_m^0 is defined via the relation

$$m_q^{0\prime} = \zeta_m^0 m_q^0 \,, \tag{1}$$

where m_q^0 and $m_q^{0\prime}$ are the bare quark mass parameters in the full n_f- and effective n_l $(\equiv n_f - 1)$-flavour theory. Introducing the renormalization constants in both theories leads to the equation

$$m_q^\prime(\mu) = \frac{Z_m}{Z_m^\prime}\zeta_m^0 m_q(\mu) = \zeta_m m_q(\mu), \tag{2}$$

which relates finite quantities and defines ζ_m. Note that primed quantities depend on $\alpha_s^{(n_l)}$ and non-primed quantities on $\alpha_s^{(n_f)}$. Four-loop results for Z_m and $Z_m^\prime$ can be found in Refs. [3,4,7] and ζ_m^0 can be computed with the help of

$$\zeta_m^0 = \frac{1-\Sigma_S^{0h}(0)}{1+\Sigma_V^{0h}(0)} \,, \tag{3}$$

where $\Sigma_S^{0h}(0)$ and $\Sigma_V^{0h}(0)$ are the scalar and vector parts of the light-quark self energy evaluated at zero external momentum. The superscript "h" reminds that one has to consider only the hard part which involves at least one propagator of the heavy quark.

In the next section we discuss the calculation of ζ_m^0 and its renormalization to arrive at ζ_m. Section 3 applies a low-energy theorem to derive, from the four-loop result of ζ_m, the effective Higgs-fermion coupling constant to five-loop order. We summarize our findings in Section 4.

2. Decoupling for light quark masses

In this section, we compute the decoupling constant ζ_m^0 and combine it with the four-loop result for Z_m to obtain the finite

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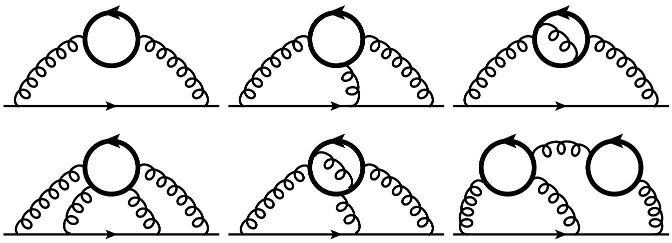

Fig. 1. Sample Feynman diagrams contributing to the hard part of the light-quark propagator up to four loops. Solid and curly lines denote quarks and gluons, respectively. At least one of the closed fermion loops needs to be the heavy quark.

quantity ζ_m. The computation of ζ_m^0 requires the knowledge of the hard contribution to the scalar and vector part of the light-quark propagator, see Fig. 1 for sample Feynman diagrams. The first non-vanishing contribution arises at two loops where one diagram contributes. At three-loop order there are 25 and at four loops we have 765 Feynman diagrams.

The perturbative expansion of Eq. (3) to four loops leads to

$$\zeta_m^0 = 1 - \Sigma_S^{0h}(0) - \Sigma_V^{0h}(0) + \Sigma_V^{0h}(0)\left[\Sigma_S^{0h}(0) + \Sigma_V^{0h}(0)\right] + \dots, \qquad (4)$$

where in the last term on the right-hand side only two-loop expressions for $\Sigma_S^{0h}(0)$ and $\Sigma_V^{0h}(0)$ have to be inserted.

We generate the Feynman diagrams with the help of QGRAF [12]. FORM [13,14] code is then generated by passing the output via q2e [15,16], which transforms Feynman diagrams into Feynman amplitudes, to exp [15,16]. After processing the latter one obtains the result as a linear combination of scalar functions which have a one-to-one relation to the underlying topology of the diagram. The functions contain the exponents of the involved propagators as arguments. At this point one has a large number of different functions. Thus, in the next step one passes them to a program which implements the Laporta algorithm [17] and performs a reduction to a small number of so-called master integrals. We use, for the latter step, the C++ program FIRE [18]. Our four-loop result is expressed in terms of 13 master integrals which we take from Ref. [19] (see also [20–22] and references therein). All ϵ coefficients are known analytically in the literature except the ϵ^3 term of integral $J_{6,2}$ (in the notation from Ref. [19]) which has been provided from [23].

Note that for our calculation we have used a general gauge parameter ξ of the gluon propagator. At four loops, in intermediate steps terms up to order ξ^6 are present, however, in the final result for ζ_m^0 all ξ terms drop out. The last term on the right-hand side of Eq. (4) is separately ξ-independent since at two loops $\Sigma_S^{0h}(0)$ and $\Sigma_V^{0h}(0)$ are individually ξ-independent. The results up to three-loop order have been checked with the help of MATAD [24] which avoids the use of the program FIRE since it implements the explicit solution of the recurrence relations.

To obtain ζ_m^0 we have to renormalize α_s and the heavy quark mass m_h to two-loop order. The corresponding $\overline{\rm MS}$ counterterms are well-known (see, e.g. Ref. [7]). ζ_m^0 still contains poles in ϵ which are removed by multiplying with the factor Z_m/Z_m' (see, Eq. (2)) which is needed to four-loop order [3,4,7]. Note that Z_m' depends on the strong coupling constant of the effective theory, $\alpha_s^{(n_l)}$, whereas Z_m and ζ_m^0 are expressed in terms of $\alpha_s^{(n_l+1)}$. In order to achieve the cancellation of the ϵ poles the same coupling constant has to be used in all three quantities. We have decided to replace $\alpha_s^{(n_l)}$ in favour of $\alpha_s^{(n_l+1)}$ which is done using the corresponding decoupling constant ζ_{α_s} up three-loop order [11]. Note, however, that higher order terms in ϵ are also needed since ζ_{α_s} gets multiplied by poles present in Z_m'. Up to two-loop order they can be found in Refs. [25,26]; the three-loop terms of order ϵ can be extracted from Refs. [9,10].

Our final result for the decoupling constant parametrized in terms of the $\overline{\rm MS}$ heavy quark mass, $m_h \equiv m_h(\mu)$, reads

$$\begin{aligned}
\zeta_m^{\overline{\rm MS}} = 1 &+ \left(\frac{\alpha_s^{(n_f)}}{\pi}\right)^2 \left(\frac{89}{432} - \frac{5}{36}\ln\frac{\mu^2}{m_h^2} + \frac{1}{12}\ln^2\frac{\mu^2}{m_h^2}\right) \\
&+ \left(\frac{\alpha_s^{(n_f)}}{\pi}\right)^3 \left[\frac{2951}{2916} + \frac{1}{9}\zeta(2)\ln^2 2\right. \\
&- \frac{1}{54}\ln^4 2 - \frac{407}{864}\zeta(3) + \frac{103}{72}\zeta(4) - \frac{4}{9}a_4 \\
&- \left(\frac{311}{2592} + \frac{5}{6}\zeta(3)\right)\ln\frac{\mu^2}{m_h^2} + \frac{175}{432}\ln^2\frac{\mu^2}{m_h^2} \\
&+ \frac{29}{216}\ln^3\frac{\mu^2}{m_h^2} + n_l\left(\frac{1327}{11664} - \frac{2}{27}\zeta(3) - \frac{53}{432}\ln\frac{\mu^2}{m_h^2}\right. \\
&\left.\left. - \frac{1}{108}\ln^3\frac{\mu^2}{m_h^2}\right)\right] \\
&+ \left(\frac{\alpha_s^{(n_f)}}{\pi}\right)^4 \left[\frac{131\,968\,227\,029}{3\,292\,047\,360} - \frac{1\,924\,649}{4\,354\,560}\ln^4 2\right. \\
&+ \frac{59}{1620}\ln^5 2 + \frac{1\,924\,649}{725\,760}\zeta(2)\ln^2 2 \\
&- \frac{59}{162}\zeta(2)\ln^3 2 - \frac{353\,193\,131}{40\,642\,560}\zeta(3) + \frac{1061}{576}\zeta(3)^2 \\
&+ \frac{16\,187\,201}{580\,608}\zeta(4) - \frac{725}{108}\zeta(4)\ln 2 \\
&- \frac{59\,015}{1728}\zeta(5) - \frac{3935}{432}\zeta(6) - \frac{1\,924\,649}{181\,440}a_4 \\
&- \frac{118}{27}a_5 + \left(-\frac{2\,810\,855}{373\,248} - \frac{31}{216}\ln^4 2\right. \\
&+ \frac{31}{36}\zeta(2)\ln^2 2 - \frac{373261}{27648}\zeta(3) \\
&\left. + \frac{4123}{288}\zeta(4) + \frac{575}{72}\zeta(5) - \frac{31}{9}a_4\right)\ln\frac{\mu^2}{m_h^2} \\
&+ \left(\frac{51\,163}{10\,368} - \frac{155}{48}\zeta(3)\right)\ln^2\frac{\mu^2}{m_h^2} \\
&+ \frac{301}{324}\ln^3\frac{\mu^2}{m_h^2} + \frac{305}{1152}\ln^4\frac{\mu^2}{m_h^2} \\
&+ n_l\left(-\frac{2\,261\,435}{746\,496} + \frac{49}{2592}\ln^4 2 - \frac{1}{270}\ln^5 2\right. \\
&- \frac{49}{432}\zeta(2)\ln^2 2 + \frac{1}{27}\zeta(2)\ln^3 2 \\
&- \frac{1075}{1728}\zeta(3) - \frac{1225}{3456}\zeta(4) + \frac{49}{72}\zeta(4)\ln 2 \\
&+ \frac{497}{288}\zeta(5) + \frac{49}{108}a_4 + \frac{4}{9}a_5 \\
&+ \left(\frac{16\,669}{31\,104} + \frac{1}{108}\ln^4 2 - \frac{1}{18}\zeta(2)\ln^2 2\right. \\
&\left. + \frac{221}{576}\zeta(3) - \frac{163}{144}\zeta(4) + \frac{2}{9}a_4\right)\ln\frac{\mu^2}{m_h^2}
\end{aligned}$$