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Nucleon–nucleon resonances at intermediate energies using a complex energy formalism

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ABSTRACT

We apply our method of complex scaling, valid for a general class of potentials, in a search for nucleon–nucleon S-matrix poles up to 2 GeV laboratory kinetic energy. We find that the realistic potentials JISP16, constructed from inverse scattering, and chiral field theory potentials N^3LO and N^2LO_{opt} support resonances in energy regions well above their fit regions. In some cases these resonances have widths that are small when compared with the real part of the S-matrix pole.

1. Introduction

There is a long-standing interest and considerable recent progress in the theoretical characterization of nuclear resonant states. A resonant state is fully characterized by its position in the energy plane and its width, which determines how fast the state will decay. One could in general solve the time-dependent Schrödinger equation to study the characteristics of resonant states [1–3], which is a demanding computational process. On the other hand, the time-independent many-body methods that deal with the description of resonant states in nuclei are under development and exhibit appealing computational features. These time-independent methods can be divided into real energy and complex energy approaches.

The spectrum of a real nuclear Hamiltonian consists of negative and positive energy states. While the negative energy spectrum is discrete (bound states), the positive energy spectrum may have a richer structure with resonant states among scattering or continuum states. Hence real-energy approaches require criteria for identifying a resonant structure and for assigning a position and a width to them.

In the domain of $\mathcal{L}^2$ integrable basis expansion methods this is usually achieved through $\mathcal{L}^2$ stabilization methods [4,5] or methods that evaluate the real Continuum Level Density (CLD) [6]. The CLD usually produces an approximate Breit–Wigner distribution in the region of the resonant state whose parameters could be determined by a fit. The CLD method has been successfully applied to atomic systems [7], nuclear clusters [8] and in mean-field approaches for describing quasiparticle resonant states [9].

The name stabilization, arises from the fact that one does not need the knowledge of the asymptotic wavefunction in order to determine the resonant parameters. On the reaction side, based on R-matrix considerations [10–12], and assuming the single channel approximation, resonant parameters can be determined by the behavior of the phase-shift function of energy $\delta(E)$ around the resonant position; in particular the position is defined as the inflection point of $\delta(E)$ (maximum energy derivative of $\delta(E)$) and the width is defined as $\Gamma = \frac{2}{d\delta/dE}|_{E=E_r}$, where E_r is the inflection point. Such formulas where employed recently in microscopic R-matrix calculations [13] to extract widths from realistic nucleon–nucleus phase-shifts. Though the R-matrix parameterizations have been very successful, the formulas become less transparent in the multi-channel case and when they are applied for the description of broad resonances (see discussion in [14]). Furthermore, for broad resonances, R-matrix analysis become more dependent on channel radii and boundary conditions [15]. Finally, combining formulas and assumptions from different theories/models for the calculation of an observable increases the possibility of uncontrollable errors.

The complex energy formalism serves as a potentially fruitful alternative for the characterization of the resonant parameters. It has been shown that once the R-matrix, S-matrix and T-matrix are analytically continued to the complex energy plane, the extraction of resonant parameters becomes independent of boundaries and radii [16]. Apart from this practical issue, some physical phenomena may have a more natural interpretation once the theory is developed in the complex energy plane (e.g. thermo-nuclear

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reactions [17,18]). In the complex energy formalism, the Gamow (resonant) states, i.e. the solutions of the Schrödinger equation which satisfy purely outgoing boundary conditions (complex wave number k), play a dominant role. It was shown by Berggren [19] that resonant states, when accompanied by non-resonant continuum states, form a complete set, an important property that gives rise to Berggren basis expansion methods either in a Configuration Interaction (CI) framework [20–25], Coupled Cluster framework [26–28] or reaction theory framework [29–35]. Expressing the Hamiltonian in such a complex energy, orthonormal non $\mathcal{L}^2$ integrable basis, automatically allows its spectrum to support resonant and also non-resonant continuum states. In addition, when the Berggren basis is used in a reaction framework the detailed knowledge of the boundary condition at large distances is not crucial.

The Complex Scaling (CS) method also belongs in the category of complex energy formalisms. The Aguilar–Balslev–Combes (ABC) theorem [36,37] establishes that once the Hamiltonian coordinates are rotated, the resonant states are independent of the rotation and behave asymptotically as bound states. Consequently, one could use the technology that has been established for bound states in order to describe resonant and scattering phenomena. Furthermore, the CS method has been successfully applied in nuclear physics [38–44] (see also [45] for an application of CS in a deformed nuclear mean-field). We recently showed [46] that this method may be applied to the most general cases of non-local nuclear potentials.

In this work we apply the CS method to nucleon–nucleon (NN) scattering spanning the range from threshold to 2 GeV laboratory kinetic energy, which exceeds the fitting range of most NN potentials. We elect to retain non-relativistic kinematics throughout as the interactions are derived for a non-relativistic scattering framework. We employ three different realistic NN interactions and we find resonant poles at laboratory kinetic energy of about 600 MeV, or at about 2.2 GeV in the total center of mass energy. Some of these poles have imaginary parts that are much smaller than their real parts.

According to the SAID data analysis group [47–49] (see also [50]) there exist resonance-like structures, poles of the S-matrix, in the $^1D_2$, $^3F_3$ uncoupled and coupled $^3P_2$–$^3F_2$ channels. Recently a resonant-like structure was also found by the WASA-at-COSY collaboration and the SAID analysis group in the $^3D_3$–$^3G_3$ coupled channel [51,52]. Our CS calculations, in addition to showing resonances in these channels, also reveal resonance-like structures in the $^3P_0$ and $^3P_1$ channels. We searched other channels up to and including $L = 4$ without any additional signals of resonance-like structures.

The study of dibaryon resonances could shed light on the reaction mechanism and aid in the interpretation of excited nucleonic states. It is also valuable to pin down the properties of dibaryon resonances as a potential link between Quantum Chromodynamics, hadron models and traditional low energy nuclear physics. In the work of [47–49] the resonant-like structures where identified by analytically continuing the T-matrix of the available data in the complex energy plane. Our goal is to simply identify resonant structures with the CS method but not to study in depth the characteristics of the NN scattering at intermediate energies, something that would require the use of NN interactions that fit scattering data at higher energies, such as CDBonn [53] or AV18 [54]. Such in-depth studies would be done relativistically [55] and by treating properly Δ and/or Roper resonances degrees of freedom (see for example [56]). Furthermore, we will not provide information on the possible decay paths that the resonant structures will follow, since we do not consider couplings to inelastic channels such as, $NN \to \pi d$ or $NN \to \Delta N$ etc. In addition, the interactions we use are modeling the short-range (high-energies) NN sector in different approaches and are fitted at lower laboratory energies (≤ 350 MeV). Hence, we are not aiming at making predictions for the existence or absence of broad dibaryonic states. For the same reason, we also do not compare resonance parameters produced by the different NN interactions. Nevertheless, it is worthwhile to discover that the NN interactions we employ, support high energy resonant-like states above the Δ production threshold (1232 MeV). The consequences of these resonances for nuclear structure are not entirely clear. Simply stated, our goal at this point is to demonstrate that the CS method locates these resonances using three different realistic NN interactions in the conventional non-relativistic framework.

2. Method and results

We apply the CS transformation to our Hamiltonian which consists of the relative kinetic energy T and the realistic NN interaction V between the nucleons. The complex rotated Hamiltonian has the form:

$$H(r,\theta) = e^{-2i\theta}T + V(re^{i\theta}), \tag{1}$$

where θ is the real CS rotation parameter and nuclear potential matrix elements are calculated according to [46]. The time-independent non-relativistic Schrödinger equation then becomes:

$$H(r,\theta)\Psi(r,\theta) = E(\theta)\Psi(r,\theta), \tag{2}$$

where E is the energy in the Center of Mass (CoM) frame here and throughout this work. To be more precise, the rotated non-Hermitian Hamiltonian operator and the Hermitian one, are related through the formula:

$$H(r,\theta) = U(\theta)H(r)U(\theta)^{-1}, \tag{3}$$

where $U(\theta)$ stands for the non-Unitary CS transformation operator.

In order to solve Eq. (2) we assume that the solution is a linear combination of orthonormal Harmonic Oscillator (HO) basis states and we solve a complex symmetric Hamiltonian eigenvalue problem by diagonalization. The spectrum of the Hamiltonian contains resonant (bound states, resonances) and non-resonant continuum states. According to the ABC theorem, once the resonant state is revealed it remains invariant under CS rotations, whereas the non-resonant continuum states follow an approximate 2θ path in the complex energy plane. This is the complex stabilization criterion that is used in CS for the identification of the resonant state. In practice, due to the truncation of the underlying basis, there is a small variation of the resonant position with θ. It is then a consequence of the complex virial theorem [57] that the resonant state will be the one that corresponds to the minimal change of the real part of the energy with respect to θ. The method is also known as θ-trajectory method and it is a common practice in CS applications (see for example [39,44,58]). We also apply this stabilization technique and we check convergence of our results as a function of the basis dimension and the variations with the rotation angle.

In Fig. 1 we present the spectrum of the complex scaled Hamiltonian for the $^3P_1$ (1^-) channel of the neutron-proton (np) system, for the JISP16 [59] and the two chiral effective field theory interactions N^{3}LO [60] and N^{2}LO$_{opt}$ [61]. The HO basis was characterized by $\hbar\omega = 40$ MeV for JISP16 and N^{2}LO$_{opt}$ and by $\hbar\omega = 28$ MeV for N^{3}LO. For the N^{3}LO potential we varied the rotation from $\theta = 0.1$ rad to 0.2 rad, for N^{2}LO$_{opt}$ from $\theta = 0.14$ to 0.24 rad and for JISP16 from $\theta = 0.2$ to 0.3 rad. The step in the θ discretization was 0.004 rad. As we start rotating the coordinates and momenta of the Hamiltonian, solutions that initially inhabit the