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QED as the tensionless limit of the spinning string with contact interaction

James P. Edwards, Paul Mansfield *

Centre for Particle Theory, Department of Mathematical Sciences, Durham University, Durham DH1 3LE, UK

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ABSTRACT

QED with spinor matter is argued to correspond to the tensionless limit of spinning strings with contact interactions. The strings represent electric lines of force with charges at their ends. The interaction is constructed from a delta-function on the world-sheet which, although off-shell, decouples from the world-sheet metric. Integrating out the string degrees of freedom with fixed boundary generates the super-Wilson loop that couples spinor matter to electromagnetism in the world-line formalism. World-sheet and world-line, but not spacetime, supersymmetry underpin the model.

1. Introduction

Quantum Electrodynamics is perhaps the most successful physical theory to confront experiment, and so it might seem redundant to consider an alternative formulation. However, as an Abelian gauge theory it is a simpler version of the non-Abelian gauge theory of the Standard Model to which new approaches may still be of interest. In this letter we treat QED by taking the electric lines of force as the basic degrees of freedom of the electromagnetic field. This immediately requires the technology of string theory but applied to a non-standard setting in which the ends of the lines of force are electrically charged particles and the electromagnetic interaction becomes a contact interaction described by δ-functions on the world-sheet. We will show that even though these interactions are off-shell they can be constructed to be independent of the scale of the world-sheet metric because of the non-standard boundary conditions. Unwanted divergences that might occur when there is more than one interaction on each world-sheet are eliminated when the model has world-sheet supersymmetry and this allows the interaction to be exponentiated thus generating the super-Wilson loops that couple spinor matter to the electromagnetic field on the world-sheet boundaries. Integrating over the boundaries after having included supersymmetric boundary terms in the action quantises the spinor matter in the world-line formalism. We will impose the tensionless limit, so that the strings representing the lines of force are potentially large, although working with a non-zero, but small, tension would result in a model where more conventional string-like behaviour would set in at large length scales.

Conventionally, the first step in the passage to the quantum theory from the classical Maxwell equations

$$\epsilon^{\mu\nu\lambda\rho}\,\partial_\nu F_{\lambda\rho}=0, \quad \partial^\mu F_{\mu\nu}=J_\nu\,, \tag{1}$$

is to solve the first set by introducing a gauge potential, A, and then construct a Lagrangian with this as the dynamical variable (modulo gauge transformations) so that the second set appear as Euler–Lagrange equations. We choose the alternative starting point by solving the second set. For simplicity we consider a system consisting of particle anti-particle pairs created and then mutually annihilating, so the current density is

$$J^\mu(x)=\sum q\int_B \delta^4(x-w)\,dw^\mu \tag{2}$$

where the world-lines B are closed. *One* solution is to take

$$F_{\mu\nu}(x)=\sum -q\int_\Sigma \delta^4(x-X)\,d\Sigma_{\mu\nu}(X)\,, \tag{3}$$

where $d\Sigma_{\mu\nu}$ is an element of area on a surface Σ spanning B. This field-strength, which vanishes away from Σ, may be interpreted as that of a single line of force. We will take this surface Σ as the dynamical degree of freedom instead of the gauge potential. Treating this as the basic physical object is reminiscent of Faraday's

* Corresponding author.
E-mail addresses: J.P.Edwards@durham.ac.uk (J.P. Edwards), P.R.W.Mansfield@durham.ac.uk (P. Mansfield).

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approach to electromagnetism [1] in which lines of force are the fundamental degrees of freedom. This was echoed in Dirac's 1955 proposal [2] that creation operators for electric charges should simultaneously create this part of the electromagnetic field so that the radially symmetric Coulomb field for a single charge would emerge from quantum mechanical averaging of (3). An equivalent expression was used to describe the polarisation vector of charged matter for molecular electrodynamics [3] and in the context of non-linear electrodynamics by Nielsen and Olesen [4] to form a field theory describing the dual string. Its dual is also present in theories of electromagnetism with magnetic monopoles [5] and has been used [6,7] to derive an effective string theory describing the evolution of the Dirac string linking two such poles.

Substituting into the classical electromagnetic action $\int d^4x\, F_{\mu\nu}F^{\mu\nu}/4$ gives the formal expression

$$\frac{q^2}{4}\delta^2(0)\,\text{Area} + \frac{q^2}{4}\int_\Sigma d\Sigma^{\mu\nu}(\xi)\,\delta^4\left(X(\xi)-X(\xi')\right)d\Sigma_{\mu\nu}(\xi')\Big|_{\xi\neq\xi'}$$

the first term [8] is the Nambu–Goto action, albeit with a divergent coefficient, whilst the second is a self-intersection interaction. Clearly, to proceed further requires the machinery of string theory but with non-standard contact interactions rather than conventional splitting and joining. Similar interactions have previously been discussed by Kalb and Ramond [9] and our proposal satisfies the consistency constraints they derive. This action has been applied classically [10] to the problem of confinement but without self-intersections or quantisation.

In [11] it was shown that the average of (3) over Σ constructed according to Polyakov's approach to the bosonic string [12] does in fact yield the electromagnetic field generated by J^μ, Wick rotated to Euclidean signature where the functional integrals behave better:

$$4\pi^2\langle\int_\Sigma \delta^4(x-X)\,d\Sigma_{\mu\nu}(X)\rangle$$

$$=\partial_\mu\int_B\frac{dw_\nu}{||x-w||^2}-\partial_\nu\int_B\frac{dw_\mu}{||x-w||^2}$$

where the average over Σ of any functional $\Omega[\Sigma]$ is

$$\langle\Omega\rangle=\frac{1}{Z}\int\mathscr{D}(g,X)\,\Omega\,\exp\left(-\frac{1}{4\pi\alpha'}\int_D g^{ab}\frac{\partial X^\mu}{\partial\xi^a}\frac{\partial X^\mu}{\partial\xi^b}\sqrt{g}\,d^2\xi\right).$$

Remarkably this result is independent of the scale of the world-sheet metric despite the δ-function being off-shell and is also independent of the string tension, α'. Integrating over a different surface Σ' spanning the fixed closed loop B' gives

$$\langle\int_{\Sigma\Sigma'}d\Sigma'^{\mu\nu}\delta^4(x-X)\,d\Sigma^{\mu\nu}(X)\rangle=\frac{1}{2\pi^2}\int_{BB'}\frac{dw'\cdot dw}{||w'-w||^2} \tag{4}$$

(since the right-hand-side is independent of this second surface we could obtain a more symmetrical looking result by also averaging over Σ'). The right-hand-side is the electromagnetic interaction between the two loops of charges B and B'. If it were possible to show that this exponentiates then we would be able to express the expectation value of Wilson loops in Maxwell theory, i.e.

$$\int\frac{\mathscr{D}A}{N}e^{-S_{gf}}\prod_j e^{-iq\oint_{B_j}dw\cdot A} \tag{5}$$

(where S_{gf} is the usual gauge-fixed action for the electromagnetic field) as the partition function of first quantised strings with fixed boundaries and which interact on contact:

$$\int\left(\prod_j\frac{\mathscr{D}(X_j,g_j)}{Z_0}\right)e^{-S}, \tag{6}$$

where

$$S=\sum_j S[X_j,g_j]+\sum_{jk}q^2\int_{\Sigma_j\Sigma_k}d\Sigma_j^{\mu\nu}\,\delta^4(X_j-X_k)\,d\Sigma^k_{\mu\nu} \tag{7}$$

effectively replacing the quantised electromagnetic field by quantised strings with fixed boundaries. Integrating over the boundaries with appropriate weights quantises the charged sources along the lines of Strassler's world-line approach [13] (see also [15] and [16] for recent applications) so we would arrive at a reformulation of QED in terms of strings with unusual boundary terms and contact interactions. This programme is pursued in detail in [14] where it is shown that with bosonic matter the programme is difficult to implement, but that for spinor matter the additional structure resulting from a spinning world-sheet renders the approach tractable, and it is this, actually more realistic, case that we describe in this letter.

Evaluating the conventional QED functional integral by first integrating over spinor matter results in the fermionic determinant depending on the gauge field A_μ. Strassler represents this determinant by a world-line functional integral. We will use a reparametrisation invariant formulation based on the action of Brink, di Vecchia and Howe [17] (for further details see [14])

$$\ln\text{Det}\left(-(\gamma\cdot(\partial+iA))^2+m^2\right)$$

$$\propto\int\mathscr{D}(h,w,\chi,\psi)\,W[A]\,e^{-S_{BdVH}} \tag{8}$$

where

$$S_{BdVH}=\frac{1}{2}\oint\left(\frac{1}{\sqrt{h}}\left(\frac{dw}{dx}\right)^2-i\psi\cdot\frac{d\psi}{dx}-i\frac{\chi}{\sqrt{h}}\frac{dw}{dx}\cdot\psi\right)dx \tag{9}$$

(for simplicity we drop the mass term) and W is the supersymmetric Wilson loop

$$W[A]=\exp\left(-q\oint\left(i\frac{dw}{dx}\cdot A-\frac{1}{2}F_{\mu\nu}\psi^\mu\psi^\nu\sqrt{h}\right)dx\right). \tag{10}$$

Here χ is the fermionic partner of h which is an intrinsic metric on the world-line parametrised by ξ, and ψ^μ are fermionic partners of the co-ordinates, w^μ in d-dimensional space–time. w, $h^{1/4}$ and χ have dimensions of length but ψ is dimensionless, so S_{BdVH} is dimensionless as well. As is well-known, the action S_{BdVH} and the exponent of W have the worldline supersymmetry

$$\delta w=i\alpha\psi\,,\quad \delta\psi=\frac{\alpha}{\sqrt{h}}\left(\frac{dw}{dx}-\frac{i}{2}\chi\psi\right),\quad \delta\sqrt{h}=i\alpha\chi\,,$$

$$\delta\chi=2\frac{d\alpha}{dx}\,, \tag{11}$$

despite the absence of supersymmetry in the spacetime theory of QED. Curiously the fermionic Green function may also be expressed in the same form of the right-hand-side of (8) but using open worldlines [14] with appropriate conditions at their ends. In (8) the gauge-field, A, appears only in W so to complete the quantisation of QED it just remains to functionally integrate over A using the super-Wilson loop equivalent of (5). It is our purpose to show that this last step can be replaced by a functional integral over spinning strings spanning the closed loops B, where our string theory contains the unusual features of the boundary action (9), contact interactions, and a tensionless limit.