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Anomaly-induced effective action and Chern–Simons modification of general relativity

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ABSTRACT

Recently it was shown that the quantum vacuum effects of massless chiral fermion field in curved space–time leads to the parity-violating Pontryagin density term, which appears in the trace anomaly with imaginary coefficient. In the present work the anomaly-induced effective action with the parity-violating term is derived. The result is similar to the Chern–Simons modified general relativity, which was extensively studied in the last decade, but with the kinetic terms for the scalar different from those considered previously in the literature. The parity-breaking term makes no effect on the zero-order cosmology, but it is expected to be relevant in the black hole solutions and in the cosmological perturbations, especially gravitational waves.

1. Introduction

The derivation and properties of conformal (trace) anomaly are pretty well known (see, e.g., [1] and also [2,3] for the technical introduction related to the present work). At the one-loop level the anomaly is given by an algebraic sum of the contributions of massless conformal invariant fields of spins 0, 1/2, 1 in a curved space–time of an arbitrary background metric. Recently, it was confirmed that the quantum effects of chiral (L) fermion produce an imaginary contribution which violates parity [4]. As a result, the anomalous trace has the form

$$\langle T^\mu_\mu \rangle = -\beta_1 C^2 - \beta_2 E_4 - a' \Box R - \tilde{\beta} F^2_{\mu\nu} - \beta_4 P_4 \,. \tag{1}$$

Here we have included the external electromagnetic field $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ for generality, also

$$C^2 = C_{\mu\nu\alpha\beta}\, C^{\mu\nu\alpha\beta} = R^2_{\mu\nu\alpha\beta} - 2R^2_{\alpha\beta} + \frac{1}{3}R^2 \tag{2}$$

is the square of the Weyl tensor in four-dimensional space–time and

$$E_4 = \frac{1}{4}\varepsilon^{\mu\nu\alpha\beta}\,\varepsilon^{\rho\sigma\lambda\tau}\, R_{\mu\nu\rho\sigma}\, R_{\alpha\beta\lambda\tau} = R^2_{\mu\nu\alpha\beta} - 4R^2_{\alpha\beta} + R^2 \tag{3}$$

is the integrand of the Gauss–Bonnet topological term.

The β-functions are given by algebraic sums of the contributions of N_s scalars, N_f Dirac fermions and N_v massless vector fields. The explicit form is well known,

$$(4\pi)^2\,\beta_1 = \frac{1}{120}N_s + \frac{1}{20}N_f + \frac{1}{10}N_v \,,$$
$$(4\pi)^2\,\beta_2 = -\frac{1}{360}N_s - \frac{11}{360}N_f - \frac{31}{180}N_v \,,$$
$$(4\pi)^2\,\beta_3 = \frac{1}{180}N_s + \frac{1}{30}N_f - \frac{1}{10}N_v \,. \tag{4}$$

One can assume that a' in (1) is equal to β_3, but there is ambiguity, as will be discussed below. $\tilde{\beta}$ is the usual β-function of QED or scalar QED etc, depending on the model.

Furthermore, there is a parity-violating Pontryagin density term $\beta_4 P_4$, where

$$P_4 = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\, R_{\mu\nu\rho\sigma}\, R_{\alpha\beta}{}^{\rho\sigma} \,. \tag{5}$$

By dimensional reasons the term with P_4 is possible, but for a long time it was believed that this term, in fact, does not show up. However, in a recent paper [4] this term was actually found with a purely imaginary coefficient $\beta_4 = i/(48 \cdot 16\pi^2)$, as a contribution of chiral (left) fermions. The chirality is important here, because the contribution of the right-hand fermions is going to cancel the one of the left-hand fermions, so taking them in a pair would kill

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the effect. Let us also note that much earlier, in [5], the possibility of such a term coming from integrating out antisymmetric tensor field has been considered, also some general considerations were presented even earlier in [6] and more recently in [7].

Some questions arise due to the result of [4] and its physical interpretation. First, does the parity-violating term in the anomaly mean that the dynamics of gravity is affected in a significant way? Second, in case of a positive answer to the last question, does it mean that the chiral fermions are disfavoured theoretically, since they produce imaginary component in the gravitational field equations? The last possibility was discussed in [4] as a theoretical argument in favor of massive neutrino. The third question is whether the parity-odd terms in the anomaly have some relation to the Chern–Simons modification of 4d-gravity suggested in [8,9]. The theories of this sort were extensively investigated in the last decade, as one can see from the review [10] and other works on the subject. This question looks really natural, because the Chern–Simons-gravity is based on the action which includes the P_4-term with an extra scalar factor inside the integral. Let us note that the relation between parity-odd terms and anomalies in $D=4$ was discussed, i.e., in [11] in relation to gravitational anomalies, so the novelty of the term (5) concerns only the trace anomaly.

The purpose of the present work is to address the questions formulated above. In order to do so, we derive the effective action of gravity by integrating conformal anomaly, and show that the result is a new version of the Chern–Simons 4d-gravity with a special form of the kinetic term for the scalar and some extra higher-derivative terms which are typical for this action. From the technical side most of the consideration is pretty well known, but we present full details in order to make it readable for those who are not familiar with the subject. The paper is organized as follows. In Section 2 we review the well-known scheme of deriving anomaly-induced effective action, with an extra parity-odd term corresponding to Pontryagin density. The anomaly-induced action provides a specific form of the kinetic term for the auxiliary scalar in Chern–Simons modified gravity. For this reason, in the last subsection we present a short review of the previous version of kinetic terms, which are known in the literature. Section 3 includes a general, mainly qualitative, discussion of the physical interpretation of the new parity-violating term. Finally, in Section 4 we draw our conclusions and suggest possible perspectives of a further work on the subject.

2. Integration of anomaly with parity-violating term

The integration of conformal anomaly (1) in $d=4$ means solving the equation similar to the one for the Polyakov action in $d=2$,

$$\frac{2}{\sqrt{-g}}\,g_{\mu\nu}\,\frac{\delta\,\bar{\Gamma}_{ind}}{\delta g_{\mu\nu}} = -\,\langle T^{\mu}_{\mu}\rangle$$
$$= \frac{1}{(4\pi)^2}\left(\omega C^2 + bE_4 + c\Box R + \tilde{b}F^2_{\mu\nu} + \epsilon P_4\right). \qquad (6)$$

Here we introduced useful notations $(\omega,\, b,\, c,\, \tilde{b},\, \epsilon) = (4\pi)^2\left(\beta_1,\, \beta_2,\, a',\, \tilde{\beta},\, \beta_4\right)$. The coefficient ϵ derived in [4] is imaginary, but we will not pay attention to this until the solution is found. The first reason for this is that this is technically irrelevant, and also it is, in principle, possible to have a real coefficient of the same sort at the non-perturbative level.

2.1. Conformal properties of Pontryagin term and anomaly

The solution of Eq. (6) is technically is not very complicated [12] in the usual theory without Pontryagin term, and it remains equally simple when this term is present. In order to understand this, let us make an observation that this term is conformal invariant in $d=4$, simply because one can recast (5) in the form when the Weyl tensor replaces the Riemann tensor,

$$P_4 = \frac{1}{2}\,\varepsilon^{\mu\nu\alpha\beta}\,C_{\mu\nu\rho\sigma}\,C_{\alpha\beta}{}^{\rho\sigma}\,. \qquad (7)$$

The proof of this statement is well known (see [13] for further developments), but for the convenience of the reader we present a proof in Appendix A. One can easily see that the *r.h.s.* of Eq. (6) consists of the three different terms, which can be classified according to [14]. One can distinguish *(i)* conformally invariant part $\omega C^2 + \tilde{\beta}F^2_{\mu\nu} + \beta_4 P_4$; *(ii)* the topological term bE_4 and *(iii)* surface term $c\Box R$.

In fact, the last division is not unambiguous. For example, in $d=4$ both P_4 and bE_4 can be presented as total derivatives, and the term P_4 is not only topological, but also conformal, according to Eq. (7). Hence, the Gauss–Bonnet invariant can be attributed to two groups of terms and the Pontryagin density even to all three groups *(i)*, *(ii)* and *(iii)*. In any case, as the reader will see shortly, the conformal invariance of P_4 makes the inclusion of this term into anomaly-induced action a very simple exercise. We shall present some details only to achieve a self-consistent exposition of the consideration.

The simplest part is the $\Box R$-term, which can be directly integrated by using the relation

$$-\,\frac{2}{\sqrt{-g}}\,g_{\mu\nu}\,\frac{\delta}{\delta g_{\mu\nu}}\int d^4x\sqrt{-g}\,R^2 \;=\; 12\Box R\,. \qquad (8)$$

It is easy to see that in this case the solution is a local functional, that gives rise to the well-known ambiguity in the coefficient a' of the $\Box R$-term, which was discussed in details in [15].

Now, let us concentrate on the non-local part of anomaly-induced action.[1] The solution of (6) can be presented in the simplest, non-covariant form, in the covariant non-local form and in the local covariant form with two auxiliary fields. Let us start from the simplest case. By introducing the conformal parametrization of the metric

$$g_{\mu\nu} = \bar{g}_{\mu\nu}\,e^{2\sigma(x)} \qquad (9)$$

one can use an identity

$$-\,\frac{2}{\sqrt{-g}}\,g_{\mu\nu}\,\frac{\delta\,A[g_{\mu\nu}]}{\delta\,g_{\mu\nu}} = -\,\frac{1}{\sqrt{-\bar{g}}}\;\frac{\delta\,A[\bar{g}_{\mu\nu}\,e^{2\sigma}]}{\delta\sigma}\Bigg|_{\bar{g}_{\mu\nu}\to g_{\mu\nu},\,\sigma\to 0}\,. \qquad (10)$$

Here and below the quantities with bars are constructed using the metric $\bar{g}_{\mu\nu}$, in particular

$$\bar{F}^2_{\mu\nu} = F_{\mu\nu}\,F_{\alpha\beta}\,\bar{g}^{\mu\alpha}\,\bar{g}^{\beta\nu}\,, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad (11)$$

Furthermore, we will need the conformal transformation rules

$$\sqrt{-g}\,W_k = \sqrt{-\bar{g}}\bar{W}^2_k\,, \qquad \text{where} \qquad \left(W_k = C^2,\; P_4,\; F^2\right), \qquad (12)$$

and

$$\sqrt{-g}(E - \frac{2}{3}\Box R) = \sqrt{-\bar{g}}(\bar{E} - \frac{2}{3}\bar{\Box}\bar{R} + 4\bar{\Delta}_4\sigma)\,,$$
$$\sqrt{-\bar{g}}\,\bar{\Delta}_4 = \sqrt{-g}\,\Delta_4\,, \qquad (13)$$

where

$$\Delta_4 = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}\,R\Box + \frac{1}{3}\,R_{;\mu}\,\nabla^\mu \qquad (14)$$

[1] The non-localities due to anomaly was first discussed in [16].