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On the electroweak contribution to the matching of the strong coupling constant in the SM

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ABSTRACT

The effective renormalizable theory describing electromagnetic and strong interactions of quarks of five light flavors ($n_f = 5$ QCD $\times$ QED) is considered as a low-energy limit of the full Standard Model. Two-loop relation between the running strong coupling constants α_s defined in either theories is found by simultaneous decoupling of electroweak gauge and Higgs bosons in addition to the top-quark. The relation potentially allows one to confront "low-energy" determination of α_s with a high-energy one with increased accuracy. Numerical impact of new $\mathcal{O}(\alpha_s\alpha)$ terms is studied at the M_Z scale. It is shown that the corresponding contribution, although being suppressed with respect to $\mathcal{O}(\alpha_s^2)$ terms, is an order of magnitude larger than the three-loop QCD corrections $\mathcal{O}(\alpha_s^3)$ usually taken into account in four-loop renormalization group evolution of α_s. The dependence on the matching scale is also analyzed numerically.

The strong coupling constant α_s being a fundamental parameter of the Standard Model (SM) Lagrangian is not predicted by the model and should be determined from experiment. The theory of strong interactions, Quantum Chromodynamics (QCD), embedded in the SM, should allow one to relate different observables measured at different scales. In perturbation theory (PT) one usually employs the notion of running coupling $\alpha_s(Q^2)$ (see Ref. [1] for a recent discussion on its determination) depending on some characteristic scale Q of the considered process, so that predictions are typically given by (truncated) series in $\alpha_s(Q^2)$. The dependence of α_s on Q^2 is given through the renormalization group (RG) equation

$$\frac{d\alpha_s(Q^2)}{d\ln Q^2} = \beta\big(\alpha_s(Q^2)\big). \quad (1)$$

A proper choice of Q^2 allows one to sum up a certain type of logarithmic corrections appearing at each order of PT. In the minimal $\overline{\text{MS}}$ renormalization scheme [2] beta-functions have a simple polynomial form and are known up to the four-loop level [3,4]. However, it is a well-known fact (see, e.g., the pioneering work [5] and reviews in Refs. [6,7]) that in the models with very different mass scales $m \ll M$ one needs to employ the "running-and-decoupling" procedure to re-sum large logarithms involving ratios of particle masses $\log m/M$ in the "low-energy" observables.[1] Application of this technique to perturbative calculations results in the absorption of leading effects due to heavy degrees of freedom with mass $\mathcal{O}(M)$ in the parameters of the effective theory (ET). This procedure is usually applied in QCD [8–12] to decouple ("integrate out") heavy quarks and define running $\alpha_s^{(n_f)}(\mu)$ in the effective n_f-flavor theory.[2] In addition to this, a study of matching corrections [14–16] is unavoidable if one is interested in high-energy behavior of the SM (see, e.g., [16,17]). The latter is analyzed with the help of the RGEs [18–25] which take into account all the interactions of the SM.

It is also worth mentioning that the same method can be applied to the supersymmetric (SUSY) extensions of the SM: SUSY–QCD corrections are considered in Refs. [26–29] and leading corrections due to electroweak interactions are calculated in Refs. [30,31].

The value of the running strong coupling $\alpha_s(Q)$ can be determined (see Ref. [32] for a comprehensive review and the references therein) from a bunch of experiments with a characteristic scale Q ranging from the tau-lepton mass $m_\tau = 1.77682(16)$ GeV up to about 1 TeV. In addition, electroweak precision fits and lattice QCD calculations can be used to yield a value for α_s. In order

[1] Related to processes with characteristic scale $Q \lesssim m$.

[2] Quark running masses are also affected by decoupling (see Ref. [13] and the references therein).

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to compare different measurements and determinations with each other and use them in a global analysis of QCD, effective couplings extracted from experiments are converted to the $\overline{\text{MS}}$-scheme and evolved by means of RGEs (1) to the reference scale which is chosen to be M_Z.

In the above-mentioned RGE analysis one usually neglects the influence of electroweak interactions on the strong coupling. However, it is obvious that since QCD is embedded in the SM, virtual electroweak bosons can also modify the strength of the strong interactions (and vice versa). This effect, however, starts at the two-loop level. The aim of the present paper is to apply the decoupling procedure to find a two-loop relation between the strong coupling defined in the $n_f = 5$ QCD $\times$ QED effective theory and that of the full SM. Additional terms due to integrated W-, Z-, and Higgs bosons can be used to decrease an uncertainty in the prediction of the SM strong coupling $\alpha_s(M_Z)$ from, e.g., $\alpha_s(m_\tau)$ extracted from tau-lepton decays, or, vice versa, a more accurate estimate of the effective α_s can be made given the SM input.

Let us give a brief description of the calculation techniques together with the approximations employed. It is convenient to carry out matching at the level of Green functions[3] of light fields by comparing the results obtained in the effective and a more fundamental theory. By integrating out heavy degrees of freedom (the top-quark with mass m_t, the W- and Z-bosons with masses m_W and m_Z, respectively, and the Higgs boson with mass m_h) we obtain an effective ("low-energy") description of the SM valid far below the electroweak scale. The latter is parametrized by an effective Lagrangian, which involves non-renormalizable operators in addition to the renormalizable ones. Both types of operators contribute to the Green functions of ET. However, the latter are suppressed by the inverse power of some large mass scale. The couplings for the ET operators can be deduced from the parameters of a more fundamental theory, the SM in our case, by means of matching (decoupling) procedure.

In theories with spontaneously broken symmetries the application of the decoupling procedure suffers from the following subtlety. Since particle masses are related to the corresponding Higgs couplings, a large mass limit can be obtained either by setting the Higgs field vacuum expectation value (VEV) v to infinity or by assuming that v is fixed but (some of) the couplings (e.g., the top-quark Yukawa coupling) tend to infinity (see, e.g., [33] and the references therein). In literature, one usually utilizes the latter option and speaks of "non-decoupling" feature of the top-quark since the corresponding Yukawa coupling is expressed in terms of its mass.

In this paper, we assume that the decoupling limit is obtained by setting $v \to \infty$, so that non-renormalizable Fermi-type operators are neglected. Nevertheless, a certain hierarchy in the Higgs couplings is assumed. Light quarks, which are not integrated out and are "left" in ET, are considered to be massless and, as a consequence, have vanishing Yukawa couplings to the Higgs boson. All other Higgs couplings are treated on equal footing.

An additional comment regarding the neglected interactions of the Higgs boson is in order. If we want to take, e.g., b-quark Yukawa coupling into account, we inevitably have to consider the matching of non-renormalizable operators in ET (e.g., Fermi-type operators). This is due to the fact that both the dimensionless Yukawa couplings in the full theory and the non-renormalizable ET interactions, which are formally suppressed by m_W^2, can lead to comparable contributions $\mathcal{O}(m_b^2/m_W^2)$ to the Green functions utilized for matching. Our setup allows us to circumvent this difficulty and avoid matching of non-renormalizable ones, which are suppressed by the ratio of "soft" (in our setup it is either some external momentum or the mass of a light quark) and "hard" (electroweak) scale.

In the considered problem electroweak interactions can only appear in loops involving quarks so that the electroweak bosons contribute starting with the two-loop level. Due to this, the relation between the strong coupling constants defined in the effective five-flavor QCD $\times$ QED (denoted by α_s') and the full SM (α_s) has the following form:

$$\alpha_s' = \alpha_s \zeta_{\alpha_s} = \alpha_s \left(1 + \frac{\alpha_s}{4\pi} \delta\zeta_{\alpha_s}^{(1)} + \frac{\alpha_s^2}{(4\pi)^2} \delta\zeta_{\alpha_s}^{(2)} + \frac{\alpha_s \alpha}{(4\pi)^2} \delta\zeta_{\alpha_s\alpha}^{(2)} + \ldots \right), \quad (2)$$

where the running couplings are assumed to be renormalized in the $\overline{\text{MS}}$-scheme and the dependence on the decoupling scale μ is implied. The strong coupling α_s and the fine-structure constant α in the right-hand side (RHS) of Eq. (2) are defined in the full SM. Pure QCD corrections to the decoupling constant ζ_{α_s} were calculated quite a long time ago [34] and are given at the two-loop level by the expressions ($C_A = 3$, $C_F = 4/3$, $T_F = 1/2$):

$$\delta\zeta_{\alpha_s}^{(1)} = X_{\alpha_s}^{(1)} \ln \frac{m_t^2}{\mu^2}, \qquad X_{\alpha_s}^{(1)} = \frac{4}{3} T_f = \frac{2}{3} \quad (3)$$

$$\delta\zeta_{\alpha_s}^{(2)} = X_{\alpha_s^2}^{(0)} + X_{\alpha_s^2}^{(1)} \ln \frac{m_t^2}{\mu^2} + X_{\alpha_s^2}^{(2)} \ln^2 \frac{m_t^2}{\mu^2} \quad (4)$$

$$X_{\alpha_s^2}^{(0)} = \left(\frac{32}{9} C_A - 15 C_F \right) T_f = -\frac{14}{3}$$

$$X_{\alpha_s^2}^{(2)} = \frac{16}{9} T_f^2 = \frac{4}{9}, \qquad X_{\alpha_s^2}^{(1)} = \left(\frac{20}{3} C_A + 4 C_F \right) T_f = \frac{38}{3} \quad (5)$$

in which m_t corresponds to the top-quark pole mass. The latter can be expressed in terms of the running mass $\hat{m}_t$ in perturbation theory. However, the advantage of m_t lies in the fact that it corresponds to an "observable" (modulo subtleties mentioned in Refs. [1,35]) quantity. In addition, this choice allows one to keep all the dependence of $\delta\zeta_{\alpha_s}^{(i)}$ on the matching scale μ explicit. Since we are interested in the electroweak corrections, the relation between $\hat{m}_t$ and m_t should be considered in the full SM. Let us mention a crucial role of tadpole diagrams rendering the corresponding running mass $\hat{m}_t$ a gauge-independent quantity (see, e.g., Ref. [36] and the references therein). Initially, the result for $\delta\zeta_{\alpha_s\alpha}^{(2)}$ has been obtained in terms of running parameters in the $\overline{\text{MS}}$-scheme (with the account of tadpole diagrams as in Ref. [37]) and latter recalculated with the on-shell counter-term for the top-quark mass. As expected, the tadpole contribution to the considered quantity was canceled by the counter-term allowing one from the very beginning to ignore the tadpole issue. The price to pay for this kind of simplifications is gauge-dependence of the top-quark mass counter-term. Having this in mind, in what follows we present the expression for $\delta\zeta_{\alpha_s\alpha}^{(2)}$ in terms of "physical" masses referring to the well-known one-loop relation between the pole and running quark masses [14,38,39] in the Standard Model.[4]

For the calculation of $\delta\zeta_{\alpha_s\alpha}^{(2)}$ we have used the fact that the renormalized decoupling constant can be deduced from the relation between the bare couplings[5] [8], denoted by α_{s0}' and α_{s0}, after proper renormalization, i.e.,

[3] One can also consider observables for matching.

[4] For consistency one should neglect all masses but m_t^2, m_W^2, m_Z^2 and m_h^2 when expressing m_t in terms of $\hat{m}_t$.

[5] Considered in dimensionally regularized theory with space–time dimension $d = 4 - 2\varepsilon$.