Physics Letters B 741 (2015) 138–144

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Lifshitz effects on holographic p-wave superfluid

Ya-Bo Wu *, Jun-Wang Lu, Cheng-Yuan Zhang, Nan Zhang, Xue Zhang, Zhuo-Qun Yang, Si-Yu Wu

Department of Physics, Liaoning Normal University, Dalian, 116029, China

ARTICLE INFO

Article history:
Received 28 October 2014
Received in revised form 8 December 2014
Accepted 9 December 2014
Available online 13 December 2014
Editor: J. Hisano

Keywords:
Gauge/gravity duality
Holographic superconductor
Lifshitz black hole
Maxwell-complex vector field

ABSTRACT

In the probe limit, we numerically build a holographic p-wave superfluid model in the four-dimensional Lifshitz black hole coupled to a Maxwell-complex vector field. We observe the rich phase structure and find that the Lifshitz dynamical exponent z contributes evidently to the effective mass of the matter field and dimension of the gravitational background. Concretely, we obtain that the Cave of Winds appeared only in the five-dimensional anti-de Sitter (AdS) spacetime, and the increasing z hinders not only the condensate but also the appearance of the first-order phase transition. Furthermore, our results agree with the Ginzburg–Landau results near the critical temperature. In addition, the previous AdS superfluid model is generalized to the Lifshitz spacetime.

1. Introduction

The gauge/gravity duality [1] builds a powerful relationship between the many-body system in the quantum mechanics with the strong interaction and the classical dynamical black hole with a higher dimension of spacetime, hence, in the past decades, it has been widely used to study various strongly coupled systems. In particular, via this holographic duality, various high-temperature superconductors[1] were constructed, which involve different gravitational backgrounds as well as matter fields, see, for example, Refs. [2–15] and the references therein.

All the above superconductor models are almost based on the isotropic gravitational backgrounds. Due to the various anisotropies of superconductors in the condensed matter system, the authors of Ref. [16] proposed a $(d+2)$-dimensional gravity dual to the Lifshitz anisotropic scaling of the space and time, $ds^2 = L^2(-r^{2z}dt^2 + r^2 d\vec{x}^2 + \frac{dr^2}{r^2})$, where $d\vec{x}^2 = dx_1^2 + \cdots + dx_d^2$, $r \in (0, \infty)$, z is the dynamical critical exponent, and L is a cosmological constant. In particular, the Lifshitz fixed points scale the space and time as $t \to b^z t, \vec{x} \to b\vec{x}$ $(z \neq 1)$. For related works, see also, for instance, Refs. [17–19]. In addition, the gravity duality with the anisotropy between two spatial directions exists, see, for example, Refs. [20,21]. In the remainder of this paper, we will set for simplicity $L = 1$. Subsequently, the Lifshitz spacetime was extended to a $(d+2)$-dimensional finite-temperature system [22]

$$ds^2 = -r^{2z} f(r)dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 \sum_{i=1}^{d} dx_i^2,$$

$$f(r) = 1 - \frac{r_0^{z+d}}{r^{z+d}}, \qquad (1)$$

where r_0 denotes the location of the event horizon. Moreover, the Hawking temperature can be written as $T = \frac{(z+d)r_0^z}{4\pi}$. To see the anisotropic effect, some holographic superconductors were constructed in the Lifshitz black hole backgrounds, see, for example, Refs. [23–33], where the results showed that the larger Lifshitz parameter z hinders the condensate. More than that, the Lifshitz parameter z contributes to the effective dimension of the gravitational background.

In order to generalize the above superconductor models to the ones with a steady current, holographic superfluid solutions were constructed by performing a deformation to the superconducting black hole [34,35], and were further investigated in Refs. [36–42]. It follows that below the critical temperature T_0 with the vanishing superfluid velocity, there is a special value of T, beyond (below) which the phase transition is of second (first) order. We call the critical superfluid velocity corresponding to this special

* Corresponding author.
E-mail address: ybwu61@163.com (Y.-B. Wu).

[1] According to the gauge/gravity duality, there is no dynamical gauge field in the dual field theory [5]. Therefore, the current induced by the applied magnetic field cannot produce an equal and opposite canceling field in the superconductor to exclude the external magnetic field, which is different from the ordinary superconductor but rather similar to thin superconducting films or wires.

http://dx.doi.org/10.1016/j.physletb.2014.12.026

temperature as the translating superfluid velocity. Moreover, in the five-dimensional (5D) anti-de Sitter (AdS) black hole background, the authors of Ref. [40] found that when the temperature decreases, the second-order transition occurs before the first-order transition to a new superconducting phase.

Recently, motivated by Ref. [2], a new holographic p-wave superconductor model was built by coupling a Maxwell-complex vector (MCV) field with the four-dimensional (4D) Schwarzschild AdS black hole [43], for related works, see Refs. [44–50]. It was shown that only the external magnetic field can induce the condensate, which is similar to result of the QCD vacuum phase transition in Ref. [51] compared with ones in Ref. [52]. In addition, even in the Lifshitz spacetime, this MCV model is still a generalization of the usual p-wave model realized by the SU(2) Yang–Mills (YM) gauge field [3]. Because of the anisotropic properties of the superfluid model in the real world, for example, the He_3 superfluid, it is valuable to construct the holographic p-wave superfluid model by coupling the MCV model in the 4D Lifshitz black hole. More interesting questions are whether we can see (i) the Cave of Winds only existed in the 5D AdS black hole when considering the effects of Lifshitz parameter z on the dimension of the gravitational background; (ii) the disappearance of the first-order phase transition due to the fact that the larger parameter z hinders the condensate. Answering these questions is just the purpose of this paper.

Based on the above mentioned, we will build a holographic superfluid model in the 4D Lifshitz black hole coupled with the MCV field in the probe limit. Interestingly, we obtain the rich structure, especially the Cave of Winds, which means that the Lifshitz parameter z contributes evidently to the effective mass of the matter field and the dimension of the background spacetime. Moreover, the larger z not only decreases the critical temperature, but also hinders the emergence of the first-order phase transition.

The paper is organized as follows. In Section 2, we obtain the equations of motion and the grand potential for the superfluid model. We numerically study the condensate and the supercurrent in Sections 3 and 4, respectively. The last section is devoted to the conclusions and further discussions.

2. Equations of motion and the grand potential

In this section, we derive the equations of motion in terms of the MCV field, following which we obtain the grand potential.

The MCV matter action including a Maxwell field and a complex vector field reads [47]

$$\mathcal{S}_m = \frac{1}{16\pi G_4}\int dx^4\sqrt{-g}\Big(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\rho^{\dagger}_{\mu\nu}\rho^{\mu\nu} - m^2\rho^{\dagger}_{\mu}\rho^{\mu} + iq\gamma\rho_{\mu}\rho^{\dagger}_{\nu}F^{\mu\nu}\Big), \tag{2}$$

where $F_{\mu\nu}$ is the strength of the Maxwell field A_μ and $\rho_{\mu\nu} = D_\mu\rho_\nu - D_\nu\rho_\mu$ with the covariant derivative $D_\mu = \nabla_\mu - iqA_\mu$, while m and q are the mass and the charge of the vector field ρ_μ, respectively. The last term with a coefficient γ stands for the interaction between ρ_μ and A_μ, which is crucial to the effect of the magnetic field in the holographic model [43–46]. However, in this paper we do not consider the magnetic field, hence, it will not contribute to our work. Moreover, we will work in the probe limit that can be realized by taking $q \to \infty$ with $q\rho_\mu$ and qA_μ fixed.

By varying the action (2), we obtain the equations of motion

$$D^\nu\rho_{\nu\mu} - m^2\rho_\mu + iq\gamma\rho^\nu F_{\nu\mu} = 0, \tag{3}$$

$$\nabla^\nu F_{\nu\mu} - iq\big(\rho^\nu\rho^{\dagger}_{\nu\mu} - \rho^{\nu\dagger}\rho_{\nu\mu}\big) + iq\gamma\nabla^\nu\big(\rho_\nu\rho^{\dagger}_{\mu} - \rho^{\dagger}_{\nu}\rho_\mu\big) = 0. \tag{4}$$

As Ref. [50], we turn on the following ansatzs for ρ_μ and A_μ

$$\rho_\nu dx^\nu = \rho_x(r)dx, \qquad A_\nu dx^\nu = \phi(r)dt + A_y(r)dy. \tag{5}$$

Thus the concrete equations of motion in terms of the matter field are given by

$$\rho_x'' + \left(\frac{z+1}{r} + \frac{f'}{f}\right)\rho_x' - \frac{\rho_x}{r^2 f}\left(m^2 - \frac{\phi^2}{r^{2z}f} + \frac{A_y^2}{r^2}\right) = 0, \tag{6}$$

$$\phi'' + \frac{3-z}{r}\phi' - \frac{2\rho_x^2}{r^4 f}\phi = 0, \tag{7}$$

$$A_y'' + \left(\frac{z+1}{r} + \frac{f'}{f}\right)A_y' - \frac{2\rho_x^2}{r^4 f}A_y = 0. \tag{8}$$

When we turn off the spatial component $A_y(r)$, Eqs. (6) and (7) reduce to the ones in Ref. [45], while Eqs. (6), (7) and (8) with $z = 1$ are the same with the ones in Ref. [50].

Due to the difficulty to solve the above equations analytically, here we turn to the numerical approach, i.e., the shooting method [34,35,37–40]. Before the numerical calculation, we should impose some boundary conditions on Eqs. (6), (7) and (8). In particular, at the horizon, $\rho_x(r_0)$ and $A_y(r_0)$ are required to be regular, while $A_t(r_0)$ vanishing in order for the normal form of $g^{\mu\nu}A_\mu A_\nu$. At the infinity boundary $r \to \infty$, the general falloffs of the fields are of the forms

$$\rho_x(r) = \frac{\rho_{x-}}{r^{\Delta_-}} + \frac{\rho_{x+}}{r^{\Delta_+}} + \cdots, \qquad \phi(r) = \mu - \frac{\rho}{r^{2-z}} + \cdots,$$
$$A_y(r) = S_y - \frac{J_y}{r^z} + \cdots \tag{9}$$

with $\Delta_\pm = \frac{1}{2}(z \pm \sqrt{z^2 + 4m^2})$. According to the gauge/gravity duality, ρ_{x-} and ρ_{x+} are usually interpreted as the source and the vacuum-expectation value of the boundary operator O_x, respectively, while μ, ρ, S_y, and J_y as the chemical potential, the charge density, the superfluid velocity, and the supercurrent, respectively. To satisfy the requirement that the symmetry is broken spontaneously, we impose the source-free condition, i.e., $\rho_{x-} = 0$.

There is a scaling symmetry for the asymptotical solutions (9) as $(r, S_y) \to \lambda(r, S_y)$, $(T, \mu) \to \lambda^z(T, \mu)$, $\rho_{x+} \to \lambda^{\Delta_+ + 1}\rho_{x+}$, $J_y \to \lambda^{z+1}J_y$ and $\rho \to \lambda^2\rho$ with λ a positive real constant, by using which we can fix the chemical potential and thus work in the grand canonical ensemble. As we know from Refs. [34,35,40], when the critical superfluid velocity increases beyond a translating value, the second-order phase transition will switch to the first-order one in the grand canonical ensemble. To determine which phase is more thermodynamically favored in this case, we should calculate the grand potential Ω of the bound state, which is identified with the Hawking temperature times the Euclidean on-shell action. From the action (2), the on-shell action $\mathcal{S}_{os}$ reads

$$\begin{aligned}\mathcal{S}_{os} &= \int dxdydtdr\sqrt{-g}\Big(-\frac{1}{2}\nabla_\mu\big(A_\nu F^{\mu\nu}\big) - \nabla_\mu\big(\rho^{\dagger}_{\nu}\rho^{\mu\nu}\big) + \frac{1}{2}A_\nu\nabla_\mu F^{\mu\nu}\Big)\\ &= \frac{V_2}{T}\Big(-\sqrt{-\gamma}n_r\rho^{\dagger}_{\nu}\rho^{r\nu}\big|_{r\to\infty} - \frac{1}{2}\sqrt{-\gamma}n_r A_\nu F^{r\nu}\big|_{r\to\infty} + \frac{1}{2}\int_{r_0}^{\infty}dr\sqrt{-g}A_\nu\nabla_\mu F^{\mu\nu}\Big)\end{aligned}$$